NUMERICAL SIMULATION OF HEAT TRANSFER CHARACTERISTICS IN THIN FILM FLOW OF MHD DISSIPATIVE CARREAU NANOFLUID PAST A STRETCHING SHEET WITH $CoFe_2O_4$ NANOPARTICLES

G.P. Ashwinkumar[1], C. Sulochana[2]

[1]*Research Scholar, Department of Mathematics, Gulbarga University, Kalaburagi-585106, India*
[2]*Professor, Department of Mathematics, Gulbarga University, Kalaburagi-585106, India*

Abstract

In present days, the thermal and physical properties of magnetic-nanofluids are more effective by the influence of external magnetic fields and it also regulates the flow and heat transfer characteristics. With this incentive, this article addresses the heat transfer characteristics in thin liquid film flow of magnetic-nanofluid towards a stretching surface in the presence of viscous dissipation and aligned magnetic field with non uniform heat source/sink. For this study, we considered $CoFe_2O_4$ nanoparticles embedded in water. Numerical results are computed by adopting Runge-Kutta based shooting technique. The influence of various pertinent parameters on velocity and temperature profiles along with local Nusselt number and friction factor are thoroughly examined and discussed with the assistance of graphs and tables. It is found that aligned magnetic field regulates the momentum boundary layer and heat transfer rate. It is also observed that increasing the volume fraction of nanoparticles effectively enhances the thermal conductivity of $CoFe_2O_4$-water nanofluid.

Keywords: *film flow, viscous dissipation, non-uniform heat source/sink, radiation, inclined magnetic field.*

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NOMENCLATURE

A^*	space dependent heat source/sink
u, v	velocity components along the x and y- axes [m/s]
B^*	temperature dependent heat source/sink
x, y	Cartesian coordinates [m]
b	positive constant
τ_w	Skin friction from the surface
B_o	magnetic field strength
n	power-law index parameter
c_p	specific heat at constant pressure [J/kg K]
C_f	Skin friction co efficient
υ_f	kinematic viscosity of the fluid [m^2/s]
Ec	Eckert number
$h(t)$	thickness of the film [m]
θ	dimensionless temperature
k	thermal conductivity [W/m K]
σ_{nf}	electrical conductivity of the nanofluid [S/m]
k^*	mean absorption coefficient [m^{-1}]
$(\rho c_p)_f$	heat capacity of the base fluid
M	dimensionless magnetic field parameter
$(\rho c_p)_s$	effective heat capacity of nanoparticle material [J/K]
Nu_x	local Nusselt number
$(\rho c_p)_{nf}$	heat capacitance of the nanofluid
$\Pr$	Prandtl number
k_{nf}	thermal conductivity of the nanofluid
q_w	heat flux from the surface [Js^{-1}m^{-2}]
μ_f	dynamic viscosity of the base fluid
q_r	thermal radiative heat flux
μ_{nf}	dynamic viscosity of the nanofluid
q'''	non uniform heat source/sink
ρ_{nf}	effective density of the nanofluid
R	radiation parameter
Re_x	local Reynolds number
σ^*	Stefan-Boltzmann constant[W/m^2 K^4]
S	unsteadiness parameter
η	similarity variable
T_s	surface temperature [K]
T	fluid temperature [k]
s	condition on the surface

T_0 Free stream temperature [k]

T_{ref} reference temperature [k]

1. INTRODUCTION

The study of momentum and heat transfer characteristics in boundary layer flow past a stretching sheet has attracted the researchers and it plays a vital role due to their enormous applications in the field of science and engineering such as, extrusion of plastics sheets, cooling and drying of papers, electronic cooling, nuclear reactors, metal spinning, wire drawing, hot rolling, glass fiber industries and many more. Initially, Crane [1] has proposed the idea of the two dimensional boundary layer flows over a stretching sheet. Later, Mukhopadhyay et al. [2], Nadeem et at. [3] extended the concept of Crane [1] for various fluids by including variable Brownian motion and thermophoresis effects.

Now a day the problem of thin liquid film flow over a stretching sheet has gained the attention of many investigators due to their enhanced applications in various branches of science and technology. These applications includes in heat exchangers, wire and fiber coating, chemical processing, transpiration cooling etc. Initially the problem of thin liquid film flow over an unsteady stretching sheet was discussed by Wang [4]. Later, this concept was used by Abel et al. [5] to study the flow and heat transfer in a liquid film past a stretching sheet in the presence of viscous dissipation effects. Recently, Sandeep and Malvandi [6] discussed the problem of enhanced heat transfer in thin film flow over stretching sheet with non-uniform heat source/sink effects. Very recently, Pal et al. [7] elaborated the effect of thermal radiation on the thin film flow and heat transfer of a fluid over a stretching sheet with convective boundary conditions.

