MODAL ANALYSIS OF A SIMPLY SUPPORTED SANDWICH BEAM

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Abstract

Every mechanical structure exhibits natural modes of vibration. Beams with variable cross section and material properties are frequently used in aeronautical, mechanical and civil engineering. Given the elastic and inertia characteristics of the structures, modes of vibration can be computed, the study being known as modal analysis. This paper presents modal analysis of simply supported beam using different materials. Comparison of natural frequency of the beam considering various materials is done analytically and also using ANSYS APDL. Effect of change of length and cross sectional area on natural frequency is also studied. Comparative study on natural frequency of sandwich beam using various materials is done analytically and also using ANSYS APDL.

Key words: *Natural Frequency, Mode Shape, Sandwich Beam.*

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1. INTRODUCTION

Vibration is mechanical phenomenon whereby oscillations occur about the equilibrium point. The oscillations may be periodic such as the motion of pendulum or random such as the motion of tire on gravel road. Every structure which is designed is subjected to some amount of vibrations. Unwanted vibrations may cause loosening of parts and cause accidents or heavy loss.[1] Mostly all materials exhibit some amount of internal structural damping. Most of the time it is not substantially effective to minimize the vibration around resonant frequencies.[3].Due to faulty design and poor manufacturing there is unbalanced and unpleasant stresses developed and creating unwanted noise. Careful designing usually minimize unwanted vibrations. Hence keeping in view all useful and devastating effects of the vibrations the study of the vibration is of immense importance. [1].

2. THEORITICAL ASPECTS

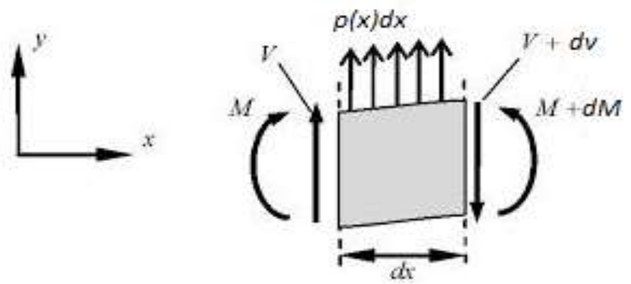


Fig-1: Forces and moments acting on beam element

To determine the differential equation for the lateral vibrations of beams, consider the forces and moments acting on an element of the beam shown in figure 1

V and M are the shear and bending moments respectively, and p(x) represents the loading per unit length of the beam. The equation for the lateral vibration of the beam reduces to

$$\frac{d^2}{dx^2}\left(EI\left(\frac{d^2y}{dx^2}\right)\right) - \rho\,\omega^2 y = 0$$

In the special case where the flexure rigidity EI is a constant, the preceding equation can be written as :

$$EI\left(\frac{d^4y}{dx^4}\right) - \rho\,\omega^2 y = 0$$

On substituting

$\beta^4 = \rho\,\omega^2 / EI$

We obtain the fourth order equation

$\left(\frac{d^4y}{dx^4}\right) - \beta 4y = 0$ for the vibration of uniform beam

The natural frequencies of vibration are found from equation (3) to be

$$\omega_n = \beta_n{}^2 \sqrt{\frac{EI}{\rho}} = (\beta_n l)^2 \sqrt{\frac{EI}{\rho A l^4}}$$

Where, the constant β_n depends on the boundary conditions of the problem.

2.1 Derivation to find β_n value:

Considering general equation

Y= Acosβx + Bsinhβx + Ccosβx + Dsinβx

$Y=A\left(\frac{e^{\beta x}+e^{-\beta x}}{2}\right) + B\left(\frac{e^{\beta x}-e^{-\beta x}}{2}\right) + C\cos\beta x + D\sin\beta x$

$y'=A\beta\left(\frac{e^{\beta x}-e^{-\beta x}}{2}\right) + B\beta\left(\frac{e^{\beta x}+e^{-\beta x}}{2}\right) - C\beta\sin\beta x + D\beta\cos\beta x$

