# EFFECT OF PITCH AND NOMINAL DIAMETER ON LOAD DISTRIBUTION AND EFFICIENCY IN MINIATURE LEAD SCREW DRIVES

**Vikrant Karmarkar[1], S.G.Jadhav[2], Rajesh Parab[3]**

[1]*PG Student, Mechanical Engineering Department VJTI Mumbai, Maharashtra, India*
[2]*Assistant Professor, Mechanical Engineering Department VJTI Mumbai, Maharashtra, India*
[3]*Deputy Manager, Portescap India Pvt Ltd, India*

## Abstract

*Lead screws are the devices which are used for power transmission or to have linear motion. It is theoretically assumed that applied load is evenly distributed among the thread pair in contact. However, practically it is observed that load is not uniformly distributed among threads. The first thread carries the maximum load and later the load on each thread reduces. Numerous studies have been carried out for analytical calculation of the load distribution using spring stiffness method. But these studies are for screw and nut combination. Not much study has been done to find load distribution on threads of a lead screw. The maximum load acting on one thread is an important parameter in lead screw design. The load decides the fatigue life of the screw and nut. To have better life of threads, the load distribution should be uniform to have fewer loads on single thread. The load is also important to know the deflection of thread which affects the positional accuracy of the lead screw drives. This paper focuses on analyzing mathematically the various thread parameters which affects the load distribution in threads and the corresponding effect on efficiency. The spring model method proposed in [1], [4] has different constant coefficient which are depending on thread geometry and material. If there are n numbers of threads in contact, there will be (n-1) number of equations in (n-1) unknowns. These are linear difference equations and can be solved by matrix elimination method. The results obtained from analytical solution are validated with the FEM (Finite Elements Method) results.*

***Keywords:*** *Lead Screws, Load Distribution, Thread Parameters, Efficiency, Linear Drives*

---------------------------------------------------------------------***---------------------------------------------------------------------

## 1. INTRODUCTION

Lead screw drives are simple, efficient, accurate and less costly devices for force transmission and linear motion applications. The basic principle behind lead screw / power screw is simple as that of nut and bolt mechanism. However, the thread angle used in nut-bolt is such that it increases the frictional contact so that it can be used ideally for fastening purposes. However, lead screws are devices for linear force transmission or for linear motion. The threads used in lead screws are either square threads or trapezoidal threads. Square threads have best efficiency but are weaker at the root. Also they are costly for manufacturing. Trapezoidal threads on the other hand are easy to manufacture and thicker at the root of thread so they can carry higher load.. In this paper, too, trapezoidal threads are analyzed. Various studies have been carried out to find load distribution in the threads in the screws. D. Miller [4] proposed spring model for load distribution in threaded connectors. He developed mathematical equations which were formulated assuming thread as a spring and calculated the stiffness in bending. The mathematics was verified using finite element analysis. He formulated the equations assuming non yielding threads. W. Wang & K. M. Marshek [1] modified the equations formulated by D. Miller [4] and proposed a model for yielding thread conditions. Hua Zhao [3] proposed a new virtual contact loading method in order to simulate the threaded connections. The model was found to be satisfactorily efficient and accurate. David J Murphy [5] studied the load distribution in lead screw wearing under varying operating conditions. W. Wang & K. M. Marshek [2] studied the load distribution in threaded connector having dissimilar material and varying thread stiffness. Not much study has been carried out to study the effect of various thread parameters on load distribution in lead screw threads. This paper focuses on how thread parameters affect load distribution and efficiency of a miniature lead screw. The mathematical data obtained from analytical solution has been verified with Finite element methods.

## 2. MATHEMATICAL MODELLING

The efficiency of a lead screw is a function of coefficient of friction. Lower the coefficient better is the efficiency. Steel – Bronze combination is observed to have least frictional coefficient. Since screw has to withstand the buckling along with the other body stresses as in the nut, lead screw must be stronger than nut. This facilitates the fact that when nut wears out after long use, it is easy and cheap to replace a nut than a screw. In this current paper, steel for screw and bronze for nut is used. There are two cases of tension and compression as used in [1] & [4].

## 2.1 Case 1: Compression

In this case, there will be compression of screw spindle. This will happen when nut is moving away from the driving end of the motor or gearbox under loading conditions. For representing screw and nut in a spring model, various sections in screw and nut has to be represented in terms of springs. Under loading conditions, the thread of the screw acts as a cantilever beam under uniformly distributed load. The stud section between two consecutive helixes acts as simple circular bar of constant cross section (equal to minor diameter of screw) under compression. Similar is the case for nut. When one thread of a nut is in contact with the respective thread of screw, the axial deflection for threads is same. So the resulting stiffness is the combined stiffness of nut & screw thread in series.
This is shown in figure below.