Nanofluids are the mixture of solid particles in base fluid. Past few decades, the study of flow and heat transfer in nanofluids is an active research among the researchers because of its enhanced properties and tremendous applications in almost every field of science, technology and biomedicine viz. nuclear reactors, solar energy, cancer therapy, machining space technology etc. Initially, Choi [8] reported on enhancing the thermal conductivity of base fluid using the suspension of solid nano-meter sized particles into the base fluids commonly named as nanofluids. Buongiorno [9] developed a model to enhance the thermal conductivity of base fluid and found that Brownian motion and thermophoresis effects are more prominent in nanofluids. Further, a comprehensive study on enhancing the thermal conductivity of a fluid by using micro and nano meter scaled solid particles are found in book by Das et al. [10] and in research articles by Sulochana et al. [11], Vajravelu et al. [12]. Very recently, Sandeep and Sulochana [13] illustrated the flow and heat transfer behavior of dusty nanofluid past stretching sheet in the presence of suction/injection effects.

The study of non-Newtonian fluid flow over a stretching/shrinking sheet has received the attention of several researchers because of its immense applications in science and technology, especially in biomedicine, such as neurological treatment, food and polymer processing, in cancer treatment, blood flows, treatment of diagnostic diseases etc. shear stress and shear rate are indirectly proportional to non-Newtonian fluids. Carreau fluid is a type of Newtonian fluid. At high shear rate the Carreau fluid acts as power-law fluid and at low shear rate it behaves as Newtonian fluid. Carreau fluid model was developed by Carreau in 1972. Akbar et al. [14] numerically analysed the electrically conducting Carreau nanofluid flow in an asymmetric channel. Various studies are available on Carreau fluid flow analysis over different channels, few of them are, Raju and Sandeep [15], Sulochana et al. [16].

Since, the external magnetic fields are very effective to set the thermal and physical properties of magnetic-nanofluids and regulate the flow and heat transfer characteristics. Motivations of the above studies, the present investigation address the flow and heat transfer characteristics of thin liquid film flow of $CoFe_2O_4$–water nanofluid over stretching sheet by considering the Carreau model in the presence of viscous dissipation, thermal radiation and aligned magnetic field. Numerical computations are carried out and results are discussed with the aid of graphs and tables.

2. MATHEMATICAL FORMULATION

Consider unsteady, two dimensional boundary layer flow of an electrically conducting Carreau nanofluid over a stretching sheet bounded by a thin liquid film of uniform thickness $h(t)$ on a horizontal elastic sheet which emerges from a narrow slit at the origin of Cartesian coordinate system which is schematically represented in Fig.1. The sheet is stretched along the x –axis with stretching velocity $U(x,t)$ and y–axis is normal to it. An inclined magnetic field B_0 is applied to the stretching sheet at an angle γ . The effects of non-uniform heat source/sink, thermal radiation, viscous dissipation and volume fraction are taken into consideration. We assume that the surface temperature T_s of the stretching sheet varies with respect to distance x from the slit as

$$T_s = T_0 - T_{ref}\,(bx^2 / 2\upsilon_f)(1-\alpha t)^{-\frac{3}{2}} \quad (1)$$

$$U(x,t) = bx / (1-\alpha t) \quad (2)$$

The Eqn. (2) is for the sheet velocity $U(x,t)$ reflects that the elastic sheet, the elastic sheet is fixed at the origin and stretched by applying a force in the positive x -direction. We used $\alpha > 0$ because the stretching rate $b/(1-\alpha t)$ increases with time. Similarly, Eqn. (2) represents the decrease in sheet temperature from T_0 at the slit in proportion to x^2.

