ROTOR POSITION TRACKING AND SPEED CONTROL OF PMSM AUGMENTED WITH NONLINEAR ESTIMATOR UNDER MICROSTEPPING EXCITATION

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Abstract

Stepper motors are the perfect tools where accurate locomotion, movement and positioning are needed. The major applications of permanent magnet stepper motors are in scanners, robotic controls and most widely, these days, in 3D-printers. The accurate rotor position tracking of permanent magnet stepper motor (PMSM) has hence become a very important task. This paper approaches this issue from the process control point of view. Consequently, controlling the rotor position requires complete state information for investigation. The rotor position tracking and speed control of PMSM is advanced from the perspective of Lyapunov-based control method. The nonlinear estimation technique employs a nonlinear observer along with Lyapunov-controller to estimate accurately the rotor position through the information from motor phase current and angular velocity. The investigation through simulation demonstrates the effectiveness of the estimation technique, as the improved position tracking is achieved upon using Lyapunov-controller in comparison with the current control method under microstepping excitation of the motor.

Keywords: *PMSM, Nonlinear Estimation - Lyapunov Controller, Microstepping, Stepper Motor, Robotics*

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1. INTRODUCTION

According to the robotiksistem, a stepper motor (or step motor) is a brushless DC electric motor that can divide a full rotation into a large number of steps. The shaft or spindle of a stepper motor rotates in discrete step increments when electrical command pulses are applied to it in the proper sequence, and the motor's position can be controlled precisely without any feedback mechanism (an open-loop controller), as long as the motor is carefully sized to the application [1]. Stepper motors are formed by coils and magnets and incorporate a shaft that moves when power is applied. Since permanent magnet stepper motors can rotate in small step angles and have high precision, they are widely used in bio-medical applications, robotics, computer peripherals and automotive drive actuators. In general, a stepper motor provides discrete movement by angular displacement with successive manner of equal displacement. The final position of the rotor is given by the total angular displacement which results the number of steps achieved. Stepper motor provides precision and high resolution of positioning which is the primary requirement of robotic automations and controls [2]. The operational efficiency is improved on employing various excitation schemes such as full stepping, half stepping and micro stepping, as discussed by Bellini et al [3]. For high performance stepper motors, nonlinear state feedback control is employed with DQ transformation.

The stepper motor speed can be estimated using motor back emf voltage and observer under certain operating conditions with damping control technique [4]. A number of control schemes were developed for rotor position and speed control of stepper motor ([5]-[9]). The present work, Lyapunov-based control scheme is developed ensuring asymptotic stability via better convergence of position tracking error. In general, position estimation based on current measurement is inefficient under mid-frequency operating conditions [10] and hence, Lyapunov-based control technique is employed. The investigation of simulation results confirms that nonlinear observer with Lyapunov controller performs efficiently under micro stepping condition without performing DQ transformation of the PMSM model.

2. MATHEMATICAL MODEL OF PMSM

The electromechanical dynamic equations of the PMSM can be represented in the state space form, [2] and [3]

$\dot{X} = AX + BU, Y = CX + DU$, as follows,

$$\dot{\theta} = \omega$$

$$\dot{\omega} = [-K_m i_a sin(N_r\theta) + K_m i_b cos(N_r\theta) - B\omega]/J \quad (1)$$

$$\dot{i_a} = [v_a - Ri_a + K_m\omega \sin(N_r\theta)]/L$$

$$\dot{i_b} = [v_b - Ri_b - K_m\omega \cos(N_r\theta)]/L$$

Where $X = [\theta, \omega, i_a, i_b]^T$ is the state variable
$U = [v_a, v_b]^T$ is the input variable
v_a, v_b and i_a, i_b are the voltages (in Volts) and currents (in Amperes) of the two phases respectively
θ is the rotor angular position of the motor in rad
ω is the rotor angular velocity in rad/s
B is the viscous friction coefficient in [N.m.s/rad]
J is the inertia of the motor in [Kg.m2]
Km is the motor torque constant in [N.m/A]
R is the resistance of the phase winding in [Ω]
L is the inductance of the phase winding [H]
Nr is the number of rotor teeth.

The detent torque and magnetic coupling are assumed to be zero. The model also ignores the variation in inductance due to magnetic saturation.

