STATISTICAL ANALYSIS OF RAINFALL DATA AND ESTIMATION OF PEAK FLOOD DISCHARGE FOR UNGAUGED CATCHMENTS

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Abstract

Estimation of Peak Flood Discharge (PFD) at a desired location on a river is important for planning, design and management of hydraulic structures. For ungauged catchments, rainfall depth becomes an important input in derivation of PFD. So, rainfall depth can be estimated through statistical analysis by fitting probability distribution to the rainfall data. In the present study, the series of annual 1-day maximum rainfall derived from the daily rainfall data recorded at Manjuhi Khad is used for estimation of 1-day extreme rainfall adopting Extreme Value Type-1 (EV1) distribution. Maximum Likelihood Method (MLM) is used for determination of parameters of the distribution. Anderson-Darling is applied for checking the adequacy of fitting of the distribution to the recorded data. The estimated 1-day extreme rainfall obtained from EV1 distribution is used to compute the 1-hour maximum value of distributed rainfall that is considered as an input to estimate the PFD by rational formula. The study suggests the estimated PFD could be used for design of flood protection measures for ungauged catchments of Bhul and Manjuhi Khad, Himachal Pradesh.

***Keywords**: Anderson-Darling Test, Extreme Value Type-1, Rainfall, Peak Flood Discharge, Maximum Likelihood Method*

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INTRODUCTION

Estimation of Peak Flood Discharge (PFD) at a desired location on a river is important for planning, design and management of hydraulic structures such as dams, bridges, barrages and design of storm water drainage systems. These include different types of flood such as standard project flood, probable maximum flood and design basis flood. In case of large river basins, the hydrological and stream flow series of a significant duration are generally available. However, for ungauged catchments, more data is not available other than rainfall. The rainfall data is also of shorter duration and may pertain to a neighbouring basin. Rainfall depth thus becomes an important input in derivation of PFD (Singh et al., 2001). For arriving at such design values, statistical analysis by fitting probability distribution to the rainfall data is carried out.

Out of a number of probability distributions, the Extreme Value Distributions (EVDs) include Generalised Extreme Value, Extreme Value Type-1 (Gumbel), Extreme Value Type-2 (Frechet) and Extreme Value Type-3 (Weibull) is generally adopted for EVA of rainfall (Casas et al., 2011). EVDs arise as limiting distributions for the sample of independent, identically distributed random variables, as the sample size increases. In the group of EVDs, Gumbel (EV1) distribution has no shape parameter as when compared to other distributions and this means that there is no change in the shape of Probability Distribution Function (PDF). Moreover, the EV1 distribution has the advantage of having only positive values, since the data series of rainfall are always positive (greater than zero); and therefore EV1 distribution is important in statistics. Lee et al. (2012) applied EV1 and Weibull probability distributions for estimation of extreme wind speed for Korea region. They have observed that the EV1 distribution gives better results than Weibull.

Daneshfaraz et al. (2013) carried out frequency analysis of wind speed adopting 2-parameter log-normal, truncated extreme value, truncated logistic and Weibull probability distributions and found that the truncated extreme value is the most appropriate distribution for Iran. Olumide et al. (2013) applied normal and EV1 distributions for prediction of rainfall and runoff at Tagwai dam site in Minna, Nigeria. They have also expressed that the normal distribution is better suited for rainfall prediction while Log-Gumbel for runoff. Rasel and Hossain (2015) applied EV1 distribution for development of intensity duration frequency curves for seven divisions in Bangladesh. In view of the above, EV1 distribution is used in the present study. Parameters of the EV1 distribution are determined by Maximum Likelihood Method (MLM) and used to estimate the 1-day extreme rainfall. For quantitative assessment on rainfall data within the recorded range, Anderson-Darling (A^2) is applied. The estimated 1-day extreme rainfall from EV1 distribution is used to estimate the PFD for ungauged catchments of Bhul and Manjuhi Khad. The methodology adopted in analysing the Annual 1-Day Maximum Rainfall (ADMR) using EV1 distribution, estimation of PFD using rational formula and GoF test are briefly described in the ensuing sections.