$y''= A\beta^2\left(\frac{e^{\beta x}+e^{-\beta x}}{2}\right) + B\beta^2\left(\frac{e^{\beta x}-e^{-\beta x}}{2}\right) - C\beta^2\sin\beta x - D\beta^2\cos\beta x$

Therefore

Y=Acoshβx + Bsinhβx + Ccosβx + Dsinβx -----------------(1)

y'=Aβsinhβx + Bβcoshβx - Cβsinβx + Dβcosβx --------- (2)

$y''=A\beta^2\cosh\beta x + B\beta^2\sinh\beta x - C\beta^2\sin\beta x - D\beta^2\cos\beta x$------ (3)

Boundary conditions for simply supported beam

Y=0 x=0

X=l y=0

X=l $\frac{d2y}{dx2}=0$

Now

Y=0 x=0

Substitute in 1

We get 0=A+C

Now,

X=0 $\frac{d2y}{dx2}=0$

0=A-C

A=C=0

Substitute this in 1

Y=Bsinhβx + Dsinβx ---------------------------------(4)

$y''= B\beta^2\sinh\beta x - D\beta^2\cos\beta x$ ------------------------(5)

Now at x=l y=0 and $y''=0$

We get 0= Bsinhβl + Dsinβl --------------------(6)

And substitute in y''

0= $B\beta^2\sinh\beta l - D\beta^2\sin\beta l$

0= Bsinhβl – Dsinβl ---------------------------------(7)

From 6[th] and 7[th] equatioin we have

Bsinhβl=Dsinβl=0

D≠0 therefore βl=0

Therefore $\beta=0,\frac{\pi}{l},\frac{2\pi}{l},\frac{3\pi}{l}$

Since βl≠0, sinh βl≠0, β=0

Also Dsinβl=0 since D≠0 otherwise y=0 for all x, sinβl=0. Hence y= Bsinhβx and the solutions to sin βl=0 gives the natural frequency

They are,

$\beta=0,\frac{\pi}{l},\frac{2\pi}{l},\frac{3\pi}{l}$,…….

Therefore

The βn values are calculated and shown in Table 1

Table 1: βn values

Beam configuration	**$(B_1l)^2$ Fundamental**	**$(B_2l)^2$ Second mode**	**(B_3l) Third mode**
Simply supported	9.87	39.5	88.9
Cantilever	3.52	22.0	61.7
Free -Free	22.4	61.7	121.0
Clamped-Clamped	22.4	61.7	121.0
Clamped Hinged	15.4	50.0	104.0
Hinged Free	0	15.4	50.0

3. MATERIAL DETAILS

The beam is analysed considering various materials such as steel aluminium and carbon FRP. The material properties are shown in Table 2 below.

Table 2: Material Properties

Name of material	Young modulus (GPa)	Density (Kg/m^3)
STEEL	200	7850
ALLUMINIUM	70	2700
CARBON FRP	150	1800

3.1 Beam Dimension

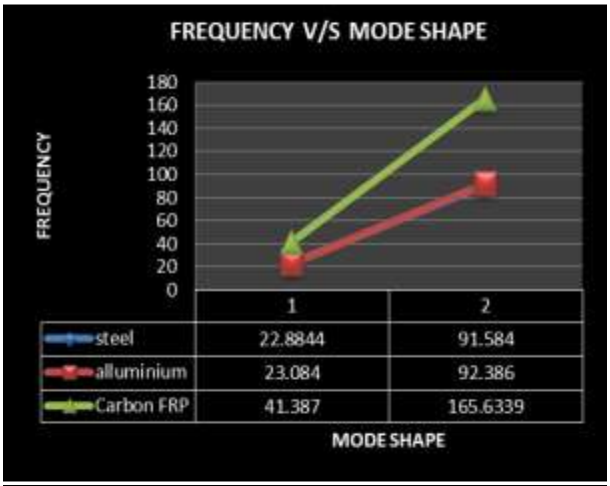
The various cross sections of the beam considered are 10mm X 10mm, 20mmX 20mm, 30X30mm and 40mmX40mm for vaious length varying from 1m to 2m. Considering the cross section constant the natural frequencies are found for varying length theoretically and then compared with Ansys APDL The values for first and second mode shapes for the different materials used considering cross section as 10mmX10mm are shown in Table 3,Table 4 and Table 5 respectively.