3. MICROSTEPPING

The three common modes of excitation are full step, half step and microstepping. Microstepping is preferred over the previous two for its effectiveness in improving low speed smoothness and minimizes low speed resonance effects. Since the stepper motors move step by step, their smoothness is low. Therefore, an input which sends the steps so quickly that it results in the fluid rotation of the motor is required. This can be achieved by microstepping which is essentially a series of step inputs sent immediately one after the other within one complete cycle. It can have 16 or more steps within one cycle.

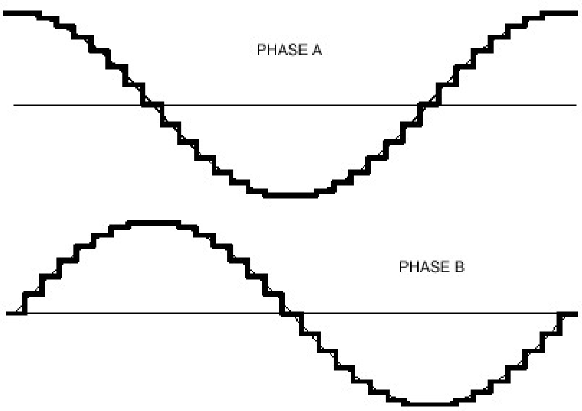


Fig - 1: Stepping sequence of microstepping

If the number of steps per cycle are increased, then it almost results in a sine wave (as can be seen from Figure 1). Since there are two phases, the inputs are designed to be two sine waves which are 90^0 phase shifted from one another. Hence, the inputs can be designed as follows:

$$V_a = V_{max} \cos(N_r \theta)$$

$$V_b = V_{max} \sin(N_r \theta) \quad \text{.........................(2)}$$

Where V_{max} is the maximum amplitude of the microstepping input voltage.

To achieve the desired position θ^d the inputs are modified as follows:

$$V_a^{\ d} = V_{max} \cos(N_r \theta^d) \text{ and } \quad \text{.........................(3)}$$

$$V_b^{\ d} = V_{max} \sin(N_r \theta^d)$$

Where $V_a^{\ d}$ and $V_b^{\ d}$ are the desired input voltages in phases a and b respectively.

This ensures that the states of PMSM finally converges to $[\theta^d, 0, i_a^d, i_b^d]$

Or,

$$\lim_{t\to\infty} \theta(t) = \theta^d, \lim_{t\to\infty} \omega(t) = 0, \lim_{t\to\infty} i_a(t) = i_a^{\ d}, \lim_{t\to\infty} i_b(t) = i_b^{\ d} \quad \text{.........................(4)}$$

Where $i_a^{\ d}$ and $i_b^{\ d}$ are the desired input currents in phases a and b respectively.

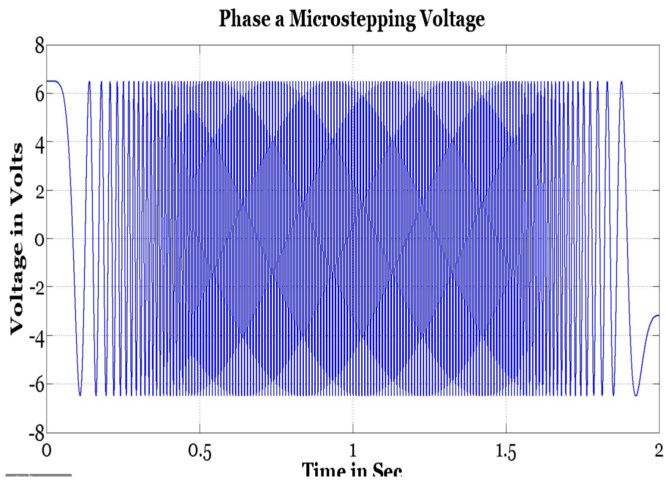


Fig - 2: Microstepping voltages corresponding to reference position (Phase a)