METHODOLOGY

The study is to estimate PFD for ungauged catchments of Bhul and Manjuhi Khad, Himachal Pradesh. Thus, it is required to process and validate the data for various application such as (i) estimate the 1-day extreme rainfall adopting EV1 distribution (using MLM); (ii) assess the adequacy of fitting of EV1 distribution to the series of ADMR using GoF test; (iii) compute the 1-hour maximum rainfall from the estimated 1-day extreme rainfall using CWC guidelines; (iv) compute the PFD by rational formula; and (v) analyze the results obtained thereof.

PDF and CDF of EV1 Distribution

The PDF and Cumulative Distribution Function (CDF) of EV1 distribution is given as below:

$$\left.\begin{aligned} \text{PDF}: f(R) &= \frac{e^{-(R-\alpha)/\beta} e^{-e^{-(R-\alpha)/\beta}}}{\beta} \\ \text{CDF}: F(R) &= e^{-e^{-(R-\alpha)/\beta}}, \beta > 0 \text{, where } (R = R_1, R_2, R_3, \ldots, R_N) \end{aligned}\right\} \quad \ldots (1)$$

where α and β are the location and scale parameters of the distribution (Gumbel, 1960). The parameters are computed by MLM through Equations (2) and (3), and used to estimate the rainfall (R_T) for different return periods from $R_T = \alpha + Y_T\beta$. Here, $Y_T = -\ln(-\ln(1-(1/T)))$ is called as a reduced variate for a given return period T (in year).

$$\alpha = -\beta \ln\left[\sum_{i=1}^{N} \exp(-R_i/\beta) \Big/ N\right] \quad \ldots (2)$$

$$\beta = \overline{R} - \left[\sum_{i=1}^{N} R_i \exp(-R_i/\beta) \Big/ \sum_{i=1}^{N} \exp(-R_i/\beta)\right] \quad \ldots (3)$$

$$SE(X_T) = (\beta/\sqrt{N})(1.15894 + 0.19187\, Y_T + 1.1 Y_T^2)^{0.5} \quad \ldots (4)$$

where R_i is the recorded ADMR of i^{th} sample and $\overline{R}$ is the average value of ADMR. The lower and upper confidence limits (LCL and UCL) of the estimated rainfall are obtained from LCL=ER-1.96(SE) and UCL=ER+1.96(SE). Here, ER is the Extreme Rainfall and SE the Standard Error

Goodness-of-Fit Test

GoF test is essential for checking the adequacy of probability distribution to the recorded series of ADMR. Out of a number GoF tests available, the widely accepted GoF test is A^2, which is used in the study. The theoretical description of A^2 test statistic (Charles Annis, 2009) is as follows:

A^2 statistic:

$$A^2 = (-N) - (1/N) \sum_{i=1}^{N} \{(2i-1)\ln(Z_i) + (2N+1-2i)\ln(1-Z_i)\} \quad \ldots (5)$$

Here, $Z_i = F(R_i)$ for i=1,2,3,…,N with $R_1 < R_2 < \ldots < R_N$, $F(R_i)$ is the CDF of i^{th} sample (R_i) and N is the sample size. If the computed value of GoF test statistic given by EV1 distribution is less than that of the theoretical value at the desired significance level then the distribution is considered to be acceptable for modelling the series of ADMR.

APPLICATION

In this paper, a study on estimation of PFD for different return periods for ungauged catchments of Bhul and Manjuhi Khad was carried out. The series of ADMR was extracted from the daily rainfall data recorded at Manjuhi Khad during the period 2009 to 2015 and used for estimation of 1-day extreme rainfall. The descriptive statistics such as average, standard deviation, coefficient of variation, coefficient of skewness and coefficient of kurtosis of the recorded ADMR was determined as 100.4 mm, 29.3 mm, 29.2%, 1.589 and 3.438 respectively. The estimated 1-day extreme rainfall obtained from EV1 distribution (using MLM) was used as an input to estimate the PFD.

RESULTS AND DISCUSSIONS

Statistical Analysis of Rainfall using EV1 Distribution

By applying the procedures of EV1 distribution, parameters were determined by MLM and used for estimation of 1-day extreme rainfall for different return periods. Table 1 gives the 1-day extreme rainfall estimates with confidence limits for different return periods adopting EV1 distribution. The observed and estimated ADMR are presented in Figure 1 along with confidence limits.

