

A REVIEW ON INVESTIGATION OF MIXED MODE FRACTURE IN BRITTLE MATERIALS

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Abstract

A series of mixed mode fracture tests were conducted on specimens made up of brittle materials to find out fracture parameters. Some of the parameters of interest are stress intensity factor (K), fracture toughness (K_{IC}) and crack growth direction. In order to minimize catastrophic effects of mixed mode brittle fracture, investigation of mixed mode fracture is necessary. A number of researches have been carried out utilizing universal testing machine with suitable fixtures. Some of the experiments conducted used techniques like indentation techniques, tension tests, compression tests, bending and so on. Each one has its own advantages and disadvantages. The experimental results obtained were theoretically verified by many mixed mode fracture criteria. Some researchers also came up with empirical relations to approximate results which wouldn't have been possible by existing famous fracture criteria. Even numerical methods like FEM analysis were carried out using commercially available softwares like ABACUS.

Keywords— Mixed mode; Brittle fracture; Fracture toughness; Universal testing machine.

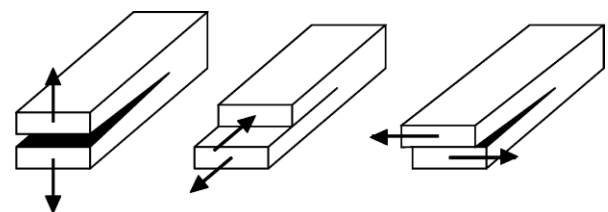
1. INTRODUCTION

Brittle material components like glass, ceramics, rocks, and some polymers products are often subjected to catastrophic fracture and sudden failure when pre-existing cracks are present. Brittle rocks are considered in many applications like mining, tunneling, rock cutting, hydraulic fracturing of oil and gas wells, rock slopes stability analysis, etc. The brittle failure of rock masses are mainly due to both mode I (opening) and mode II (sliding) i.e. mixed mode deformation. Even glasses used in aerospace fail due to mixed mode fracture in presence of pre existing cracks. In the engineering components made up of brittle materials, cracks can occur during manufacturing or during usage. These cracks under applied loads may lead to brittle failure due to the mixed mode fracture which may happen. All of these results in breakdown of the machine or equipment.

Basically under applied load condition, a crack propagates as per any one of the three fundamental failure modes or under mixed mode. Mode I is also called as tensile opening mode or simply opening mode. In this mode the two crack surfaces move perpendicular to crack plane. Mode II is also called as plane sliding mode or in-plane shear mode or simply shear mode. In this mode the direction perpendicular to crack front faces of crack are mutually sheared. Mode III is also called as tearing mode or out of plane mode. Here in the direction parallel to the crack front faces of crack are mutually sheared. (Fig. 1)

Mixed mode conditions are nothing but any combinations of these three modes. As per literature generally the brittle

materials fail in mixed mode conditions. They fail mainly in combinations of mode I and mode II. In order to design and manufacture advanced components made from brittle materials the investigation of fracture toughness and crack growth direction are crucial issues. A review of the work done by researchers on the mode I, mode II and mainly mixed mode fracture of brittle materials is given in this paper.



Mode I Mode II Mode III
Fig. 1 Three basic modes of crack propagation [13]

In engineering components and structures cracks are inevitable in most of the situations. They can be formed during manufacturing, handling, due to environmental causes or due to repetitive loads, etc. Fracture is nothing but unstable development of crack resulting in failure of component in turn machine breakdown. When load is applied, the cracks, which are present earlier, results in fracture. Thus For a cracked component, in presence of in-plane loading, pure mode I or pure mode II fracture takes place. In practical situations most commonly the components are subjected to mixed mode I / II fracture i.e combination of mode I mode II.

In order to prevent catastrophic failure of components and structures studies have to be carried out on causes of failure like mixed mode failure. By finding out direction of crack propagation and fracture load, the necessary control action can be taken in future in order to avoid it in future in corresponding application. Also using mixed mode fracture criteria Fracture toughness can be calculated. There are many experimental, numerical and FEA methods available for fracture analysis. A review of work done on mixed mode brittle fracture is made here.

