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AN ANALYSIS OF FREE UNDAMPED TRANSVERSAL VIBRATION OF A CANTILEVER BEAM WITH A C-SECTION VARYING ACROSS LENGTH

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Abstract

Vibration of a beam with a varying C-section across the length is investigated. Analytical solutions of undamped free vibration of the cantilever beam are obtained using MATLAB and validated using the commercial finite element software ANSYS. Weak form of the governing equation is used; considering the beam to be Euler-Bernoulli beam. A Ritz-Galerkin approximation is resorted to for the evaluation of resulting stiffness, mass matrix and transversal natural frequencies.

KeyWords: *Cantilever beam, Ritz-Galerkin approximation, Varying C-section, Euler-Bernoulli beam, Transversal Vibration, MATLAB, ANSYS, Natural frequencies.*

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1. INTRODUCTION

Beams with uniformly varying cross sections are widely used in engineering applications to save weight and/or to satisfy the engineering requirements. A varying C-cross section across the length; not so widely discussed in the literature is preferred for analysis.

Galerkin finite element method used [1] for studying non-linear vibrations of beams considered the transverse displacement term together with certain assumptions regarding the nature of the vibration.

An approximate solution presented in [2] used a Galerkin's procedure for a beam subjected to axial loads. Results provide a comparison of increasing rate of fundamental frequency with amplitude for beams with different end conditions.

Response of beams and plates undergoing large amplitude free oscillations wasinvestigated byreducing the governing partial differential equations and use of one term approximation and elliptic functions [4-5];while a Ritz-Galerkin procedure was employed to present equally good results [6].

A Euler -Bernoulli beam of varying C-section along the length is analyzed here using Ritz-Galerkin approximation to derive analytical solutions of undamped free transversal vibrations. The results are the validated with a finite element model using the commercial software ANSYS.

2. ANALYTICAL MODEL

The cantilever beam considered has

Length= 0.6 m
Material Properties: Young's Modulus:210 GPa
Density :7800 kg/m^3

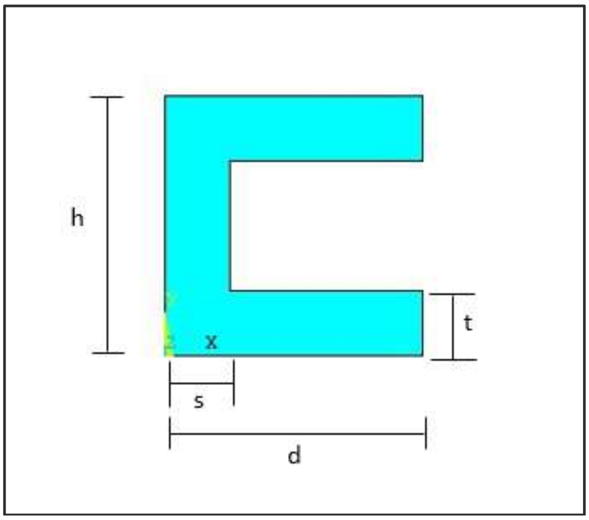

Fig -1: Dimensions of the C-section

Dimensions at Fixed end:

h=0.02 m, s=0.005 m, d=0.02 m, t=0.005 m

Dimensions at Free end

h=0.01 m, s=0.0025 m, d=0.01 m, t=0.0025 m

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The principle of virtual work for the dynamic problem presented is

$$\int_0^l \delta w^T{}_{/xx}\, EJ_y(x) w_{/xx}\, dx = -\int_0^l \delta w^T m(x)\ddot{w}\ dx$$

An approximate solution for the transverse displacement is

$$w(x,t) = \sum_{i=1}^{N} N_i(x) u_i(t)$$

where

$$N_i(x) = \frac{1}{i(i+1)(i+2)}\left(\frac{x}{l}\right)^{i+2} + \frac{1}{(i+1)(i+2)(i+3)}\left(\frac{x}{l}\right)^{i+3} + \frac{1}{2i(i+1)}\left(\frac{x}{l}\right)^{i+2}$$

is an approximate comparison function which satisfiesthe natural boundary conditions.