Table 1: Estimated 1-day extreme rainfall with lower and upper confidence limits

Return period (year)	1-day Extreme Rainfall (mm)	Standard Error (mm)	Confidence limits at 95% level	
			Lower	Upper
2	95.6	8.5	79.1	112.2
5	117.3	13.0	91.8	142.7
10	131.6	16.7	98.9	164.2
15	139.7	18.8	102.7	176.6
20	145.3	20.4	105.4	185.3
25	149.7	21.6	107.3	192.0
50	163.1	25.3	113.4	212.8
75	170.9	27.6	116.9	224.9
100	176.4	29.1	119.3	233.5

From Figure 1, it can be seen that the recorded ADMR data are within the confidence limits of the estimated 1-day extreme rainfall using EV1 distribution.

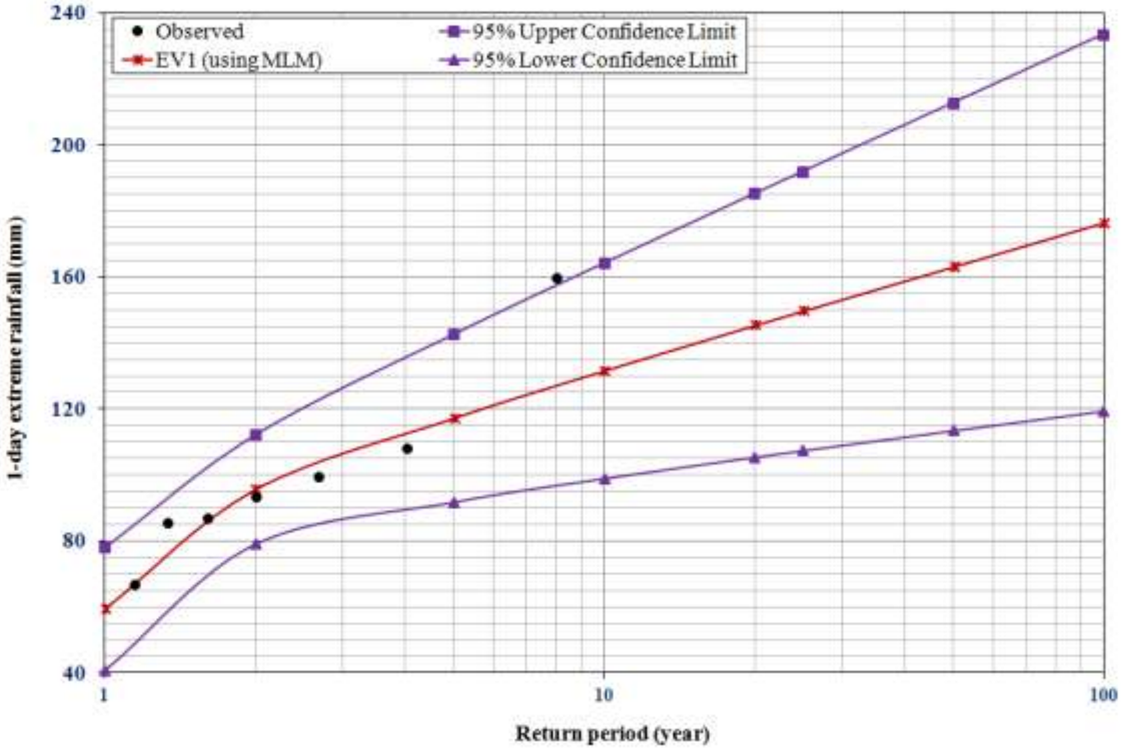

Figure 1: Plots of recorded and estimated 1-day extreme rainfall with confidence limits

Analysis Based on GoF Test

The adequacy of fitting of EV1 distribution for statistical analysis of rainfall was performed by adopting A^2 test, as described above. From the result, it was observed that the computed value of A^2 test statistic is 0.286, which is not greater than the theoretical value of 0.757 at 5% significance level, and at this level, the A^2 test confirmed the applicability of EV1 distribution for modelling the series of ADMR.