2. LITERATURE ON MIXED MODE BRITTLE FRACTURE

2.1 Literature

Ergodan et al. (1963) [1] investigated crack growth and direction in brittle material by conducting experiments on plexiglass plate with central thru crack. Using plexiglass sheets with inclined crack many fracture experiments in combined stress field i.e. mixed mode fracture were conducted and checked with maximum stress criterion. A large plexiglass plate having straight and thru central crack was subjected to shear loads which are equal and opposite and was applied at a distance from symmetric axis. They found out that in sliding mode condition crack grows in a direction which is 70° from crack plane and is almost orthogonal to maximum tangential stress at crack (70.5°). By theoretical calculations and experiments they concluded that for mixed mode brittle fracture analysis maximum stress theory is applicable (MTS theory).

G. C. Sih et al. (1974) [2] proposed new criterion based on strain energy density factor S to predict crack growth direction under mixed mode (mode I and mode II) condition. The predictions were based on strain energy density factor (S_{cr}) stationary factor's stationary values. He proposed critical value S_{cr} as material parameter i.e. S_{cr} does not depend on loading and crack geometry. S_{cr} can be used to determine resistance against fracture. Mixed mode tests were performed on inclined crack specimens made up of plexiglass and aluminium alloy under tension and compression. Close agreement between experimental and theoretical results were observed. This investigation provided a standard procedure for finding out critical size of crack any structure would tolerate for a given applied stress under mixed mode conditions.

Freiman et al. (1979) [3] investigated mixed mode (mode I and mode II) fracture of soda lime silica glass. Knoop micro hardness indentation technique was used. The critical stress intensity factors were calculated combining K_1 and K_2 using four analysis techniques. Also fracture toughness K_{IC} was found out experimentally by measuring the size of fracture mirror. The comparison of above two showed that K_{IC} which was determined by non-coplanar strain energy release rate analysis showed close agreement with experimental one.

D. K. Shetty et al. (1986) [4], investigated mixed mode fracture (mode I and mode II) in glass ceramic and an alumina ceramic specimens. They conducted mixed mode fracture tests of diametrical compression of angled crack specimen and four point bend test of angled crack plate specimen. In both tests cracks were induced by knoop indentation. They also theoretically compared experimental results with maximum Hoop stress theory. They concluded simple approach to study mixed mode fracture in diametrical compression test along with knoop surface flaws as any combination of mode I and mode II loading can be obtained easily by this. In this study knoop flaws extended in maximum remote tension direction.

Scott Glasemann et al. (1987) [5] used knoop indentation flaws technique to study mixed mode fracture (mode I and mode II) in soda lime specimen. Four point bending was employed for conducting strength tests. He conducted tests in liquid nitrogen water and oil environment. Fracture strength in liquid nitrogen is more than fracture strength in oil. The fracture strength in oil is more than in water. From the annealed residual stress free surface flaws optical examination showed extension of crack is in a plane orthogonal to the maximum normal stress direction. Hence during tests the subcritical crack growth caused the crack tip to realign in the direction normal to applied failure stress. (Fig.2).

He et al.(1990) [6] mixed mode fracture toughness in soda lime glass and PMMA. Experiments were carried on four point shear (FPS) specimens and over a full range of mode (mode I and mode II) mixed mode fracture resistance was found out. Even the loading point friction effects were accounted in the analysis. The theoretical calculations carried out using strain energy release rate criterion. He found PMMA as most straight forward and it has mode mixity independent fracture energy. This behavior is due to the interaction which occurs between two crack surfaces as a result of surface forces.

