SENSITIVITY ANALYSIS TO ASSESS VIBRATIONAL RESPONSE OF A SANDWICH BEAM USING RESPONSE SURFACE METHODOLOGY

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Abstract

Every mechanical structure exhibit natural mode of vibrations. Beams with variable cross-section and material properties are frequently used in aeronautical, mechanical, civil engineering .Given the elastic and inertia characteristics of the structures, mode of vibration can be computed, the study being known as modal analysis. Sandwich beams offer designers a number of advantages, such as the high strength to weight ratio, flexibility, high bending and buckling resistances.Sandwich materials are frequently used wherever high strength and low weight are important criteria.(Maha M. A. Lashin e tal, 2015)

In the present study a sandwich beam of size 10 mm X 10 mm cross-section with a length of 1 meter is considered. Analysis is done for different combinations of material and randomly varying the type of material across the depth of the cross-section of the beam. The response is calculated in terms of natural frequency. Using Monte - Carlo simulation design matrix was generated. Response surface methodology was applied in the present study using Design expert software for modeling and analysis of response parameter in order to obtain the natural frequency of the beam. A multiple regression analysis was carried out to obtain coefficients and the equations that can be used to predict the responses viz natural frequency. The sandwich beam which gave maximum frequency is Carbon-Aluminium-Carbon. The maximum frequency for the first mode was found out to be 41.22 Hz. Further, optimisation is carried and the best possiblecombination in order to get the maximum frequencyis found out.

Keywords:*Sensitivity analysis, Monte Carlo, Sandwich beam, Natural frequency,*

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1. INTRODUCTION

Sandwich beams offer designers a number of advantages, as the high strength to weight ratio, flexibility, high bending and buckling resistances. Sandwich construction results higher natural frequencies than none sandwich constructions, also it developed an adaptive tuned vibration absorber.

A Structural Sandwich is a special form of composite comprising of a combination of different materials that are bonded to each other so as to utilize the properties of each separate component to the structural advantage of the whole assembly (Nalagulla e tal ,2006).

The natural frequencies of vibration are found from equation

$$\omega_n = \beta_n 2 \sqrt{\frac{EI}{\rho}} = (\beta_n l)2 \sqrt{\frac{EI}{\rho A l^4}}$$

Where, the constant β_n depends on the boundary conditions of the problem.

2. DERIVATION TO FIND BN VALUE:

Considering general equation

Y= Acosβx + Bsinhβx + Ccosβx + Dsinβx

$$Y=A\left(\frac{e^{\beta x}+e^{-\beta x}}{2}\right) + B\left(\frac{e^{\beta x}-e^{-\beta x}}{2}\right) + C\cos\beta x + D\sin\beta x$$

$$y'=A\beta\left(\frac{e^{\beta x}-e^{-\beta x}}{2}\right) + B\beta\left(\frac{e^{\beta x}+e^{-\beta x}}{2}\right) - C\beta\sin\beta x + D\beta\cos\beta x$$

$$y''= A\beta 2\left(\frac{e^{\beta x}+e^{-\beta x}}{2}\right) + B\beta 2\left(\frac{e^{\beta x}-e^{-\beta x}}{2}\right) - C\ \beta 2\sin\beta x - D\ \beta 2\cos\beta x$$

Therefore

Y=Acoshβx + Bsinhβx + Ccosβx + Dsinβx ---------- --- (1)

y′=Aβsinhβx + Bβcoshβx - Cβsinβx + Dβcosβx ---------- (2)

y″=Aβ2coshβx + Bβ2sinhβx - C β2sinβx - Dβ2cosβx----- (3)

Boundary conditions for simply supported beam

Y=0 x=0

X=l y=0

X=l $\frac{d2y}{dx2}$=0

Now

Y=0 x=0

Substitute in 1

We get 0=A+C

Now,

X=0 $\frac{d2y}{dx2}$=0

0=A-C

Therefore

A=C=0

Substitute this in 1

Y=Bsinhβx + Dsinβx -------------------------------------- (4)

y″= Bβ2sinhβx - D β2cosβx ------------------------------- (5)

Now at x=l y=0 and y″= 0

We get 0= Bsinhβl + Dsinβl ---------------------------- (6)