Estimation of Peak Flood Discharge using Rational Formula

Estimation of PFD for ungauged catchments in Bhul and Manjuhi Khad was required for the model studies on flood protection measures for the river or stream. The size of these catchments is presented in Table 2. These streams are ungauged and hence the PFD for these catchments was computed by using rational formula, which is given below:

$$q = 0.278 * C\,I\,A \quad \ldots (6)$$

where, q is peak discharge (m^3/s), C is runoff coefficient, I is rainfall intensity (mm/hour) and A is catchment area (km^2). By studying topography of the catchments using maps and other available literature, the value of the C was considered as 0.55 while computing the flood discharge. The time of concentration (t_c) estimated was 1-hour.

Table 2: Catchment area of different streams

Sl. No.	Name of Catchment	Area (km^2)
1	Chachiayan Nallah	45
2	Sidhpur Ghar Nallah	18
3	Darkati Nallah	35
4	Paloura Nallah	15
5	Dehri Khad	30
6	Kher Khad	14
7	Kandour Khad	25

In the absence of the short duration rainfall, say, 1-hour, 2-hour, 3-hour, etc., the same could be computed from estimated 1-day extreme rainfall (100-yr return period) by using distribution factor (Figure 2), as given in Central Water Commission Report entitled 'Flood estimation report for Western Himalayas-Zone 7' (CWC, 1994) in which the study area falls. The computed 1-hour maximum values of distributed rainfall from the 1-day extreme rainfall were presented in Table 3. In the present study, the distributed 1-hour rainfall was used as input for estimation of PFD. The estimated PFD for different return periods for ungauged catchments of Bhul and Manjuhi Khad presented in Table 4 could be taken as design flood for flood protection measures.

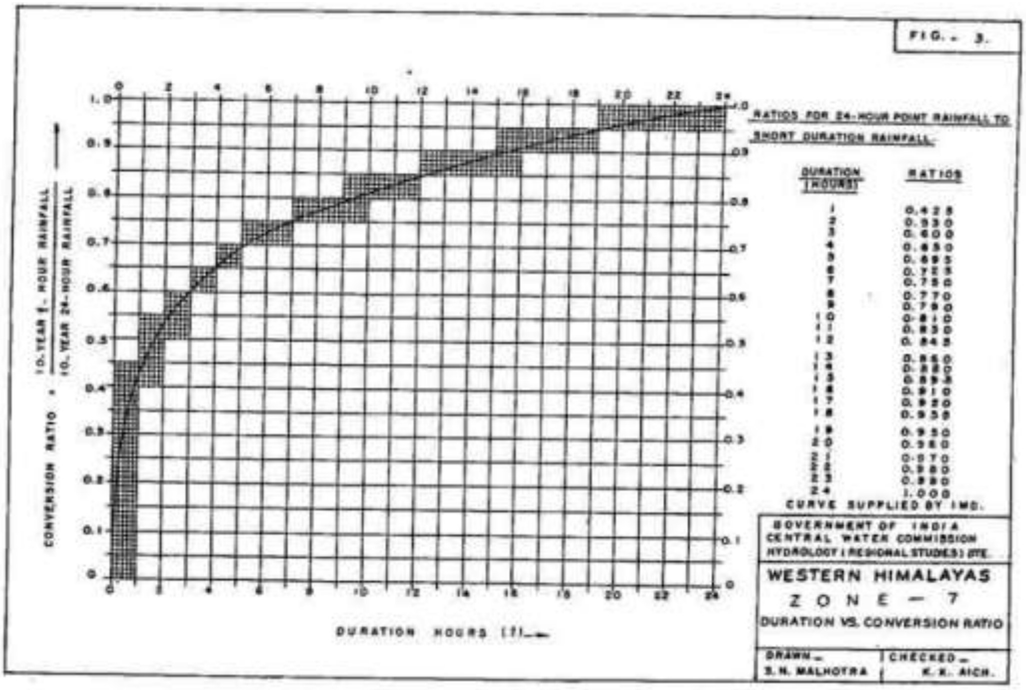

Figure 2: Conversion factor for computation of distributed rainfall for shorter duration

Table 3: Distributed rainfall for short durations

Return period (year)	Estimated 1-day extreme rainfall (mm)	1-hour maximum rainfall (mm)	Return period (year)	Estimated 1-day extreme rainfall (mm)	1-hour maximum rainfall (mm)
2	95.6	40.6	25	149.7	63.6
5	117.3	49.8	50	163.1	69.3
10	131.6	55.9	75	170.9	72.6
15	139.7	59.4	100	176.4	75.0
20	145.3	61.8			