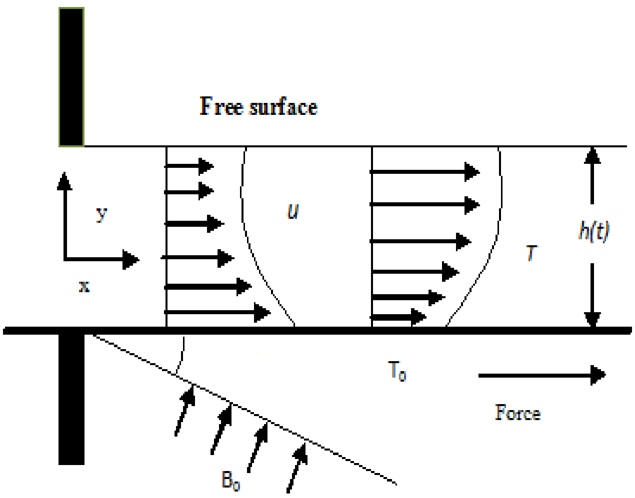

Fig. 1. Physical Configuration of the problem

The constitutive equations for a Carreau fluid is given by

$$\bar{\tau}_{ij} = \eta_0 \left[1 + \frac{(n-1)}{2} (\Gamma \bar{\dot{\gamma}})^2 \right] \bar{\dot{\gamma}}_{ij} \tag{3}$$

in which $\bar{\tau}_{ij}$ is the extra stress tensor, η_0 is the zero shear rate viscosity, Γ is the time constant, n is power-law index and $\bar{\dot{\gamma}}_{ij}$ is defined as

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2} \sum_i \sum_j \bar{\dot{\gamma}}_{ij} \bar{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}} \Pi . \tag{4}$$

Here $\prod$ is the second invariant strain tensor.

The governing boundary layer equations for momentum and thermal energy with associated boundary conditions are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{5}$$

$$\rho_{nf} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu_{nf} \left(1 + \frac{3(n-1)\Gamma^2}{2} \left(\frac{\partial u}{\partial y} \right)^2 \right) \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 cos^2 \gamma u \, , \tag{6}$$

$$(\rho c_p)_{nf} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{nf} \frac{\partial^2 T}{\partial y^2} + \mu_{nf} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{\partial q_r}{\partial y} + q''' \, , \tag{7}$$

$$\left. \begin{array}{l} u = U_w, \quad v = 0, \quad T = T_s \quad \text{at y=0}, \\ \dfrac{\partial u}{\partial y} = 0, \dfrac{\partial T}{\partial y} = 0, \text{ at y=h, } v = \dfrac{dh}{dt} \quad \text{as } y = h(t), \end{array} \right\} \tag{8}$$

Where

$$\begin{array}{l} \rho_{nf} = (1-\phi)\rho_f + \phi \rho_s, \quad \mu_{nf} = \dfrac{\mu_f}{(1-\phi)^{2.5}}, \\ k_{nf} = k_f \left[\dfrac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \right] \\ (\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s \, , \end{array} \tag{9}$$

Using Rosseland approximation of radiation we get

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \, , \tag{10}$$

Now by expanding T^4 is a linear temperature function which is expanded by using Taylor series expansion about T_0 as

$$T^4 = 4T_0^3 T - 3T_0^4 \, , \tag{11}$$

Using (10)-(11) in equation (7) we get

$$(\rho c_p)_{nf} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{nf} \frac{\partial^2 T}{\partial y^2} + \mu_{nf} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{16\sigma^* T_0^3}{3k^*} \frac{\partial^2 T}{\partial y^2} + q''' \, , \tag{12}$$

The non uniform heat generation/absorption q''' has taken as

$$q''' = \frac{k_f U_w}{x \upsilon_f} [A^* (T_s - T_0) f' + B^* (T - T_0)] \, , \tag{13}$$

Let us now introduce the similarity variables as given below

$$\begin{array}{l} u = \dfrac{bx}{(1-\alpha t)} f'(\eta), \eta = y\sqrt{\dfrac{b/\upsilon_f}{1-\alpha t}}, v = -\dfrac{f(\eta)}{\sqrt{b\upsilon_f (1-\alpha t)}}, \\ T = T_0 - T_{ref} (bx^2 / 2\upsilon_f)(1-\alpha t)^{-\frac{3}{2}} \theta(\eta), \end{array} \tag{14}$$