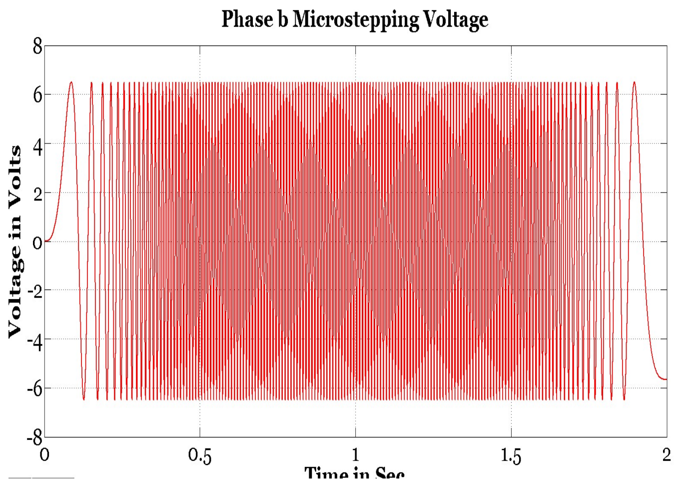


Fig - 3: Microstepping voltages corresponding to reference position (Phase b)

The Figures 2 and 3 represent the waveforms of $V_a^{\ d}$ and $V_b^{\ d}$ corresponding to the reference position discussed later.

4. CONTROLLER DESIGN

The proposed controller is designed based on the Lyapunov stability criterion as the objective is to build robust and adaptive controllers. In simple notations, the theory states that for any system, if the equilibrium point is given by s_e, then the system is said to be stable if its solutions of the system s(t) begin at s_e and stay around and at s_e for all values of t.

Furthermore, the system is said to be asymptotically stable if the solutions are initially at S_e, remain around S_e and converge at the last to S_e. This paper deals with this aspect of stability.

The Lyapunov conditions for asymptotic stability of a system states that the derivative of the function V(s) must be strictly less than zero, where V(s) is a scalar function continuous in the small region around origin in state space. [11]

$$\dot{V}(s) < 0, s \neq 0 \quad \text{……………………(5)}$$

From the previous section, it can be inferred that $i_a(t)$ and $i_b(t)$ converges to $\frac{V_a{}^d}{R}$ and $\frac{V_b{}^d}{R}$ respectively. Hence,

$$i_a{}^d = \frac{V_a{}^d}{R} \quad \text{………………….. (6)}$$

$$i_b{}^d = \frac{V_b{}^d}{R}$$

The desired currents can thus be written as, from (3) and (5)

$$i_a{}^d = \frac{V_{max}}{R}\cos(N_r\theta^d)\ and \quad \text{…………………. (7)}$$

$$i_b{}^d = \frac{V_{max}}{R}\sin(N_r\theta^d)$$

The errors can consequently be declared as

$$e_a = i_a{}^d - i_a \text{ and} \quad \text{…………………(8)}$$

$$e_b = i_b{}^d - i_b$$

By Lyapunov's direct method, a suitable Lyapunov candidate function would be

$$V_1 = \frac{1}{2}e_a{}^2 + \frac{1}{2}e_b{}^2 \quad \text{………………….(9)}$$

Which is a positive definite scalar function.

The time derivative of V1 along any trajectory is given by,

$$\dot{V_1} = \frac{dV(e_a, e_b)}{dt}$$

$$= \frac{1}{2}.2.e_a.\dot{e_a} + \frac{1}{2}.2.e_b.\dot{e_b}$$

$$= e_a(\dot{i}_a^d - \dot{i_a}) + e_b(\dot{i}_b^d - \dot{i_b}) \quad \text{……………….. (10)}$$

$$= e_a\left(\dot{i}_a^d - \frac{[v_a - Ri_a + K_m\omega\sin(N_r\theta)]}{L} +\right)$$

$$e_b\left(\dot{i}_b^d - [v_b - Ri_b - K_m\omega\cos(N_r\theta)]/L\right)$$

The function V1 is asymptotically stable if

$$\dot{V_1}(e_a, e_b) < 0, e_a, e_b \neq 0$$

i.e., $\dot{V_1}(e_a, e_b)$ is a negative definite function.

Hence the control inputs va and vb are designed such that $\dot{V_1}(e_a, e_b)$ becomes negative definite.

$$v_a = (Ri_a - K_m\omega\sin(N_r\theta)) + L(\dot{i}_a^d + \rho e_a) \quad \text{……………………….. (11)}$$

$$v_b = (Ri_b + K_m\omega\cos(N_r\theta)) + L(\dot{i}_b^d + \rho e_b)$$

Where ρ is any positive control gain.

The equation (9) therefore becomes,

$$\dot{V_1} = -\rho e_a{}^2 - \rho e_b{}^2 \quad \text{…………………… (12)}$$

This ensures that $\dot{V_1}$ is always less than zero and the errors converge to zero as$t \to \infty$.