Table 4: PFD (m^3/s) for ungauged catchments of Bhul and Manjuhi Khad

Name of the Catchment	Peak flood discharge (m^3/s) for different return periods (yr)								
	2-yr	5-yr	10-yr	15-yr	20-yr	25-yr	500-yr	75-yr	100-yr
Chachiayan Nallah	279.7	342.9	384.8	408.7	424.9	437.6	476.9	499.5	515.8
Sidhpur Ghar Nallah	111.9	137.2	153.9	163.5	170.0	175.1	190.7	199.8	206.3
Darkati Nallah	217.5	266.7	299.3	317.9	330.5	340.4	370.9	388.5	401.2
Paloura Nallah	93.2	114.3	128.3	136.2	141.6	145.9	159.0	166.5	171.9
Dehri Khad	186.4	228.6	256.5	272.5	283.3	291.8	317.9	333.0	343.9
Kher Khad	87.0	106.7	119.7	127.2	132.2	136.2	148.4	155.4	160.5
Kandour Khad	155.4	190.5	213.8	227.1	236.1	243.1	264.9	277.5	286.6

CONCLUSIONS

The paper described briefly the study carried out for statistical analysis of rainfall data adopting EV1 distribution and estimation of PFD for ungauged catchments of Bhul and Manjuhi Khad. The following conclusions were drawn from the study:

i) The A^2 test result confirmed the suitability of EV1 distribution (using MLM) for modelling the data series of ADMR.
ii) From Figure 1, it was observed that the recorded ADMR data are within the confidence limits of the estimated 1-day extreme rainfall.
iii) The estimated 1-day extreme rainfall was used to compute 1-hour maximum value of distributed rainfall adopting CWC guidelines described in Flood estimation report for Western Himalayas-Zone 7.
iv) By using the 1-hour distributed rainfall, the PFD for ungauged catchments of Bhul and Manjuhi Khad was computed by rational formula.
v) The study suggested that the PFD, as given in Table 4, may be considered for design of flood protection measures. However, considering the data length made available for the study, it was cautioned to use the PFD for return periods beyond 25-year because of uncertainty in the estimated values.

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REFERENCES

[1] Casas, M.C., Rodriguez, R., Prohom, M., Gazquez, A. and Redano, A., Estimation of the probable maximum precipitation in Barcelona (Spain), *Journal of Climatology*, 2011, Vol. 31, No. 9, pp. 1322–1327.
[2] Charles Annis, P.E., Goodness-of-Fit tests for statistical distributions, [http://www.statistical engineering. com/goodness.html], 2009.
[3] CWC, Flood estimation report for Western Himalayas-Zone 7, Central Water Commission Design Office Report No.: WH/22/1994, New Delhi, 1994.
[4] Daneshfaraz, R., Nemati, S., Asadi, H. and Menazadeh, M., Comparison of four distributions for frequency analysis of wind speed: A case study, *Journal of Civil Engineering and Urbanism*, 2013, Vol.3, No.1, pp. 6-11.
[5] Gumbel, E.J., Statistic of Extremes, 2nd edition, Columbia Univ. Press, New York, 1960.
[6] Lee, B.H., Ahn, D.J., Kim, H.G. and Ha, Y.C., An estimation of the extreme wind speed using the Korea wind map, *Renewable Energy*, 2012, Vol. 42, No. 1, pp. 4–10.
[7] Olumide, B.A., Saidu, M. and Oluwasesan, A., Evaluation of best fit probability distribution models for the prediction of rainfall and runoff volume (Case study: Tagwai Dam, Minna-Nigeria), *Journal of Engineering and Technology*, 2013, Vo1. 3, No. 2, pp.94-98.
[8] Rasel, M.M. and Hossain, S.M., Development of rainfall intensity duration frequency equations and curves for seven divisions in Bangladesh, *International Journal of Scientific and Engineering Research*, 2015, Vol. 6, No. 5, pp. 96-101.
[9] Singh, R.D., Mishra, S.K. and Chowdhary, H., Regional flow duration models for 1200 ungauged Himalayan watersheds for planning micro-hydro projects, *ASCE Journal of Hydrologic Engineering*, 2001, Vol. 6, No. 4, pp. 310-316.