$$w(0) = 0;\ w_{/x}(0) = 0;\ \ w_{/xx}(l) = 0;\ w_{/xxx}(l) = 0$$

The element of the Mass Matrix is

$$\begin{aligned}\frac{M(i,j)}{rho * 10^{-4}} = 2.5\, l^1 \Bigg[& \frac{1}{ij(i+1)(i+2)(j+1)(j+2)(i+j+5)} - \frac{2ij+3i+3j}{ij(i+1)(i+2)(i+3)(j+1)(j+2)(j+3)(i+j+6)} \\ & - \frac{1}{2ij(i+1)(i+2)(j+1)(i+5)} + \frac{1}{(i+1)(i+2)(i+3)(j+1)(j+2)(j+3)(i+j+7)} \\ & + \frac{1}{2j(i+1)(i+2)(i+3)(j+1)(i+6)} - \frac{1}{2ij(i+1)(j+1)(j+2)(j+5)} \\ & + \frac{1}{2i(i+1)(j+1)(j+2)(j+3)(j+6)} + \frac{1}{20\, ij(i+1)(j+1)}\Bigg] \\ -3.3352\, l^2 \Bigg[& \frac{1}{ij(i+1)(i+2)(j+1)(j+2)(i+j+6)} - \frac{2ij+3i+3j}{ij(i+1)(i+2)(i+3)(j+1)(j+2)(j+3)(i+j+7)} \\ & - \frac{1}{2ij(i+1)(i+2)(j+1)(i+6)} + \frac{1}{(i+1)(i+2)(i+3)(j+1)(j+2)(j+3)(i+j+8)} \\ & + \frac{1}{2j(i+1)(i+2)(i+3)(j+1)(i+7)} - \frac{1}{2ij(i+1)(j+1)(j+2)(j+6)} \\ & + \frac{1}{2i(i+1)(j+1)(j+2)(j+3)(j+7)} + \frac{1}{24ij(i+1)(j+1)}\Bigg] \\ +1.0430\, l^3 \Bigg[& \frac{1}{ij(i+1)(i+2)(j+1)(j+2)(i+j+7)} - \frac{2ij+3i+3j}{ij(i+1)(i+2)(i+3)(j+1)(j+2)(j+3)(i+j+8)} \\ & - \frac{1}{2ij(i+1)(i+2)(j+1)(i+7)} + \frac{1}{(i+1)(i+2)(i+3)(j+1)(j+2)(j+3)(i+j+9)} \\ & + \frac{1}{2j(i+1)(i+2)(i+3)(j+1)(i+8)} - \frac{1}{2ij(i+1)(j+1)(j+2)(j+7)} \\ & + \frac{1}{2i(i+1)(j+1)(j+2)(j+3)(j+8)} + \frac{1}{28ij(i+1)(j+1)}\Bigg]\end{aligned}$$

The element of the stiffness Matrix is

$$\begin{aligned}\frac{K(i,j)}{E * 10^{-8}} = \frac{1.208}{l^3}\Bigg[& \frac{1}{ij(i+j+1)} - \frac{i+j+2ij}{ij(i+1)(j+1)(i+j+2)} - \frac{1}{ij(j+1)(i+1)} + \frac{1}{(i+1)(j+1)(i+j+3)} \\ & + \frac{1}{j(i+1)(j+1)(i+2)} - \frac{1}{ij(i+1)(j+1)} + \frac{1}{i(i+1)(j+1)(j+2)} + \frac{1}{ij(i+1)(j+1)}\Bigg]\end{aligned}$$