Now by using the similarity variables the equations (6) and (12) reduces to the non- dimensional form with associated boundary conditions as

$$f''' \left(1 + \frac{3(n-1)}{2} W_e f''^2 \right) + B_1 \left\{ \begin{array}{l} B_2 \left(S \left(f' + \dfrac{\eta}{2} f'' \right) + ff'' - f'^2 \right) \\ -M \cos^2 \gamma f' \end{array} \right\} = 0 \, , \tag{15}$$

$$\left.\begin{array}{l}\left(B_3+\frac{4}{3}R\right)\theta''+\frac{Ec\Pr}{B_1}f''^2+(A^*f'+B^*\theta)-\\ B_4\Pr\left(\frac{S}{2}(\eta\theta'+3\theta)+2f'\theta-f\theta'\right)=0\end{array}\right\}, \quad (16)$$

where,

$$B_1=(1-\phi)^{2.5}, B_2=1-\phi+\phi\frac{\rho_s}{\rho_f}, B_3=\frac{k_{nf}}{k_f}, B_4=1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}, \quad (17)$$

Corresponding boundary conditions are

$$\left.\begin{array}{l}f'(0)=1, \quad f(0)=0, \quad \theta(0)=0,\\ f''(\beta)=0, \quad \theta'(\beta)=0, \quad f(\beta)=\frac{S\beta}{2}\end{array}\right\}, \quad (18)$$

here $S=\alpha/b$ is unsteadiness parameter and prime represents differentiation with respect to η. Further, β indicates the value of the similarity variable η at the free surface so that η value gives

$$\beta=\left(\frac{b}{\upsilon_f(1-\alpha t)}\right)^{\frac{1}{2}}h, \quad (19)$$

The rate at which film thickness varies can be obtained by differentiating (19) w.r.t. t,

$$\frac{dh}{dt}=-\frac{\alpha\beta}{2}\left(\frac{\upsilon_f}{b(1-\alpha t)}\right)^{\frac{1}{2}}, \quad (20)$$

where,

$$\Pr=\frac{(\mu c_p)_f}{k_f}, \quad We^2=\frac{b^3x^2\Gamma^2}{\upsilon_f(1-\alpha t)^3}, \quad M=\frac{\sigma B_0^2}{\rho_f b}, \quad R=\frac{4\sigma^*T_0^3}{k^*k_f}, \quad S=\frac{\alpha}{b}, \quad Ec=\frac{U_w^2}{c_p(T_s-T_0)}, \quad (21)$$

The physical quantities of practical interest in this problem are the skin friction coefficient C_f and the Nusselt number Nu, which are given as

$$C_f=\frac{\tau_w}{\rho_f U_w^2}, \quad Nu=\frac{q_w x}{k_f(T_s-T_0)}, \quad (22)$$

where,

$$\tau_w=\mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w=-K_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \quad (23)$$

substituting Eqn. (22) in Eqn. (23), we obtain

$$B_1 C_{fx}\operatorname{Re}_x^{-1/2}=f''(0), \ (1/B_3)Nu_x\operatorname{Re}_x^{1/2}=-\theta'(0), \quad (24)$$

where, $\operatorname{Re}_x=U_w x/\upsilon_f$ is the local Reynolds number.

3. RESULTS AND DISCUSSION

The system of coupled ordinary differential equations (6) and (12) with respect to the boundary conditions (8) are solved numerically using Runge-Kutta based shooting technique. We considered $CoFe_2O_4$ nanoparticles with water as the base fluid. The results obtained shows the influence of non-dimensional parameters such as, aligned angle γ, magnetic field parameter M, volume fraction of the nanoparticles ϕ, non-uniform heat source/sink parameters A^* and B^*, viscous dissipation parameter Ec, Weissenberg number We, and the boundary layer thickness parameter β on velocity and temperature profiles of the flow along with friction factor and local Nusselt number. For numerical calculations we considered $S=0.5; A^*=0.2; B^*=0.2; M=2; We=0.5; n=1.5; Ec=0.1; \gamma=\pi/4; R=1$. And these values are treating as common in the entire study except the varied values are displayed in the respective figure and tables.

Figs. 2 and 3 elaborate the velocity and temperature profiles of the flow for $CoFe_2O_4$-water nanofluid for various values of aligned magnetic field parameter. It is observed that for increasing values of aligned angle there is a hike in velocity and depreciation in temperature fields. Generally, increase in the value of aligned angle weakens the strength of the applied magnetic field, this result in declination of the drag forces which are acting opposite to the flow direction. We have noticed opposite results for increasing values of magnetic field parameter, which are displayed in Figs. 4 and 5. Physically, increasing values of magnetic field develops a resistive type of force known as Lorentz force. Due to this force we observed declination in the velocity profiles and hike in temperature profiles of the flow.