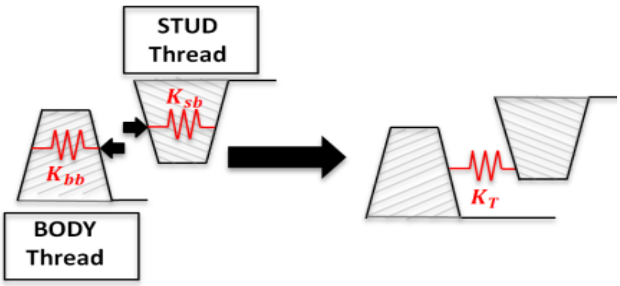


**Fig -1**: Equivalent stiffness of threads in bending

So the helixes of the screw are assumed to be like thread rings connected via the resulting spring stiffness. This is better explained in the figure.

The load F applied on the stud is carried into threads by load P, where $P_i$ represents load on $i^{th}$ thread. Si is the load on the section between the threads i& i+1. $L_i$ is the load on the body section i. Load on 1st thread will be the force applied F minus load on stud sections between threads 1&2. The stud section between 0th and 1st thread carries full load F. So$S_0$=F. Since there are 8 threads in contact the body sections between 8th& 9th thread carries no load. So $S_n$=0. These are the constants which are useful in solving the difference equation.

The load on $i^{th}$thread can be given as
$P_i = S_{i-1} - S_i \quad , \quad 1 \le i \le n \ldots$ (1a)

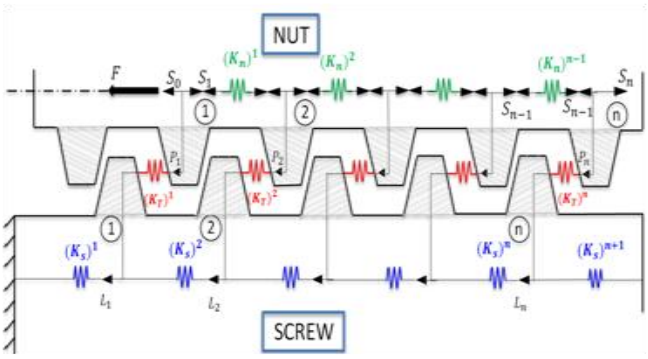


**Fig -2**: Spring model for compression case

The body sections between the threads have a spring constant$K_s = L/\delta_s$, where $\delta_s$ represents axial deflection due to load L. Similarly, stiffness of the stud section is given by$K_n = L/\delta_n$ , where $\delta_n$ represents elongation of the section under load S. The absolute axial deflection of any thread ring is the addition of the individual axial deflections of all the previous threads. So the absolute deflection of $k^{th}$ thread is given by,

$$u_s^k = \delta_T^k = \sum_{j=1}^{k} \delta_s^j \; , 1 \le k \le n \qquad (2a)$$

The difference between axial deflections of any two consecutive threads will give elongation of the stud section between those threads. Therefore

$$\delta_n^i = u_s^i - u_s^{i+1} \, , 1 \le i \le n-1 \qquad \ldots \qquad (3a)$$

Putting equation (2) in equation (3),

$$\delta_T^i - \delta_T^{i+1} - \delta_s^{i+1} = \delta_n^i \, , 1 \le i \le n-1 \qquad \ldots \qquad (4a)$$

Now, the deflections can be expressed in terms of applied loads and spring stiffness. So equation (4) yields,

$$\frac{P_i}{K_T^i} - \frac{P_{i+1}}{K_T^{i+1}} - \frac{L_{i+1}}{K_s^{i+1}} = \frac{S_i}{K_n^i} \qquad \ldots (5a)$$

The load on body section corresponding to $k^{th}$ thread is the load on the remaining threads of the section from thread ring k to last thread n. So,

$$L_k = \sum_{j=k}^{n} P_j$$

Using equation (1),

$$L_k = S_{k-1} - S_n \qquad \ldots (6a)$$

As discussed earlier, the stud section between last active thread and the next corresponding thread carries no load. So $S_n = 0$. The equation (6) gives,

$$L_k = S_{k-1} \qquad \ldots (7a)$$

Using equation (1) and (7), equation (5) can be modified as

$$S_{i-1} - S_i \left[1 + \frac{K_T^i}{K_T^{i+1}} + \frac{K_T^i}{K_s^{i+1}} + \frac{K_T^i}{K_n^i}\right] + S_{i+1}\frac{K_T^i}{K_T^{i+1}} = 0 \qquad (8a)$$

$$S_{i-1} - \beta_i S_i + \alpha_i S_{i+1} = 0 \qquad \ldots \qquad (9a)$$

Where,$\beta_i = \left[1 + \frac{K_T^i}{K_T^{i+1}} + \frac{K_T^i}{K_s^{i+1}} + \frac{K_T^i}{K_n^i}\right]$,

$$\alpha_i = \frac{K_T^i}{K_T^{i+1}}$$

For 8 threads in contact, n=8. Putting i=1 to eight in equation (9), we get corresponding equations in variable S.
For i=1,
$S_0 - \beta_1 S_1 + \alpha_1 S_2 = 0$ .