5. OBSERVER DESIGN

The PMSM model with the above mentioned controller inputs assume that all the states are observable. Let us assume that only an optical encoder is available which is used to observe the position of the motor. Hence, we need an observer which can estimate the other unknown states. The estimated value dynamics can be written as follows:

$$\dot{\hat{\theta}} = \hat{\omega} + b1(\theta - \hat{\theta})$$

$$\dot{\hat{\omega}} = [-K_m\hat{i_a}sin(N_r\theta) + K_m\hat{i_b}cos(N_r\theta) - B\hat{\omega}]/J + b2(\theta - \hat{\theta}) \quad \text{……………………. (13)}$$

$$\dot{\hat{i_a}} = \frac{[v_a - R\hat{i_a} + K_m\hat{\omega}\sin(N_r\theta)]}{L} + b3(\theta - \hat{\theta})$$

$$\dot{\hat{i_b}} = \frac{[v_b - R\hat{i_b} - K_m\hat{\omega}\cos(N_r\theta)]}{L} + b4(\theta - \hat{\theta})$$

Where b1, b2, b3 and b4 are observer gains.

The error between the actual and the estimated values is thus given by,

$$\breve{\theta} = \theta - \hat{\theta}$$

$$\breve{\omega} = \omega - \hat{\omega} \quad \text{………. (14)}$$

$\breve{\iota_a} = i_a - \hat{\iota_a}$

$\breve{\iota_b} = i_b - \hat{\iota_b}$

Substituting the above values in (12), the equation (12) can thus be rewritten in terms of error dynamics as follows:

$$\dot{\breve{\theta}} = \hat{\omega} - b1\breve{\theta}$$

$$\dot{\breve{\omega}} = [-K_m\hat{\iota_a}sin(N_r\theta) + K_m\hat{\iota_b}cos(N_r\theta) - B\hat{\omega}]/J - b2\breve{\theta}$$

$$\dot{\breve{\iota_a}} = \frac{[v_a - R\hat{\iota_a} + K_m\hat{\omega}\sin(N_r\theta)]}{L} - b3\breve{\theta} \quad \text{…………………..} (15)$$

$$\dot{\breve{\iota_b}} = \frac{[v_b - R\hat{\iota_b} - K_m\hat{\omega}\cos(N_r\theta)]}{L} - b4\breve{\theta}$$

To prove the stability of the above dynamics, consider the Lyapunov candidate function,

$$V_2 = \frac{1}{2}\breve{\theta}^2 + \frac{1}{2}\breve{\omega}^2 + \frac{1}{2}\breve{\iota_a}^2 + \frac{1}{2}\breve{\iota_b}^2 \quad \text{………..} (16)$$

According to Lyapunov's direct method, the error converges to zero if

$$\dot{V_2} < 0$$

i.e. $\breve{\theta}.\dot{\breve{\theta}} + \breve{\omega}.\dot{\breve{\omega}} + \breve{\iota_a}.\dot{\breve{\iota_a}} + \breve{\iota_b}.\dot{\breve{\iota_b}} < 0$

Or, $\breve{\theta}(\breve{\omega} - b1\breve{\theta}) + \breve{\omega}\left(\frac{[-K_m\breve{\iota_a}sin(N_r\theta) + K_m\breve{\iota_b}cos(N_r\theta) - B\breve{\omega}]}{J} - b2\breve{\theta}\right) + \breve{\iota_a}(\frac{[v_a - R\breve{\iota_a} + K_m\breve{\omega}\sin(N_r\theta)]}{L} - b3\breve{\theta}) + \breve{\iota_b}(\frac{[v_b - R\breve{\iota_a} - K_m\breve{\omega}\cos(N_r\theta)]}{L} - b4\breve{\theta}) < 0$

Or, $-b1\breve{\theta}^2 - \frac{B}{J}\breve{\omega}^2 - \frac{R}{L}{i_a}^2 - \frac{R}{L}{i_b}^2 + \breve{\theta}(\breve{\omega} - b2\breve{\omega} - b3\breve{\iota_a} - b4\breve{\iota_b}) < 0$ ………………(17)

Hence, the gains are chosen so as to satisfy the condition for negative definite; so b1 can be chosen as a positive value, b2 as 1, b3 and b4 as zero. When these values are substituted, we get

$$-b1\breve{\theta}^2 - \frac{B}{J}\breve{\omega}^2 - \frac{R}{L}\breve{\iota_a}^2 - \frac{R}{L}\breve{\iota_b}^2 < 0 \quad \text{………………} (18)$$