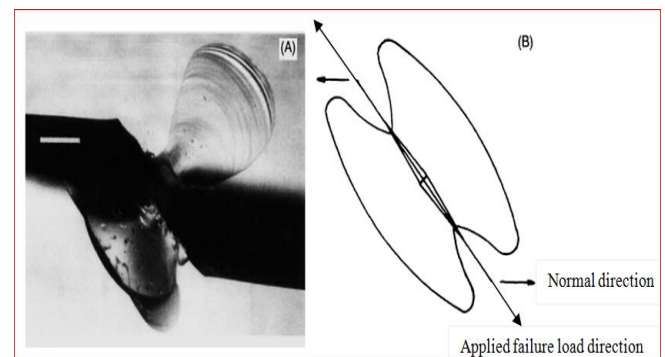


Fig. 2 Fracture patterns of annealed knoop indentation flaws [5]

R. Dal Maschio et al. (1994) [7] investigated fracture toughness in brittle materials by using Vickers indentation test. Though Vickers indentation test was very simple to conduct the multiple complex models and corresponding equations are quite complicated as per their literature survey.

So they come up with simplified and realistic model for estimating fracture toughness in brittle materials. The fracture toughness calculations done by them showed well approximation with fracture toughness investigated from other standard fracture mechanics experiments. At higher loads they found out small differences in toughness value. They developed a model taking into account only realistic hypothesis on material behavior. They concluded this approach which was done with glass can be extended to other brittle ceramic materials as well.

Lim et al (1994) [8] investigated on mixed Mode (mode I and mode II) fracture toughness on Johnstone i.e. water saturated synthetic mudstone specimen. Semicircular specimen with three point bending is used in analysis. They compared the results with theories like maximum tangential stress criterion, maximum energy release rate criterion, strain energy release rate criterion and maximum dilational strain energy density criterion. By all these criteria calculations and comparing with the experimental results he found some deviations. So he empirically derived new criteria which well agreed with experimental results.

G. R. Krishnan et al. (1997) [9] conducted a number of fracture toughness tests under mode I and mode II and combinations of these two loading conditions. He conducted mixed mode tests on straight edge notch Brazilian disk (SENBD) in order to develop an empirical failure envelop. This series of tests were first of its kind because of the unavailability of literature in mixed mode fracture toughness. Naturally weakly cemented soft antler sandstone necessarily needs extreme care in handling; accomplishing precise and dependable sample preparation was a great achievement. The failure fracture pattern was consistent in all tests. The results were matched experimentally and theoretically. They used an empirical formula to find fracture toughness envelope.

Hideo Awaji et al. (1999) [10] investigated mixed mode (mode I and mode II) fracture experimentally and theoretically. The conducted experiments of diametrical compression of Brazilian disk with central thru crack made of float glass, Alumina and PMMA. Also he conducted rectangular plate tension tests to study mixed mode fracture. Theoretically he developed criteria for mode I and mode II by extending Griffith's concept for mode I and they found out ratio of fracture toughness of mode II with mode I as 1.2. He also compared with Singh and Shetty's empirical equation C is equal to ratio of fracture toughness of mode II with mode I and c was found to be 1.2. From the experiments conducted the ratio of fracture toughness of mode II with mode I yielded a range of 1.1 to 1.3 which is very close to 1.2. Well agreement between practical and theoretical criteria was observed.

K Khan et al.(2000)[11] studied mode I, mode II and combinations of these two mixed mode using three types of specimen experimentally namely straight edge cracked round bar bend, Brazilian disk with straight notch and

semicircular specimen with thru straight crack. He used straight edge cracked round bar bend to study mode I. The Brazilian disk with notch and semicircular bend specimens were employed for mixed mode study. All specimens were prepared from a lime stone rock in central province of Saudi Arabia. The results showed diameter of specimen, type of crack have enormous influence on the values of measured fracture toughness. He found out size of crack, loading rate and thickness. He found Brazilian disc with a straight notch was convenient geometry for fracture toughness investigation. He proposed a simple method of making notch precisely inside the disk specimen using drilling machine and a wire saw.

Gong et al. (2001) [12], carried out Vickers indentation tests to find out fracture toughness on a commercial soda lime glass. Also, he has analyzed the resultant fracture toughness statistically. When carried out Vickers indentation tests it was observed a large scatter in the indentation toughness measured at different applied load levels which agreed with the literature till that time. Similar scatter was observed by other researchers in soda lime glass. The variation was even more than 10% in some results. So he used weibull statistics to check the variability in measured indentation toughness. He observed weibull modulli obtained are nearly identical with each other for the different sets of indentation toughness data measured for various load levels and hence showed the indentation toughness statistical variability is load independent.