Fig. 6 and 7 describes the velocity and temperature profiles of the flow of $CoFe_2O_4$-water nanofluid for different values of Weissenberg number We. It is seen that an increase in Weissenberg number decelerates the velocity fields and enhances temperature profiles of the flow. Physically, We is proportional to the ratio of time constant to viscosity and this ratio is larger for increasing values of We, as a result we have seen a hike in velocity and temperature profiles. We have noticed a gradual decrement in temperature profiles of $CoFe_2O_4$ –water nanofluid for rising values of Weissenberg number.

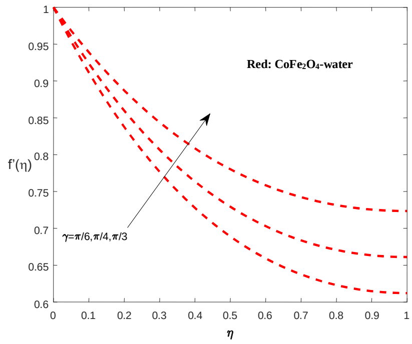

Fig-2: Velocity fields for various values of γ

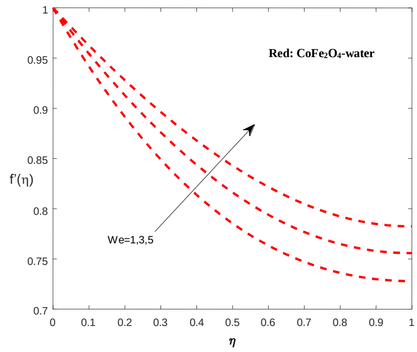

Fig-6: Velocity fields for various values of We

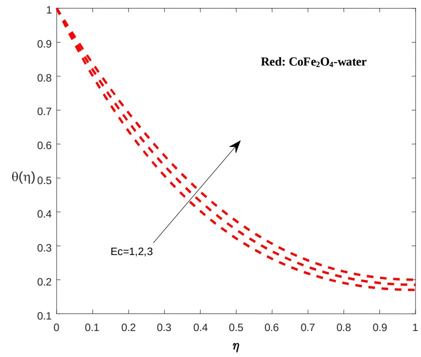

Fig-10: Temperature fields for various values of Ec

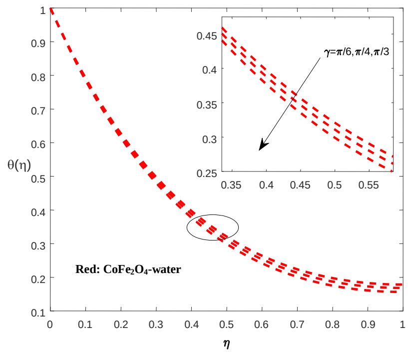

Fig-3: Temperature fields for various values of γ

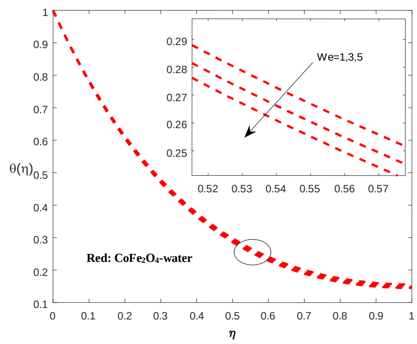

Fig-7: Temperature fields for various values of We

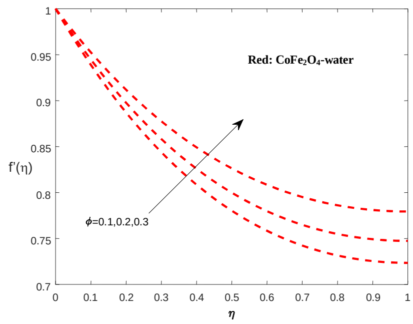

Fig-11: Velocity fields for various values of ϕ

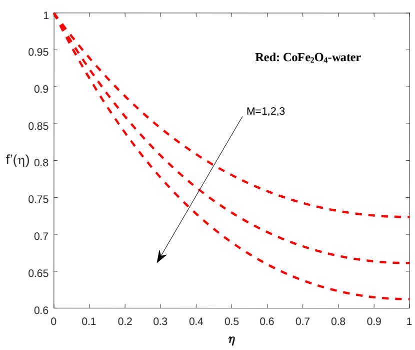

Fig-4: Velocity fields for various values of M

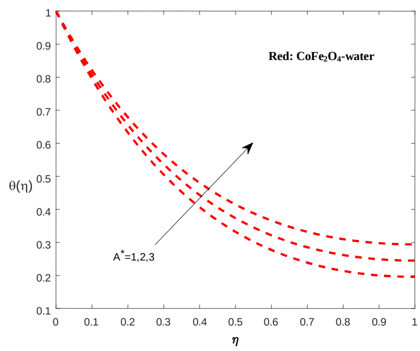

Fig-8: Temperature fields for various values of A^*

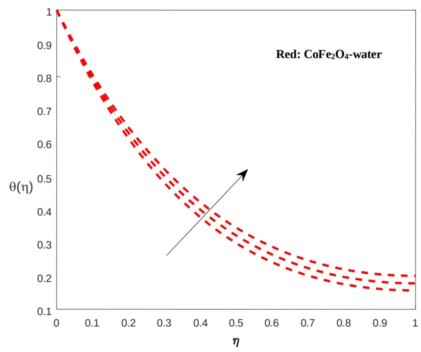

Fig-12: Temperature fields for various values of ϕ

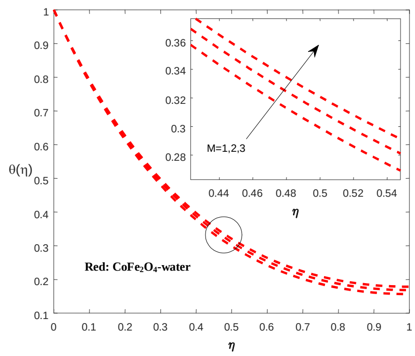

Fig-5: Temperature fields for various values of M

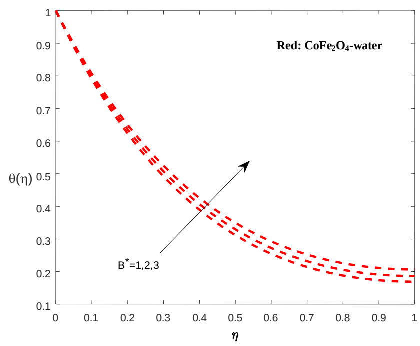

Fig-9: Temperature fields for various values of B^*