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$$- \frac{3.5428}{l^2}\left[\frac{1}{ij(i+j+2)} - \frac{i+j+2ij}{ij(i+1)(j+1)(i+j+3)} - \frac{1}{ij(j+1)(i+2)} + \frac{1}{(i+1)(j+1)(i+j+4)} + \frac{1}{j(i+1)(j+1)(i+3)} - \frac{1}{ij(i+1)(j+2)} + \frac{1}{i(i+1)(j+1)(j+3)} + \frac{1}{2ij(i+1)(j+1)}\right]$$

$$+ \frac{3.8219}{l^1}\left[\frac{1}{ij(i+j+3)} - \frac{i+j+2ij}{ij(i+1)(j+1)(i+j+4)} - \frac{1}{ij(j+1)(i+3)} + \frac{1}{(i+1)(j+1)(i+j+5)} + \frac{1}{j(i+1)(j+1)(i+4)} - \frac{1}{ij(i+1)(j+3)} + \frac{1}{i(i+1)(j+1)(j+4)} + \frac{1}{3ij(i+1)(j+1)}\right]$$

$$- \frac{1.7862}{1}\left[\frac{1}{ij(i+j+4)} - \frac{i+j+2ij}{ij(i+1)(j+1)(i+j+5)} - \frac{1}{ij(j+1)(i+4)} + \frac{1}{(i+1)(j+1)(i+j+6)} + \frac{1}{j(i+1)(j+1)(i+5)} - \frac{1}{ij(i+1)(j+4)} + \frac{1}{i(i+1)(j+1)(j+5)} + \frac{1}{4ij(i+1)(j+1)}\right]$$

$$+ \frac{0.30185}{l^{-1}}\left[\frac{1}{ij(i+j+5)} - \frac{i+j+2ij}{ij(i+1)(j+1)(i+j+6)} - \frac{1}{ij(j+1)(i+5)} + \frac{1}{(i+1)(j+1)(i+j+7)} + \frac{1}{j(i+1)(j+1)(i+6)} - \frac{1}{ij(i+1)(j+5)} + \frac{1}{i(i+1)(j+1)(j+6)} + \frac{1}{5ij(i+1)(j+1)}\right]$$

The convergence of the solution and the natural frequencies for the first 3 transversal modes are:

Table -1: Transversal Natural frequencies and convergence using MATLAB

Number of terms	Mode 1	Mode 2	Mode 3
1	72.3	-	-
2	68.1	365.2	-
3	67.8	310.8	1022.7
4	67.8	305.6	797.5
5	67.8	305.3	774.3
6	67.8	305.3	773.8
7	67.8	305.3	773.8