Chang et al. (2001) [13] obtained rock fracture toughness in mixed mode study using disk type specimens. He applied straight thru crack assumption to cracked chevron notched Brazilian disk specimen and semicircular specimen under three point bending to determine mixed mode fracture toughness of rock. After conducting tests on several type of specimen he concluded cracked chevron notched bend specimen as most preferable and versatile disc type specimen. He found out that pre-crack making is unnecessary in these types of specimen as chevron notch used in the specimen includes self pre-cracking and hence stable crack propagation was seen. His tests on semicircular bend specimen showed large scatters and patterns were inconsistent with notch length and low fracture toughness was observed. He found out reason for that as pre-cracking was not carried out. He recommended carrying out pre-cracking in SCB instead of chevron notch. He also carried out theoretical calculations and analysed with many mixed mode fracture criteria. The minimum strain energy density criterion well approximates experimental results.

Ayatollahi et al. (2006) [14] conducted a series of tests in mode I, mode II and mixed mode combinations of these two modes and found out that for brittle materials Johnstone and PMMA are used here. The pre-cracked semicircular bend (SCB) specimen subjected to three point bend is found as appropriate test specimen. Fabrication, pre-crack making on SCB is very simple. Also simple loading fixture is sufficient for a fracture test. It was found experimentally mode II fracture toughness K_{IIc} in terms of mode I fracture

toughness K_{Ic} . During theoretical calculations it was found out that almost all theoretical criteria for mixed mode brittle fracture did not predict properly experimentally determined fracture toughness K_{IIc} . He then used modified maximum tangential stress criterion for predicting values in brittle failure. He showed in his work how modified criterion prediction well approximated experimental results.

Ayatollahi et al. (2006) [15] conducted a series of mixed mode fracture tests in range from pure mode I to mode II on semicircular bend specimen with an edge crack made up of PMMA material. He found out fracture load crack growth path for different crack angles from his experiments. He showed the popular mixed mode criteria such as maximum tangential stress (MTS) criterion failed to predict mixed mode brittle fracture strength of PMMA for mode II dominant loading situation when semicircular bend specimen is used for fracture tests. When generalized MTS criterion was used predictions were very close to results. It was concluded that crack growth path predicted by generalized MTS criterion results in good agreement with the observed fracture path in SCB fractured specimens. FEM analysis also carried out using ABACUS and results were matching with experimental results.

Koike et al. (2007) [16] studied crack velocity of brittle soda lime silicate glass. He used rectangular bar with crack under diametrical compression (Double Cleavage Drilled Compression or DCDC) specimen. By using DCDC method test on specimens of soda lime glass and less brittle glass sub critical crack growth were studied with different fictive temperatures. The less brittle glass had a faster crack growth rate than brittle soda lime silicate glass. Also in both glasses faster crack growth rate was observed at lower fictive temperature. Also K_{Ic} of soda lime silicate glass is larger than the K_{Ic} of the less brittle glass.

Hasanpour et al. (2008) [17] investigated limestone specimen in mixed mode fracture behavior by experimental and numerical analysis. In order to conduct mixed mode test a modified version of arcan specimen was employed with a special testing device. By modified arcan specimen loading device mode I, Mode II and wide variety of combinations of these two modes (mixed mode) conditions can be obtained. Test procedure is simple, clamping and unclamping of specimen is easier. Single type of specimen is sufficient to obtain all loading conditions. Finite element analysis indicated for loading angle near pure mode II loading, a high ratio of mode II to mode I was observed and trend was opposite for loading angle near pure mode I loading. The fracture toughness was determined experimentally by mixed mode tests on modified arcan specimen. The results confirmed the cracked specimen is weaker in shear loading and under tensile loading conditions it is tougher.