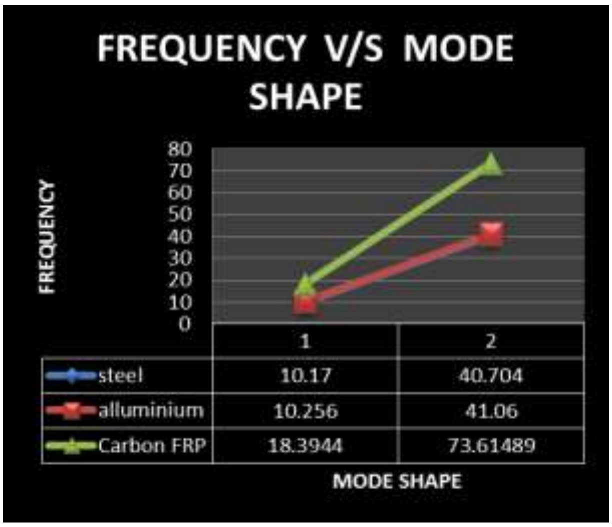
Table 3. Frequency values

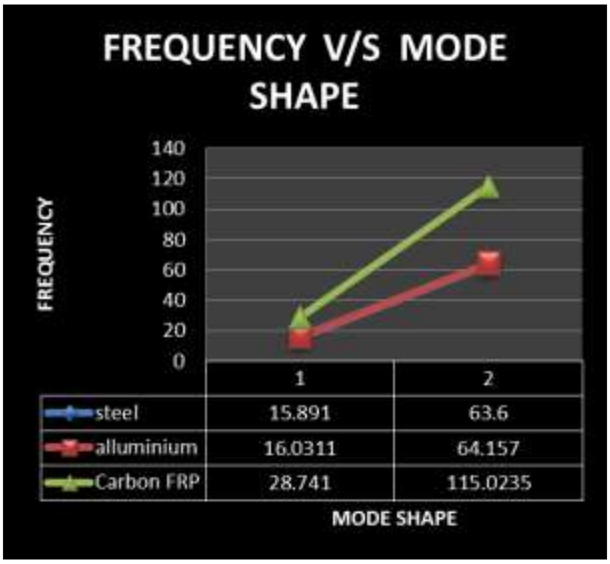
material	1 M LENGTH		1.2 M LENGTH	
	mode1	mode 2	Mode1	Mode 2
STEEL	22.88	91.584	15.89	63.6
ALLUMINIUM	23.08	92.386	16.01	64.157
CARBON FRP	41.3	165.6	28.74	115.023

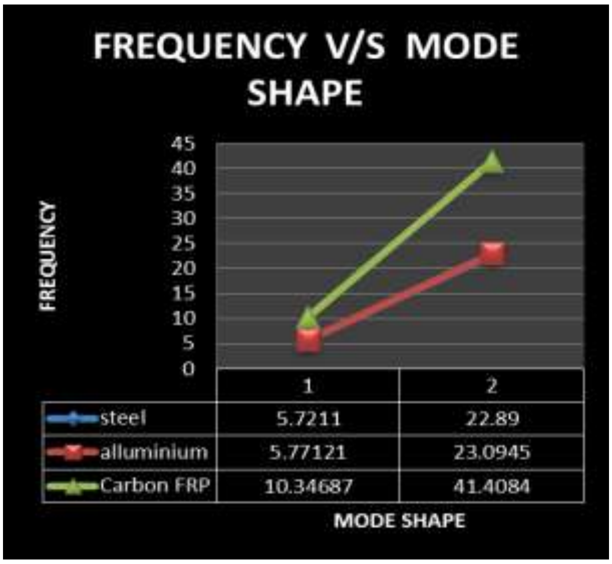
Table 4. Frequency values

material	1.5 M LENGTH		2 M LENGTH	
	mode1	mode 2	Mode1	Mode 2
STEEL	10.17	40.704	5.7221	22.89
ALLUMINIUM	10.256	41.06	5.771	23.094
CARBON FRP	18.394	73.648	10.346	41.4084