Since $S_0 = F$, the equation becomes,
$-\beta_1 S_1 + \alpha_1 S_2 = -F$ … (i)

Similarly, for other values of i,

$S_1 - \beta_2 S_2 + \alpha_2 S_3 = 0$ … (ii)
$S_2 - \beta_3 S_3 + \alpha_3 S_4 = 0$ … (iii)
$S_3 - \beta_4 S_4 + \alpha_4 S_5 = 0$ … (iv)
$S_4 - \beta_5 S_5 + \alpha_5 S_6 = 0$ … (v)
$S_5 - \beta_6 S_6 + \alpha_6 S_7 = 0$ … (vi)
$S_6 - \beta_7 S_7 + \alpha_7 S_8 = 0$

Since $S_8 = 0$ , the last equation can be modified as

$S_6 - \beta_7 S_7 = 0$… (vii)

As the lead screw material is uniform throughout,

$$K_T^i = K_T^{i+1}$$

The value of $\beta_i$ can be given as

$$\beta_i = \left[2 + \frac{K_T^i}{K_s^{i+1}} + \frac{K_T^i}{K_n^i}\right]$$

The seven equations in seven variables derived above can be represented in matrix format

$$\begin{bmatrix} -\beta_1 & \alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\beta_2 & \alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\beta_3 & \alpha_3 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\beta_4 & \alpha_4 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\beta_5 & \alpha_5 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\beta_6 & \alpha_6 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\beta_7 \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{Bmatrix} = \begin{bmatrix} -F \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Also for uniform bolt material, $\alpha_i = 1$. So the matrix becomes,

$$\begin{bmatrix} -\beta_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\beta_2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\beta_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\beta_4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\beta_5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\beta_6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\beta_7 \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{Bmatrix} = \begin{bmatrix} -F \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (10a)$$

These matrices can be solved by elimination method to get the values of S.

### 2.2 Case 2: Tension

The two cases of compression and tension are similar. In tension case, the nut is coming travelling towards motor or gearbox end, where the screw is attached, under loading condition. This will result in tension in the screw part hence the name tension case. For mathematical modeling of this case, only difference is with the location of fixing point. In compression case, the location of fixing point is near the first active thread, whereas in tension case, fixing is done near the last active thread. It is worth noting that the direction of applied force is same in both the cases. The spring model diagram for tension case is shown in the figure 3.

In tension case, load on body section corresponding to $k^{th}$ thread is the sum of all the loads from thread 1 to k. So,

$$L_k = \sum_{j=1}^{k} P_j$$

From equation (1)

$$L_k = S_0 - S_k$$

The absolute deflection of a thread ring is the combination of local thread ring deflection and sum of all the deflections of the rings from that particular ring in consideration to last active thread. Thus,

$$u_s^k = \delta_T^k + \sum_{j=k}^{n} \delta_s^j$$

Using this equation in equation (3a),
$\delta_T^i - \delta_T^{i+1} + \delta_s^i = \delta_n^i$… (1b)

The deflections can be expressed in terms of load and stiffness as

$$\frac{P_i}{K_T^i} - \frac{P_{i+1}}{K_T^{i+1}} + \frac{L_i}{K_s^i} = \frac{S_i}{K_n^i}$$

Using equations for $P_i$&$L_k$,

$$S_{i-1} - S_i\left[1 + \frac{K_T^i}{K_T^{i+1}} + \frac{K_T^i}{K_s^i} + \frac{K_T^i}{K_n^i}\right] + S_{i+1}\frac{K_T^i}{K_T^{i+1}}$$

$$= -S_0\frac{K_T^i}{K_s^i} \qquad \text{… (2b)}$$

Following the same approach as was done for compression case,

$$S_{i-1} - \beta_i S_i + \alpha_i S_{i+1} = -\gamma_i S_0 \qquad \text{… (3b)}$$