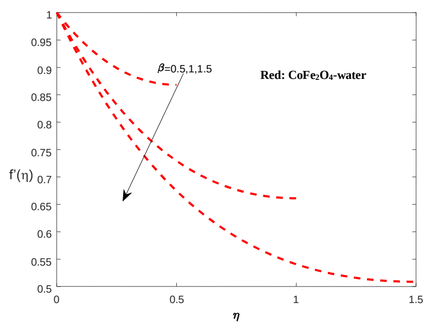

Fig-13: Velocity fields for various values of β

Table-1: Physical parameter values of $f''(0)$ and $-\theta'(0)$ for $CoFe_2O_4$-water nanofluid.

ϕ	M	S	We	n	A^*	B^*	R	Ec	γ	β	$CoFe_2O_4$-water nanofluid	
											$f''(0)$	$-\theta'(0)$
0.1											-0.660332	2.408800
0.2											-0.596309	2.279990
0.3											-0.510401	2.155543
	1										-0.660332	2.408800
	2										-0.827771	2.365280
	3										-0.965764	2.327408
		0.2									-0.799113	2.294342
		0.4									-0.710420	2.374789
		0.6									-0.605864	2.439775
			1								-0.624425	2.414312
			3								-0.477628	2.441852
			5								-0.387868	2.461439
				1							-0.675702	2.406461
				5							-0.590034	2.420128
				10							-0.534607	2.430365
					1						-0.660332	2.291149
					2						-0.660332	2.144087
					3						-0.660332	1.997023
						1					-0.660332	2.335402
						2					-0.660381	2.239563
						3					-0.660381	2.138765
							1				-0.660332	2.408800
							2				-0.660332	1.986482
							3				-0.660332	1.705787
								1			-0.660381	2.189150
								2			-0.660381	1.945214
								3			-0.660381	1.701279
									π/6		-0.965764	2.327408
									π/4		-0.827771	2.365281
									π/3		-0.660332	2.408800
										0.5	-0.565384	2.114083
										1.0	-0.827771	2.365281
										1.5	-0.930492	2.364583

Table-2: Thermo physical properties of the nano and ferro particles with base fluid (Oztop et al. [20])

Physical properties	Fluid phase (water)	$CoFe_2O_4$
$c_p(J/KgK)$	4179	700
$\rho(Kg/m^3)$	997.1	4907
$k(W/mK)$	0.613	3.7

Table-3: Comparison of values $-\theta'(0)$ for different values of S when $R = M = Ec = 0$ and $\Pr = 1$.