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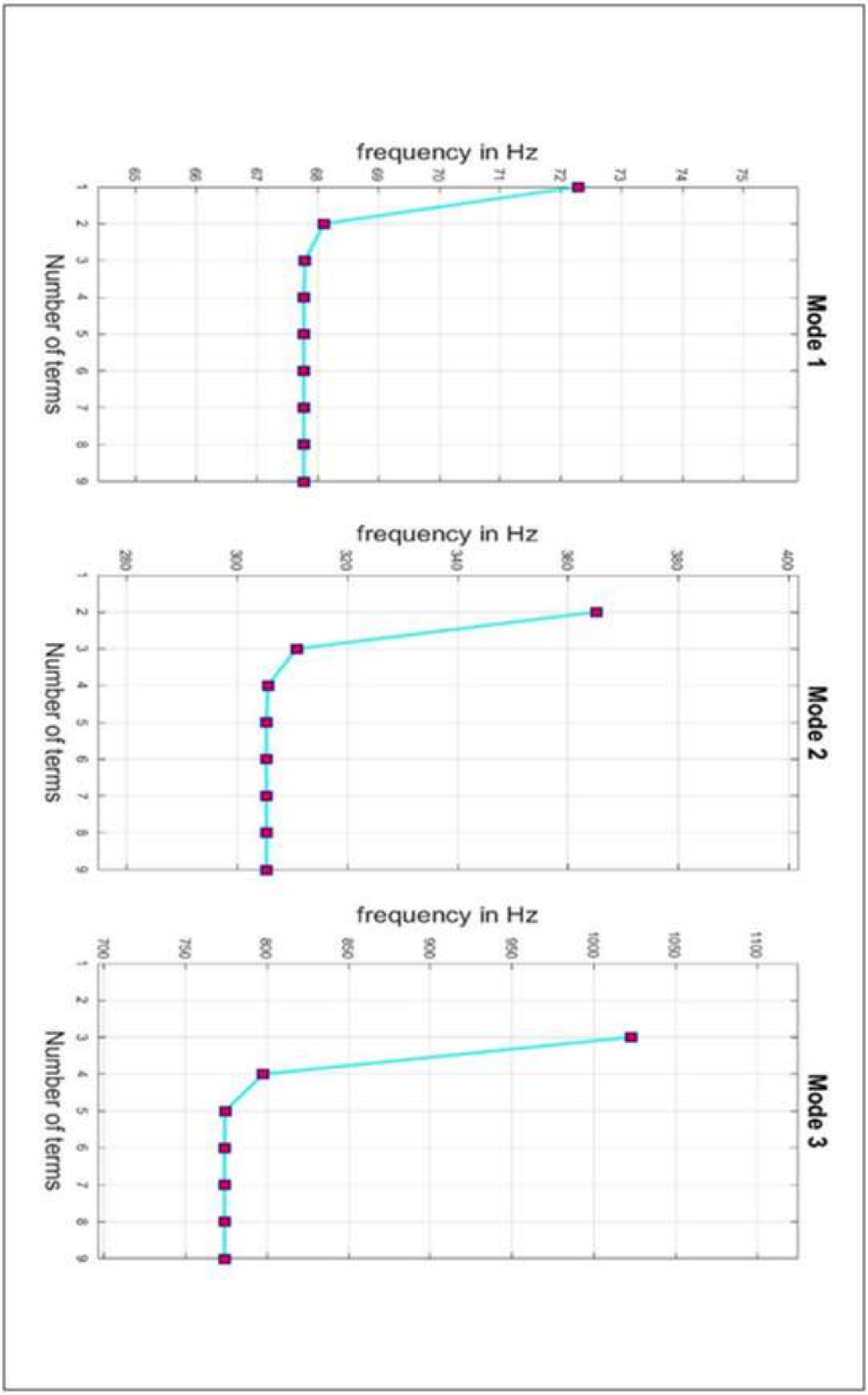

Fig -2: Convergence of solution

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3. FINITE ELEMENT ANALYSIS

The results of the finite element model built using solid45 elements are plotted below.