And substitute in y″

0= Bβ2sinhβl - D β2sinβl

0= Bsinhβl – Dsinβl ---------------------------------- (7)

From 6th and 7th equation we have
Bsinhβl=Dsinβl=0

D≠0 therefore βl=0
Therefore β=0,$\frac{\pi}{l}$,$\frac{2\pi}{l}$,$\frac{3\pi}{l}$
Since βl≠0, sinh βl≠0, β=0
Also Dsinβl=0 since D≠0 otherwise y=0 for all x, sinβl=0. Hence y= Bsinhβx and the solutions to sin βl=0 gives the natural frequency
They are,
β=0,$\frac{\pi}{l}$,$\frac{2\pi}{l}$,$\frac{3\pi}{l}$,…….
The vaues of βn are shown in Table.1
The natural frequency is calculated using

$$\omega_n = (\beta_1 * L)^2 \sqrt{\frac{EI}{\rho A L^4}}$$

ω_n-Natural frequency
β- Euler's constant
L-length of the beam
I-Moment of inertia
E- Young's modulus of beam
ρ-Density of the beam
A-Area of cross-section

Table.1βn values

Beam configuration	(B1l)2 Fundamental	(B2l)2 Second mode	(B3l) Third mode
Simply supported	9.87	39.5	88.9
Cantilever	3.52	22.0	61.7
Free -Free	22.4	61.7	121.0
Clamped-Clamped	22.4	61.7	121.0
Clamped Hinged	15.4	50.0	104.0
Hinged Free	0	15.4	50.0

3. RESPONSE SURFACE METHODOLOGY

Response Surface Methodology (RSM), introduced by Box and Wilson(1951), is a collection of techniques that were developed as means to findoptimal settings of input(predictor, independent)factors or design variables that maximize or minimize the target measured response of outcome variables. In general RSM is a collection of mathematical and statistical techniques for empirical building. The object of present study is to optimize the given models (beam) which is influenced by several independent variables. In our research paper we aim, to use RSM to develop a general equation for maximizing the frequency using the Euler's equations..While using the RSM it may be noted that RSM makes the best statistical model in approximation to reality.(Raymond H. Myers e tal)

4. MATERIALS AND METHODS

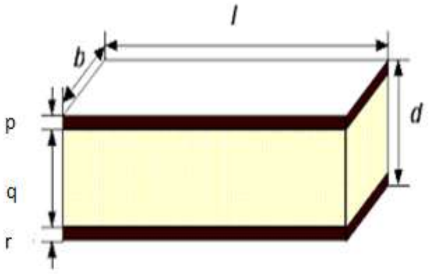

Figure 1 Details of sandwich beam

Initial analysis of sandwich beam was done by considering 1m length and for different combination of materials.The details of the dimension are
p =2mm
q=2mm
r =10mm
The beam is analysed considering various materials such as steel aluminium and carbon FRP. The material properties are shown in Table 2 below.

Table 2: Material properties

Material	Young's modulus Gpa	Density (Kg/m^3)
Steel	200	7850
Carbon FRP	150	1800
Aluminium	70	2700

The natural frequency for various material combinations are calculated and shown in Table 3

Table 3 Frequency values for different combination of material

Material	Frequecy	
	mode1	mode 2
steel-FRP-steel	28.77244365	115.075
steel-Al-steel	22.946	91.789
FRP-steel-FRP	26.0966	104.3881
FRP-Al-FRP	29.925	119.70361
Al-steel-Al	22.915	91.665
Al-FRP-Al	33.4225	134

It was seen that among various combinations, Aluminium-carbon FRP-Aluminium gave the highest frequency of 33.425 Hz followed by carbon FRP-Aluminium-carbon FRP with frequency of 29.925 Hz.

In the present work,the beam is considered of 1m length and depth as 10mm.Further analysis is done by considering various combination of materials viz: Al-C-Al, Al-Fe-Al, C-Al-C,C-Fe-C,Fe-Al-Fe, and Fe-C-Fe by randomly varying the material across the depth using MonteCarlo simulation technique.The random numbers were generated using Mat-Lab.