	$\phi = 0$	
S	Xu et al. [37]	Present Results
1.0	2.677222162	2.6772221621
1.2	1.999591426	1.9995914260
1.4	1.447754361	1.4477543611
1.6	0.956697844	0.9566978443

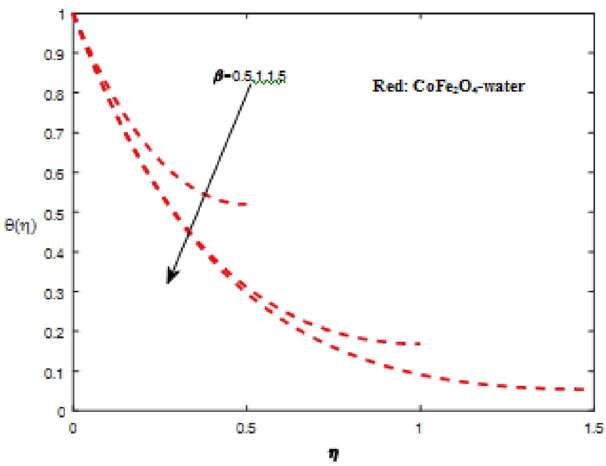

Fig-14: Temperature fields for various values of β

Fig. 8 and 9 depicts the influence of non-uniform heat source/sink parameters A^* and B^* on temperature profiles of the flow. It is evident that increasing values of non-uniform heat source/sink parameter enhances the temperature profiles of the flow. Physically, $A^* > 0$ and $B^* > 0$ acts as heat generators and $A^* < 0$ and $B^* < 0$ acts as heat absorbers. Due to this reason we observed the enhancement in temperature profiles for positive values of A^* and B^*.

Fig. 10 shows the influence of viscous dissipation parameter Ec on temperature profiles of the flow. It is clearly observed that increasing values of Ec enhances the temperature profiles of the flow. Generally, rise in the viscous dissipation improves the thermal conductivity of the fluid. Fig. 11 and 12 elaborates the effect of volume fraction parameter ϕ on velocity and temperature profiles of the flow. It is evident that rise values of ϕ enhances the velocity and temperature profiles of the flow. We observe a significant change in the velocity profiles of $CoFe_2O_4$-water for increasing values of ϕ.

Fig. 13 and 14 portrays the influence of boundary layer thickness parameter β on velocity and temperature profiles of the flow. It is observed that an increase in β depreciates the velocity and temperature profiles of the flow. Physically, rise in the value of β enlarges the thickness of momentum and thermal boundary layers.

Table 1 portrays the effect of various non-dimensional governing parameters on skin friction coefficient and local Nusselt number for $CoFe_2O_4$-water nanofluids. It is evident that volume fraction of nanoparticles reduces the heat transfer rate. Weissenberg number We, unsteadiness parameter S, inclined angle γ and boundary layer thickness parameter β enhances both local skin friction and heat transfer rate in both the solutions. Thermal radiation R, non uniform heat source/sink parameter A^* and B^* shows no effect on local skin friction, but it reduces the heat transfer rates. The nanoparticle volume fraction ϕ enhances the skin friction but decreases the local Nusselt number for both the solutions. Table 2 display the thermophysical properties of water and nanoparticles. Table 3 depict the comparison of the present results with the published results. We found a favourable agreement of the present results with the published results under some special and limited cases. This proves that present results are valid.

4. CONCLUSION

The conclusions of the present study are made as follows.

- Unsteadiness parameter have tendency to enhance both the skin friction and heat transfer rate.
- Rising values of Weissenberg number and Power- law index parameter shows the effective enhancement in heat transfer rate.
- Aligned magnetic field angle regulates the flow and heat transfer
- Rise in the boundary layer thickness enhances the heat transfer rate.
- Non-uniform heat source/sink parameters controls the thermal boundary layer thickness.
- Heat transfer rate is high in $CoFe_2O_4$–water nanofluid.

REFERENCES

[1] LJ. Crane, Flow past a stretching plate. *Z. Angew. Math. phys*. vol. 21, pp. 645-647, 1970.

[2] S. Mukhopadhyay, G.C. Layek, Sk.A. Samad, Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity, *Int. J. Heat and Mass Transfer*, vol. 48, pp. 4460-4466, 2005.