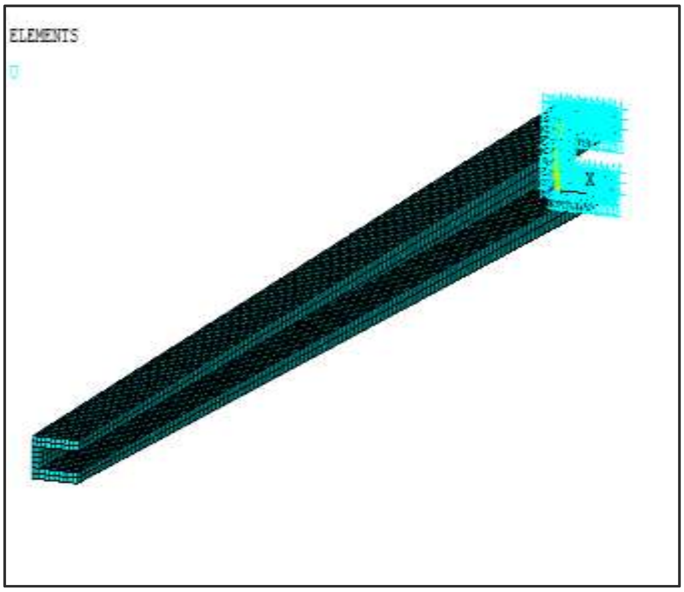

Fig -3: FEM of cantilever beam with varying C-section

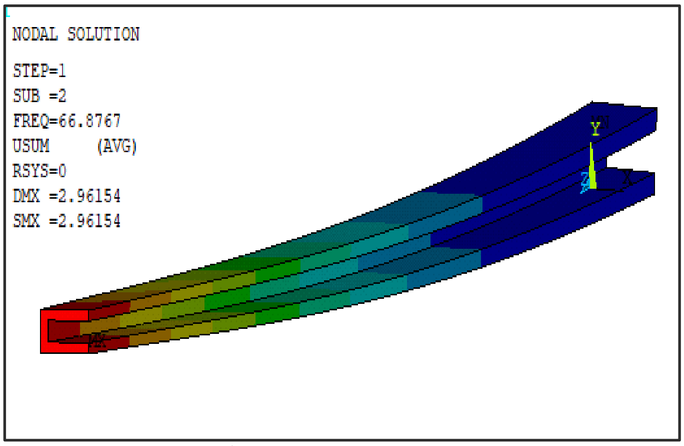

Fig -4:1st Transversal bending mode

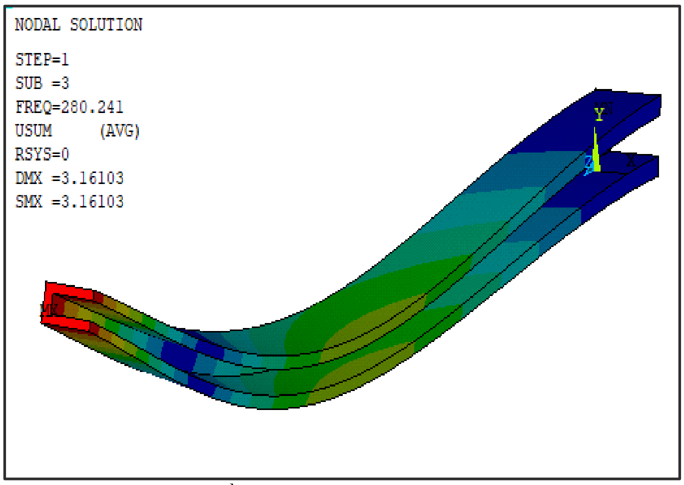

Fig -5:2nd Transversal bending mode

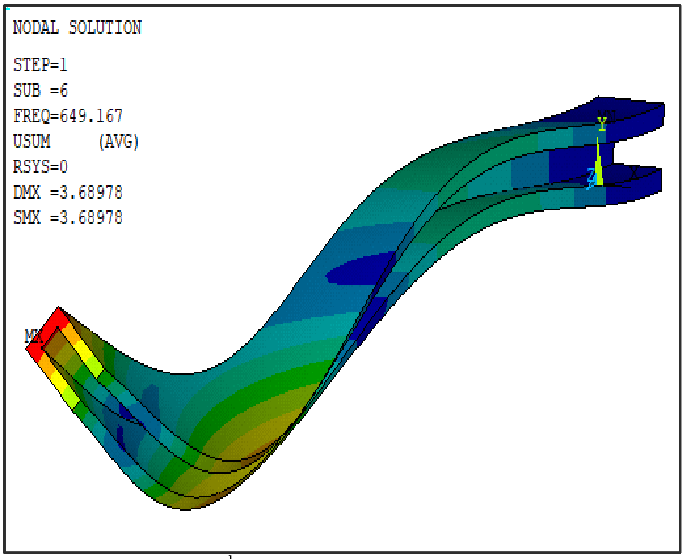

Fig -6:3rd Transversal bending mode

4. CONCLUSION

Table -2: Comparison of results

	Mode 1	Mode 2	Mode 3
Ritz-Galerkin	67.8	305.3	773.8
FEM	66.8	280.2	649.1
Error %	1.47	8.22	16.11

It can be concluded that the Ritz Galerkin approach using a Euler Bernoulli beam is accurate in predicting the first transversal bending mode while there is considerable error in the 2nd and 3rd mode as it neglects any complicated effects that arise from inhomogeneous mass distribution, variable cross section, non-uniform bending stiffness.

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