Ayatollahi et al. (2009) [18] studied mixed mode fracture in soda lime glass by using cracked Brazilian disc specimen. It was observed that the previously reported experimental

results in literature for cracked Brazilian disc specimen made up of several glasses are always underestimated by conventional mixed mode fracture criteria like maximum tangential stress (MTS) criterion. For this reason mixed mode fracture test results was approximated using generalized maximum stress (GMTS) criterion. When generalized MTS criterion was used, in the experimental results reported in literature for cracked Brazilian disc (CBD) T-stress plays important role. The GMTS criterion considers both the effects of conventional singular stress and T-stresses. The T-stress is negative always for CBD specimen and its magnitude as significant effect for calculating fracture toughness.

Aliha et al. (2011) [19] investigated mixed mode fracture toughness of an Iranian white marble specimen. Mixed mode fracture tests conducted on the above said material specimen with edge thru crack and tested in all mode mixities from pure mode I to pure mode II and different mode mixities by varying crack angles in different specimens. It was verified theoretically. The SCB specimens under mixed mode tests yielded lower fracture resistance which was mainly due to influence of large positive T-stresses that exist in SCB specimen. Generalized maximum tensile strength criterion was employed for finding out mixed mode fracture toughness data. In GMTS criterion along with stress intensity factors effect of non singular stress term (T-stress) was considered. It was shown that GMTS criterion gave significantly close estimates for mode fracture in SCB specimen made up of Hasin marble.

Aliha et al. (2012) [20] investigated brittle fracture in ceramics under mixed mode combination of mode I and II conditions. The theoretical calculations were based on generalized maximum stress criterion (GMTS). GMTS criterion was used to predict the fracture initiation angle for mixed mode I/II loading. The experimental results are obtained using centrally cracked circular disc or Brazilian disc.

Experimental and theoretical results were approximated well. According to the present fracture model fracture angle of the elastic T-stress in ceramic specimens tested. The fracture initiation angle in the investigated ceramics found to decrease due to negative T-stress that exist in the circular disc specimen. Tests were conducted on Glass, SiC, alumina and zircona.

M. R. Ayatollahi et. al. (2011) [21] suggested a test configuration newly named as Asymmetric semi circular bend specimen (ASCB) for fracture experiments on I/II combination of mixed mode on brittle materials. This ASCB specimen had number of advantages like simple geometry, simple experimental loading set up. Also PMMA material was used as material for specimen in which crack generating was easier. The main advantage of this particular ASCB specimen was that, all combination of mode mixities were easily introduced. The experimentally obtained results were matching well with theoretically calculated results using Generalised maximum tangential stress criterion. They

concluded that for mixed mode study of brittle materials ASCB specimens were very much suitable.

3. CONCLUSION

1. The most popular brittle materials of interest for mixed mode fracture are mainly rocks [8], [9], [11], [13], [14], ceramics [4], [7], [19], [20]. Glasses like plexiglass (PMMA) [1], [2], [6], [10], [14], [15], soda lime silica glass [3], [5], [6], [12], [16], [17], [18] and some polymers.
2. The mixed mode (mode I and mode II) fracture toughness found out experimentally using many types of methods and specimens. Indentation technique used are knoop indentation tests [3], [5] and Vickers indentation tests [7], [12]. The popular test methods include cracked Brazilian disc [9], [10], [11], [13], [18], [20] under diametrical compression and semicircular bend specimen [8], [11], [14], [15], [19] with thru crack under three point bending. From the literature it can be concluded that the simplicity of loading fixture, manufacturing ease and specimen pre-cracking are prime reasons for selecting specimens and testing procedure for particular mixed mode testing in brittle material [14]. The testing method chosen should be such that the full range of loading conditions for mixed mode starting from pure mode I to pure mode II and wide variety of these two modes combinations should be possible.
3. The practical experiments were approximated using several theoretical criteria. Most famous criteria used for mixed mode fracture criterion are:- maximum tangential stress criterion [3], maximum energy release rate criterion [1], strain energy density criterion [2]. Some of the researchers derived empirical equation which closely approximated experimental results for mixed mode brittle fracture. Generalized or modified maximum tangential stress criterion (GMTS) [14], [15], [18], [19], [20] used in many research works which takes into account T-stress. In cracked Brazilian disc specimen negative T-stresses [18] will be present which enhances fracture resistance in CBD. In semicircular bend (SCB) specimen positive T-stresses will be present which reduces fracture resistance [19].

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