This confirms that the error converges to zero,

i.e. $\lim_{t\to\infty}\hat{\theta}(t) = \theta$ li $_{t\to\infty}\hat{\omega}(t) = \omega$ $\lim_{t\to\infty}\hat{\iota_a}(t) = i_a$ $\lim_{t\to\infty}\hat{\iota_b}(t) = i_b$ ………………… (19)

Henceforth, the control inputs V_a, V_b are modified again to suit the Lyapunov controller augmented with the observer.

$$v_a = (Ri_a - K_m\omega\sin(N_r\theta)) + L(\dot{\iota}_a^d + \rho e_a) + (-R\breve{\iota_a} + K_m\breve{\omega}\sin(N_r\theta) + \rho L\breve{\iota_a})$$

$$v_b = (Ri_b + K_m\omega\cos(N_r\theta)) + L(\dot{\iota}_b^d + \rho e_b) + (-R\breve{\iota_b} - K_m\breve{\omega}\cos(N_r\theta + \rho L\breve{\iota_b}) \quad \text{…………………} (20)$$

The above equation guarantees that the Lyapunov candidate function is negative definite and that the error converges to zero.

6. PI CONTROLLER

A proportional-integral controller is a control loop feedback mechanism commonly used in industrial control systems. A PI controller basically takes in the error signal (difference between process variable and set point) as the input. The proportional part amplifies the error by a certain factor (Kp) and the integral portion accumulates the error of the past (Ki).This is the most conventional method of controlling and is the most widely used method until date. The performance analysis of both the conventional PI and the proposed Lyapunov controller and observer has been compared as shown in Table 1. The latter has proved to be better than the former.

Table - 1: Comparison between PID and Lyapunov Controller

Parameter	PID Controller	Lyapunov controller
ISE	6*10-4	1.52*10-4
IAE	6.25*10-4	3.25*10-5
ITAE	6.6*10-4	4*10-5

The formulae for calculating the above parameters are as follows:

$$\mathbf{IntegralStandardError} = \int \mathbf{e^2dt}$$

$$\mathbf{IntegralAbsoluteError} = \int \mathbf{edt}$$

$$\mathbf{IntegralTimeAbsoluteError} = \int \mathbf{etdt}$$

Where e is the error signal

7. RESULTS AND DISCUSSION

The Lyapunov and PID controllers were designed using MATLAB SIMULINK Software. The reference trajectory was designed using the following equations: [14]

$$\omega_{ref} = \begin{cases} k1t^2 + k2t^3 & 0 \le t \le t_1 \\ \omega_{max} & t_1 \le t \le t_2 \\ k1(t_3 - t)^2 + k2(t_3 - t)^3 & t_2 \le t \le t_3 \end{cases}$$

Where, $k1 = +3\omega_{max}/{t_1}^2$ **and** $k2 = -2\omega_{max}/{t_1}^3$

ω_{max} is the maximum angular velocity

Hence, for the desired angular velocity, the corresponding constants are calculated using the above formulae. The reference trajectory was given as inputs to both the

Lyapunov and PID controller. For the Lyapunov controller, as the gain value of ρ increases, the error keeps decreasing. However, the amplitude of input voltage is also decreased simultaneously. Since the input voltages must be high enough to compensate for the back emf and phase lags, a trade-off must be done between the two quantities and the optimal value of ρ has been calculated to be **40000** by trial and error method. The PID controller was tuned by **ZN-II** method and the **Kp** and **Ki** values were found to be **1000** and **2** respectively. The motor parameters that were used for simulation are shown in Table 2. The tracking performances of both the controllers have been shown in the graph. Also, their tracking errors have been plotted and shown. The performance of the Lyapunov controller along with the non-linear observer have also been analyzed and the actual and estimated values are plotted and compared.