Using response surface methodology predicted equations to obtain natural frequency of the sandwich beam are generated.The natural frequency has been computed for each set of material and length of beam taken as 1m.The equations are shown below.

ALUMINIUM-CARBON FRP-ALUMINIUM (mode 1)
Freq= 23.10484 -9.48850E-004* p +1.78391* q+0.000000* r -0.012437* p * q +3.39238E-004* p * r -0.013074* q * r

ALUMINIUM-CARBON FRP-ALUMINIUM (mode 2)
Freq= 92.41938 - 3.79540E-003* p +7.13565* q+ 0.000000* r- 0.049747* p * q+ 1.35695E-003* p * r- 0.052298* q * r

ALUMINIUM-STEEL-ALUMINIUM (mode 1)
Freq= 23.06960+ 4.54494E- 005* p -0.014364* q+0.000000* r- 2.91855E- 003* p * q - 5.41100E-005* p * r- 2.85774E-003* q * r

ALUMINIUM-STEEL-ALUMINIUM (mode 2)
Freq= 92.27841+ 1.81798E- 004* p- 0.057455* q+ 0.000000* r - 0.011674* p * q- 2.16440E- 004* p * r- 0.011431* q * r

CARBON FRP-ALUMINIUM-CARBON FRP (Mode 1)
Freq= 41.34089- 1.20863E- 003* p- 1.78397* q+ 0.000000* r- 0.029589* p * q- 0.029829* q * r

CARBON FRP-ALUMINIUM-CARBON FRP (mode 2
Freq= 165.36770+ 3.79785E- 003* p- 7.13322* q+ 0.000000* r- 0.12180* p * q- 1.46424E-003* p * r-0.11918* q * r

CARBON-STEEL-CARBON (mode 1)
Freq= 40.40796 + 1.02475E- 003* p- 1.19867* q+ 0.000000* r- 0.29802* p * q- 5.86125E- 003* p * r- 0.29247* q * r

CARBON FRP-STEEL-CARBON FRP (mode 2)
Freq= 161.63183 + 4.09901E- 003* p- 4.79468* q+ 0.000000* r- 1.19209* p * q- 0.023445* p * r-1.16986* q * r

STEEL-ALUMINIUM-STEEL (mode 1)
Freq= 38.97451+ 1.78953E- 003* p - 0.98771* q+ 0.000000* r+ 0.041282* p * q - 4.77645E-004* p * r+0.042383* q * r

STEEL-ALUMINIUM-STEEL (mode 2)
Freq= 155.89805+ 7.15813E-003* p- 3.95083* q+ 0.000000* r+ 0.16513* p * q- 1.91058E- 003* p * r+ 0.16953* q * r

STEEL-CARBON FRP-STEEL (Mode 1)
Freq= 23.56834- 0.29537* p+ 1.29445* q+ 0.000000* r- 0.067126* p * q+ 0.063491* p * r- 0.14434* q * r

STEEL-CARBON FRP-STEEL (Mode 2)
Freq= 91.97319- 0.033096* p+ 5.30902* q+ 0.000000* r- 0.34740* p * q + 7.53110E-003* p * r- 0.36711* q * r

Further optimization was carried out in RSM to find out which combination of materials gives maximum frequency. It was found out thatcarbon FRP-Aluminium-carbonFRP combination gave maximum frequency for the values of t=2.664mm, c=0.067mm and t=7.268mm across the depth of cross section.The frequency was found to be 41.22982 Hz and 164.9193 Hz for mode 1 and mode 2 respectively.

CONCLUSIONS

Response surface methodology was applied in the present study using Design expert software for modelling and analysis of response parameter in order to obtain the natural frequency of the simply supported sandwich beam. A multiple regression analysis was carried out and predicted equations were generated for various combination of materials.Performing sensitivity analysis of the. sandwich beam it was found out that the beam which gave maximum frequency was Carbon FRP-Aluminium-Carbon FRP. The maximum frequency was found to be 41.22982 Hz and 164.9193 Hz for mode 1 and mode 2 respectively..It was found out that carbon FRP-Aluminium-carbon-FRP combination gave maximumfrequency for the values of p=2.664mm, q=0.067mm and r =7.268mm across the depth of cross section.

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