The theorical values were then compared with Ansys APDL as shown in Table 5 and Table 6

Table 5. Frequency values for steel

Material(Steel)	1 M LENGTH		1.2 M LENGTH	
	ANSYS	THEO	ANSYS	THEO
Mode 1	22.885	22.88	15.893	15.891
Mode 2	91.512	91.58	63.563	63.60

Table 6. Frequency values for steel

Material(Steel)	1.5 M LENGTH		2 M LENGTH	
	ANSYS	THEO	ANSYS	THEO
Mode 1	10.172	10.17	5.722	5.721
Mode 2	40.687	40.704	22.889	22.890

Table 7. Frequency values for Aluminium

Material (Aluminium)	1 M LENGTH		1.2 M LENGTH	
	ANSYS	**THEO**	**ANSYS**	**THEO**
Mode 1	23.086	23.084	16.0328	16.031
Mode 2	92.313	92.386	64.1200	64.157

Table 8. Frequency values for Aluminium

Material (Aluminium)	1.5 M LENGTH		2 M LENGTH	
	ANSYS	**THEO**	**ANSYS**	**THEO**
Mode 1	10.261	10.256	5.7720	5.7712
Mode 2	41.043	41.060	23.090	23.094

Table 9. Frequency values for Carbon FRP

Material (carbon FRP)	1 M LENGTH		1.2 M LENGTH	
	ANSYS	**THEO**	**ANSYS**	**THEO**
Mode 1	41.3905	41.3870	28.7446	28.7410
Mode 2	165.519	165.633	114.963	115.023

Table 10. Frequency values for Carbon FRP

Material (Carbon FRP)	1.5 M LENGTH		2 M LENGTH	
	ANSYS	**THEO**	**ANSYS**	**THEO**
Mode 1	18.3972	18.3944	10.3487	10.346
Mode 2	73.5867	73.6148	41.3970	41.408

The results were repeated for other cross sections viz

20X20mm, 30mmmx30mmm and 40mmx40mm respectively for various materials considered. It was observed that as cross section increases the frequency increases whereas as length increases the frequency decreases

4. ANALYSIS OF SANDWICH BEAM

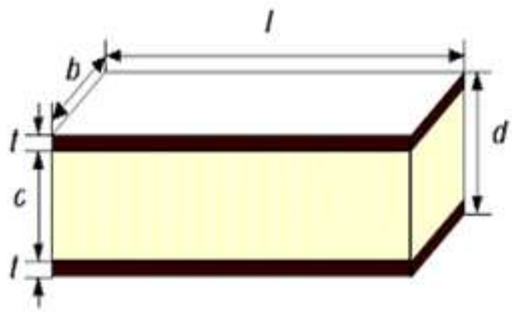


Fig- 6: Sandwich beam

Analysis of sandwich beam is done by considering 1m length and for different combination of materials.
t =2mm
c =6mm
t =2mm

Table 9. Frequency values for different combination of material

Material	Frequecy	
	mode1	mode 2
steel-FRP-steel	28.77244365	115.075
steel-Al-steel	22.946	91.789
FRP-steel-FRP	26.0966	104.3881
FRP-Al-FRP	29.925	119.70361
Al-steel-Al	22.915	91.665
Al-FRP-Al	33.4225	134

It is seen that among various combinations, Aluminium-carbon FRP-Aluminium gives the highest frequency of 33.425 Hz followed by carbon FRP-Aluminium-carbon FRP with frequency of 29.925 Hz.

6. CONCLUSIONS

- As length of beam increase its frequency decrease.
- As cross section area of the beam increases the frequency increases
- The analytical value of frequency is in good agreement with ANSYS APDL
- Carbon FRP has a highest natural frequency as compared to Aluminium and steel
- Aluminium-carbon FRP-Aluminium has the highest frequency of 33.425 Hz followed by carbon FRP-Aluminium-carbon FRP with frequency of 29.925 Hz.

7. REFERENCES

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