Table - 2: Motor and Controller Parameters

Parameter	Value	Parameter	Value
J	8 x 10-5 kg.m2	N	50
Km	0.51 N.m/A	L	40 mH
R	14.8Ω	V_{lim}	6.5V
F	0.006 N.m.s/rad	ρ	40000

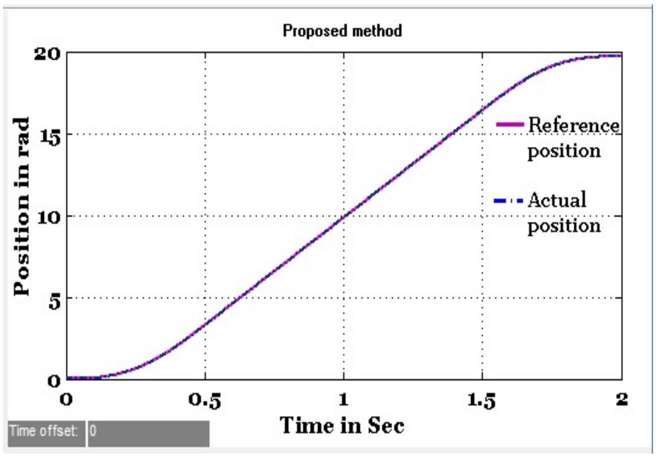
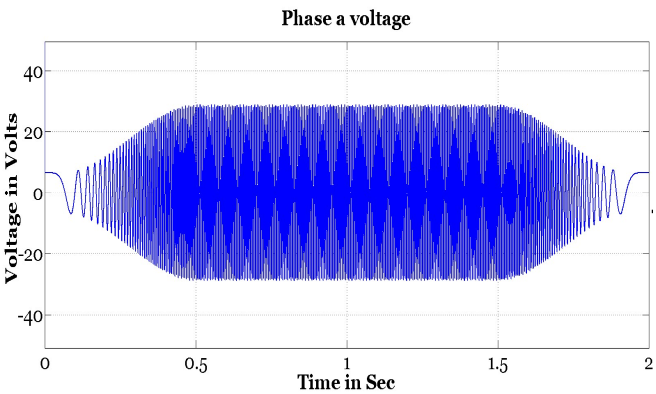


Fig - 4: Position tracking of proposed method corresponding to angular velocity 13.125 rad/s



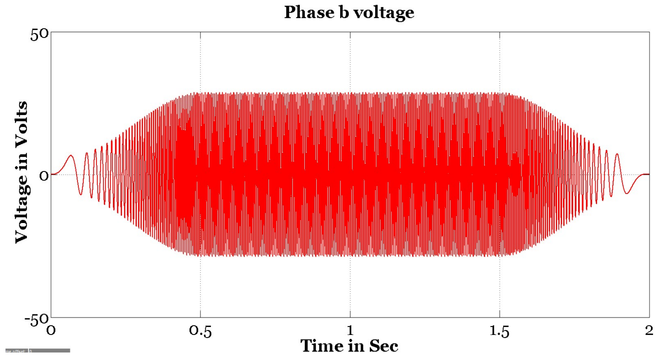


Fig - 5: Phase voltages of the proposed method

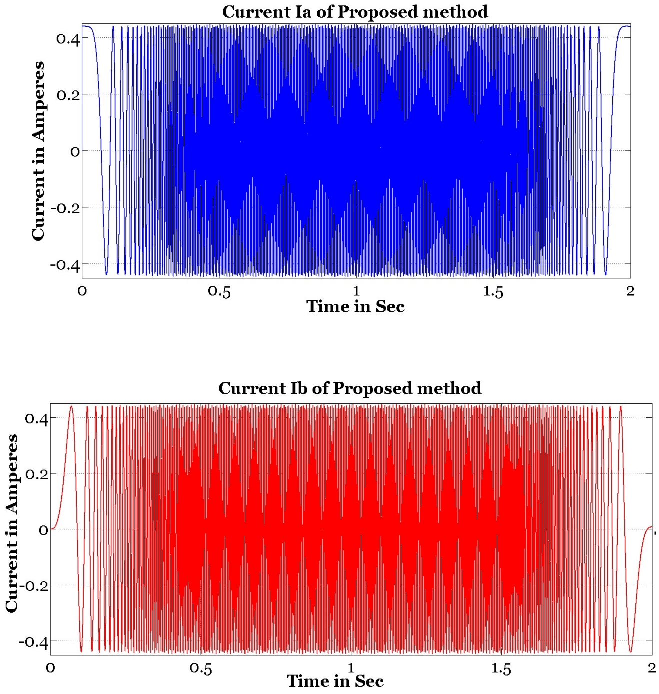


Fig - 6: Phase currents of the proposed method

The above graphs show the various parameters of the motor when it is subjected to the proposed method. The reference (desired) position and the actual position of the motor are compared in Figure 4. It can be seen that the actual position almost tracks the desired trajectory. Figure 5 represent the control inputs V_a and V_b designed by Lyapunov method and Figure 6 represents the actual motor phase currents I_a and I_b respectively.