[3] S. Nadeem, R. Ul Haq, Z.H. Khan, Numerical solution of non-Newtonian nanofluid flow over a stretching sheet, *Appl. Nanosci*., vol. 4, pp. 625-631, 2014.

[4] C.Y. Wang, Liquid film on an unsteady stretching surface, *Quarterly of applied Mathematics*, vol. 48, pp. 601-610, 1990.

[5] M.S. Abel, J. Tawade, M.M. Nandeppanavar, Effect of non-uniform heat source on MHD heat transfer in a liquid film over an unsteady stretching sheet, *International Journal of Non-Linear Mechanics*, vol. 44, no. 9, pp. 990-998, 2009.

[6] N. Sandeep, A. Malvandi, Enhanced heat transfer in liquid film flow of non-Newtonian nanofluids embedded in grapheme nanoparticles, *Advanced Powder Technology*, (2016). Doi: 10.1016/j.apt.2016.08.023.

[7] D. Pal, P. Saha, K. Vajravelu, Combined effects of nonlinear thermal radiation and internal heat generation/Absorption on heat and mass transfer in a thin liquid film on a permeable unsteady stretching surface with convective boundary condition, *Int. J. Appl. Comput. Math*, (2016) DOI 10.1007/s40819-016-0242-z

[8] S.U.S. Choi, Enhancing thermal conductivity of fluids with nanoparticles, *In: The proceedings of the 1995 ASME International Mechanical Engineering Congress and Exposition* vol. 66, pp. 99-105, 1995.

[9] J. Buongiorno, Convective transports in nanofluids, *J. Heat Transfer*, vol. 128, pp. 240-250, 2006.

[10] S. K. Das, S.U. Choi, Yu W, T. Pradeep, Nanofluids: Science and Technology, Hoboken: Wiley, 2007

[11] C. Sulochana, G.P. Ashwinkumar, N. Sandeep, Similarity solution of 3D Casson nanofluid flow over a stretching sheet with convective boundary conditions, *J. of the Nigerian Mathematical Society*, vol. 35, pp. 128-141, 2016.

[12] K. Vajravelu, K. V. Prasad, L. Changhoon, I. Pop, Robert A. Van Gorder, Convective heat transfer in the flow of Ag-water and Cu-water nanofluids over a stretching surface, *Int. J. Therm. Sci*. vol. 50, pp. 843-851, 2011.

[13] N. Sandeep, C. Sulochana, Unsteady MHD flow of Cu-water dusty nanofluid over a stretching surface, *J. Advanced Physics* vol. 5, no.3, pp. 191-198, 2016.

[14] N.S. Akbar, S. Nadeem, Z.H. Khan, Numerical simulation of peristaltic flow of a Carreau nanofluid in an asymmetric channel, *Alexandria Engineering Journal*, vol. 53, pp. 191-197, 2014.

[15] C.S.K. Raju, N. Sandeep, Heat and mass transfer in MHD non-Newtonian bio convection flow over a rotating cone/plate with cross diffusion, *J. Molecular Liquids*, vol. 215, pp. 115-126, 2016.

[16] C. Sulochana, G.P. Ashwinkumar, N. Sandeep, Transpiration effects on stagnation point flow of a Carreau nanofluid in the presence of thermophoresis and Brownian motion, *Alexandria Engineering Journal*, vol. 55, pp. 1151-1157, 2016.

[17] H. Oztop, E. Abu-nada, Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids, *Int. J. Heat Fluid Flow*, vol. 29, pp. 1326-1336, 2008.

BIOGRAPHIES

Mr. G.P. Ashwinkumar obtained his M.Sc. degree from Central University of Karnataka, Gulbarga. Presently he is pursuing Ph.D degree under the guidance of Prof. C. Sulochana in Department of Mathematics, Gulbarga University, Kalaburagi, Karnataka. His area of interest is Fluid Mechanics. He has published several papers in reputed scientific journals and presented papers in national and international conferences.

Prof. C.Sulochana obtained her M.Sc., and M.Phil., degrees from Gulbarga University, Gulbarga, Karnataka and Ph.D degree from Sri Krishna Devaraya University, Anantapur. She produced 6 Ph.D and 7 M.Phil, degrees. She published more than 35 papers in reputed scientific journals and presented papers in national and international conferences. She handled various prestigious positions at Gulbarga University. At present she is serving as a professor at Department of Mathematics, Gulbarga University, Kalaburagi. Her area of interest is Fluid Mechanics.