1. INTRODUCTION

India has experienced some of the severe earthquake events in the last few decades that sought attention of the engineers and requirement of awareness and efforts to reduce damage.[1] The Indian subcontinent is prone to earthquakes due its tectonic setup. The seismic hazard is severe along the belts of Himalaya and Western region of India.[2] Some major earthquakes have occurred in this region is past decades and reason is associated with the collision and friction between Indian and Eurasian tectonic plates. Information on ground shaking magnitude and its occurring frequency is beneficial for proper assessment of the vulnerable region.[3] A suitable database of occurring ground shaking events collected at a platform for engineers to study is very much required. The strong motion instrumentation network of India provides such details required.[4] The ground motion data for the various earthquake events are also readily available by COSMOS[5]

Modal analysis[6][7] by transformation of coordinates is performed for the MDOF system to investigate the vibration shape vectors and modal time periods. The Newmark’s Linear Acceleration Method has shown the response of structure subjected to base ground excitation.
The dynamic equilibrium condition at time instance $t$, governing the linear response of MDOF system with $[M]$, $[C]$, $[K]$ as the mass, damping and stiffness matrices, respectively, may be expressed in the form of coupled differential equation in matrix notation.

$$[M][\ddot{u}(t)] + [C][\dot{u}(t)] + [K][u(t)] = -[M][\Gamma][\phi](t) \quad (1)$$

where $[\Gamma]$ represents the influence coefficient vector showing the motion of rigid body corresponding to unit displacement of ground motion. The coordinates of the equation (1) can be transformed in terms of modal matrix $[\Phi]$ and can be shown as,

$$[u(t)] = [\Phi][q(t)] \quad (2)$$

The uncoupled differential equation using modal coordinates is now expressed as

$$[M][\Phi][\ddot{\phi}(t)] + [C][\Phi][\dot{\phi}(t)] + [K][\Phi][\phi(t)] = -[M][\Gamma][\phi](t) \quad (3)$$

Normalizing the uncoupled equation (3) with respect mass matrix $[M]$, and expressing in terms of damping ratio $\xi$ and circular frequency $\omega$, the equation can now be written as

$$\ddot{\phi}_i(t) + 2\xi\omega\dot{\phi}_i(t) + \omega^2\phi_i(t) = -\Gamma_i\ddot{u}_i(t) \quad (4)$$

where $\Gamma_i$ is the modal participation factor and $i$ being the mode number and $j$ being the floor number.

$$\Gamma_i = \frac{[\Phi]_i^T[M][\Gamma][\phi]}{[\Phi]_i^T[M][\Phi]_i} \quad (5)$$

2.2 Mode Shapes and Time Periods

To determine the vibration frequencies of the MDOF system, free vibration is considered with undamped condition. This leads to characteristic equation as follows:

$$([K] - \omega^2[M])[\Phi] = 0 \quad (6)$$

Thus transformation of coordinate makes the analysis of shear building as an eigen value problem having mode shape vector $\phi_0$. To obtain a non-trivial solution of equation (6), the determinant of the matrix $[K - \omega^2M]$ is equaled to 0.

$$\begin{vmatrix} [K] - \omega^2[M] \end{vmatrix} = 0 \quad (7)$$

The formulation of mass and stiffness matrices are as follows:

$$M = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 \end{bmatrix}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 & 0 & 0 \\ 0 & 0 & -k_4 & k_4 + k_5 & -k_5 & 0 \\ 0 & 0 & 0 & -k_5 & k_5 + k_6 & -k_6 \\ 0 & 0 & 0 & 0 & -k_6 & k_6 \end{bmatrix}$$

The eigen values $\omega_i^2$ and corresponding vibration mode shapes $\phi_i$ are determined as shown in the fig. 2 and fig.3, respectively.

This is done by normalizing the modes such that the element corresponding to the 6th story is unity. This makes the following equation (8) true with modes being mass orthonormal.

$$[\Phi]_i^T[K][\Phi]_i = \omega_i^2 \quad (8)$$

The modal time period $T_i$ of each mode are also obtained.
Considering initial values of $h_0 = 0.00$, the step by step procedure of $t_0$ calculated using this base. The algorithm parameters $\gamma$ and $\beta$ are selected as $1/2$ and $1/6$ respectively and additional system properties considered are $k$ and $\Delta \dot{q}$. Considering initial values of relative velocity and relative displacement as zero, the time stepping operation is performed using Newmark’s Method to obtain the relative acceleration $\ddot{q}$, relative velocity $\dot{q}$ and relative displacement $q$. The step by step procedure of Newmark’s Method that was implemented on a computer program is listed hereafter in equation (13) to (25).