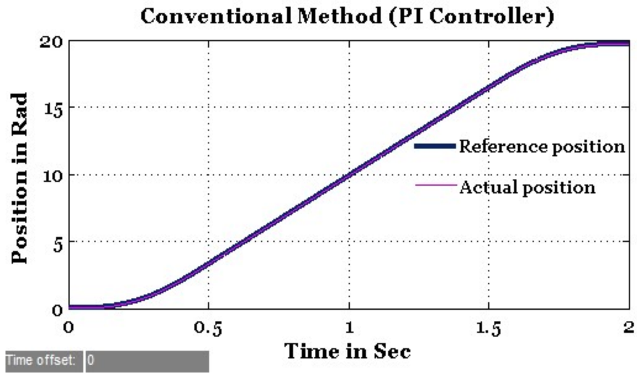


Fig - 7: Position tracking of PI Controller

Figure 7 shows the comparison of the rotor position of the motor controlled by PI controller and the desired position. From Figure 8, it can be seen that by employing the suggested method, the error in position tracking is almost halved.

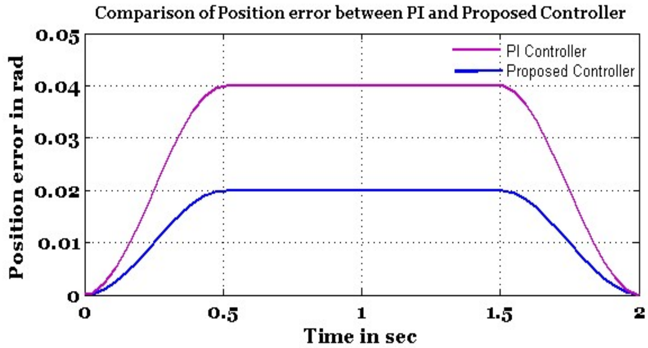


Fig - 8: Comparison of Position tracking error of PI and proposed Controller

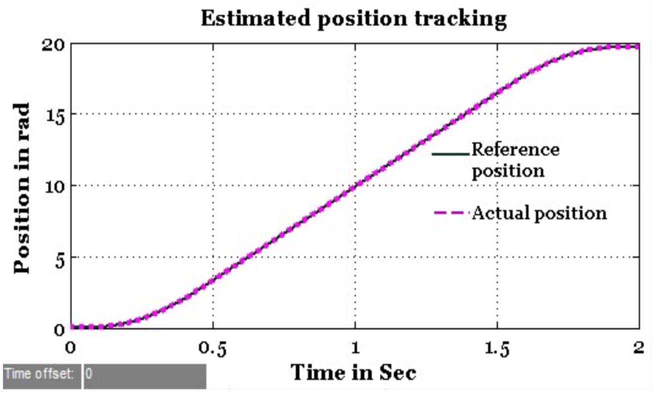


Fig - 9: Estimated position tracking of the observer

Figure 9 shows the comparison of the actual position of the motor (reference position) and the estimated value of the position using the nonlinear observer (actual position).

8. CONCLUSION

The simulation experiments analyzing Lyapunov control and the conventional methods of controlling have thus been carried out and the reference trajectories have been tracked by both the controllers. The results have verified that the proposed control scheme has proven to be more efficient and has much lesser error than the conventional (PI) controller. Also, a non-linear observer was designed and proved stable under Lyapunov stability conditions. The errors of the observer (difference between the actual and estimated value) have guaranteed to converge to zero according to Lyapunov stability conditions.

Stepper motors play an important role in major fields such as in robotics and in biomedical instrumentation. In robotics, the robotic arm, which plays an important role in the field, employs stepper motor in it if it has to perform tasks like spreading, fetching, delivering etc. In biomedical, stepper motors may be used to drive laser beams in a blood circulation scanner for example. It is also used in many other fields such as for positioning the satellite system so that they can be controlled remotely. In all the above examples, one can easily understand the importance of accurate positioning of the stepper motors and that is why, even though they can achieve precise positioning without any feedback mechanism, such advancements are made in this control field so that a high level of accuracy is maintained.

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BIOGRAPHIES

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