### Initial Calculations:
\[\ddot{q}_0 = \frac{p_0 - c_0 \dot{q}_0 - k_0 q_0}{m_i} \tag{13}\]
\[k_i = k_i + \frac{\gamma}{\beta \Delta t} c_i + \frac{1}{\beta (\Delta t)^2} m_i \tag{14}\]
\[a_i = \frac{1}{\beta \Delta t} m_i + \frac{\gamma}{\beta} c_i \tag{15}\]
\[b_i = \frac{1}{2 \beta} m_i + \Delta t \left( \frac{\gamma}{2 \beta} - 1 \right) c_i \tag{16}\]

### Calculation for each time step, $n$:
\[p_{i+1} = -\Gamma \dot{\mu}_i \tag{17}\]
\[\Delta p_{i+1} = p_{i+1} - p_i \tag{18}\]
\[\Delta \dot{p}_i = \Delta p_{i+1} - a_i \dot{q}_i + b_i \dot{q}_i \tag{19}\]
\[\Delta q_i = \frac{\Delta \dot{p}_i}{k_i} \tag{20}\]
\[\Delta q_i = \frac{\gamma}{\beta \Delta t} \Delta q_{i-1} - \frac{\gamma}{\beta} \dot{q}_{i-1} + \Delta t \left( 1 - \frac{\gamma}{2 \beta} \right) \dot{q}_i \tag{21}\]
\[
\Delta q_{i(n+1)} = q_{i(n)} + \Delta q_{i(n)}
\]

(23)

\[
\Delta \dot{q}_{i(n+1)} = \dot{q}_{i(n)} + \Delta \dot{q}_{i(n)}
\]

(24)

\[
\Delta \ddot{q}_{i(n+1)} = \ddot{q}_{i(n)} + \Delta \ddot{q}_{i(n)}
\]

(25)

The result of entire time stepping operation further provides the peak deformation \( D_i \) to identify the peak pseudo acceleration \( A_i \) for each mode. The corresponding values of equivalent lateral forces \( F_{ij} \) for each story are obtained using respective values of lateral force vector \( s_{ij} \).

The story shear forces obtained from \( F_{ij} \) for all modes are combined using the SRSS method[8]. Maximum response of each story in terms of displacement is recorded along with the time instance of peak response.

3. BUILDING MODEL IN ETABS

ETABS is the software tool used in this study for analysis of six story building system. The direction of analysis is restricted to X-Direction only so that the system represents a shear building with six translational degrees of freedom in X-Direction only.

The six story building model prepared in ETABS is shown in fig. 5. The material and dimensions of the model are similar to the sizes considered for theoretical analysis. The time history loading is applied to model as shown in fig. 6.

The ETABS model of six story shear building is analyzed by Equivalent Static Method[11] and Linear Modal Time History Method[12] to obtain base shear values and displacement response values of each floor.

4. RESULTS AND DISCUSSIONS

The MDOF system analyzed by theoretical method is compared with ETABS model analysis thus discussed in this section of the study.
The time periods of each mode are obtained with the help of eigen values identified by solving characteristic equation. These time periods are compared with modal time periods obtained from analyzing the same geometry building in ETABS software. The comparison is shown in fig.7 that indicates good conformity in the modal time periods with at variation of 1 to 2%.

Fig-9: Story Shear Force by Time History Analysis

Fig-10: Max Top Story Displacement by Equivalent Static Method

Fig-11: Max Top Story Displacement by Time History Analysis

Fig-12: Top Story Displacement Time History by Theoretical Analysis

Fig-13: Top Story Displacement Time History by ETABS

Fig-14: Top Story Displacement Time History Comparison
The mode shapes obtained from ETABS are compared with theoretical mode shapes by normalizing at the top floor for each mode. It is observed that the mode shapes obtained from ETABS are exactly matching with the theoretically calculated mode shapes.

The building system is analyzed using Equivalent Static Method theoretically and using ETABS software and the story shear force results are observed to be same in both the cases. The corresponding graph is shown in fig. 8.

Story Shear forces obtained by linear modal time history analysis of the six story shear building performed using Newmark’s Linear Acceleration Method is compared with ETABS results as shown in fig. 9. The story shear forces due to applied ground excitation by theoretical and ETABS time history analysis vary by 2%.

The response of building in terms of story displacement is also compared in fig. 10 by Equivalent Static Method. There is approximately 3% of variation in the displacement of floors.

Maximum story displacement of the six story shear building subjected to ground excitation is shown in fig. 11. The variation in the results of the floor displacement response is 1 to 3%. The maximum top floor displacement time history is obtained from theoretical analysis as shown in fig. 12 and using ETABS as indicated in fig. 13. This top floor displacement response is compared for theoretical and ETABS analysis as shown in fig. 14 which indicates that the peak top story displacement response is occurring at time instance of 45.05sec in theoretical and at 45.055sec in ETABS analysis.

5. CONCLUSION
- The modal time periods and vibrations mode shapes shown by ETABS software are almost same as obtained by theoretical calculations.
- The results obtained by ETABS software are acceptably matching with the results obtained theoretical analysis using Newmark’s Method for Equivalent Static Method of analysis.
- Displacement of the floors identified using ETABS and theoretical approach are in good agreement with each other for Equivalent Static Method.
- The time history analysis performed using the algorithm based on theoretical time stepping method called Newmark’s Linear Acceleration Method is compared with the linear modal time history analysis performed using ETABS software. The obtained base shear values from both theoretical and ETABS analyses are in good conformity with each other.
- Response of building in terms of displacement obtained using ETABS software is in good agreement with the displacement response by theoretical analysis when the Newmark’s Method fulfills stability and convergence condition.

ACKNOWLEDGEMENT
Authors are grateful to the Civil Engineering Department of Dr. Vithalrao Vikhe Patil College of Engineering Ahmednagar for motivation and supporting with laboratory facility in this study.

REFERENCES