Where $\gamma_i = \frac{K_T^i}{K_s^i}$

For all values of i from 1 to 8, we have seven equations in seven variables as

$$-\beta_1 S_1 + \alpha_1 S_2 = -\gamma_1 F \quad \ldots \text{(vi)}$$
$$S_1 - \beta_2 S_2 + \alpha_2 S_3 = -\gamma_2 \ldots \text{(vii)}$$
$$S_2 - \beta_3 S_3 + \alpha_3 S_4 = -\gamma_3 \ldots \text{(viii)}$$
$$S_3 - \beta_4 S_4 + \alpha_4 S_5 = -\gamma_4 \ldots \text{(ix)}$$
$$S_4 - \beta_5 S_5 + \alpha_5 S_6 = -\gamma_5 \ldots \text{(x)}$$
$$S_5 - \beta_6 S_6 + \alpha_6 S_7 = -\gamma_6 \ldots \text{(xi)}$$
$$S_6 - \beta_7 S_7 + \alpha_7 S_8 = -\gamma_7$$
$$S_6 - \beta_7 S_7 = -\gamma_7 \quad \ldots \text{(xi)}$$

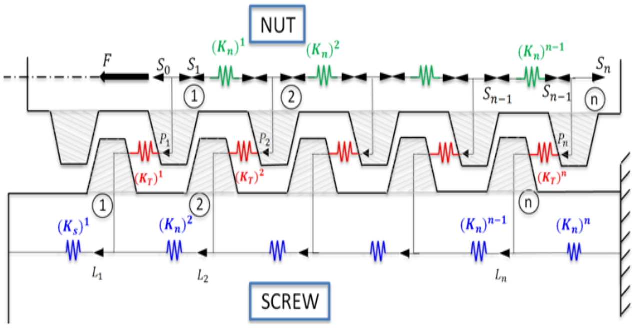


**Fig-3**: Spring model for tension case

In matrix form

$$\begin{bmatrix} -\beta_1 & \alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\beta_2 & \alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\beta_3 & \alpha_3 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\beta_4 & \alpha_4 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\beta_5 & \alpha_5 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\beta_6 & \alpha_6 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\beta_7 \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{Bmatrix} = \begin{bmatrix} -\gamma_1 F \\ -\gamma_2 \\ -\gamma_3 \\ -\gamma_4 \\ -\gamma_5 \\ -\gamma_6 \\ -\gamma_7 \end{bmatrix}$$

$$\begin{bmatrix} -\beta_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\beta_2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\beta_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\beta_4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\beta_5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -\beta_6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\beta_7 \end{bmatrix} \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{Bmatrix} = \begin{bmatrix} -\gamma_1 F \\ -\gamma_2 \\ -\gamma_3 \\ -\gamma_4 \\ -\gamma_5 \\ -\gamma_6 \\ -\gamma_7 \end{bmatrix} \quad \text{(4b)}$$

Above equations (10a & 4b) are simple difference equations in seven variables. These were solved by an online equation solver program [6], the solutions obtained from the solver were verified by Gaussian Elimination Method manually.

## 3. LEAD SCREW DESIGN:

For determining the dimensions of lead screw and nut, ASME/ANSI B 1.8-1988 [7] standards for acme screw were used. The accuracy class used was 2G. The nominal diameter was kept constant at 5mm and the pitch was varied from 1 mm to 1.5 mm. Due to tolerances given on major and minor diameters, there are two values (max & min) of each diameter. Thus for calculations, average of the diameters was taken.

The values of the obtained dimensions are summarized in the table 1.

**Table 1** : Major & minor diameters of screw & nut for different pitch (All dimensions are in mm)

| | | | SCREW | | NUT | |
|---|---|---|---|---|---|---|
| | $D_0$ | p | $D_{maj}$ | $D_{min}$ | $D_{maj}$ | $D_{min}$ |
| 1 | 5 | 1 | 4.9750 | 4.1122 | 5.2878 | 4.4250 |
| 2 | 5 | 1.1 | 4.9725 | 4.0522 | 5.2878 | 4.3675 |
| 3 | 5 | 1.2 | 4.9700 | 3.9922 | 5.2878 | 4.3100 |
| 4 | 5 | 1.3 | 4.9675 | 3.9322 | 5.2878 | 4.2525 |
| 5 | 5 | 1.4 | 4.9650 | 3.8722 | 5.2878 | 4.1950 |
| 6 | 5 | 1.5 | 4.9625 | 3.8122 | 5.2878 | 4.1375 |

The spring stiffness constants are summarized in the table 2.

**Table 2**: Spring stiffness constant for screw & nut (K in X*$10^5$ N/mm, p in mm)

| No | p | $K_{sb}$ | $K_{sc}$ | $K_{bb}$ | $K_{bc}$ | $K_T$ |
|---|---|---|---|---|---|---|
| 1 | 1.0 | 1.1943 | 8.8778 | 0.9158 | 35.199 | 0.5183 |
| 2 | 1.1 | 1.4064 | 7.8369 | 1.1147 | 31.999 | 0.6218 |
| 3 | 1.2 | 1.6280 | 6.9727 | 1.3346 | 29.3325 | 0.7334 |
| 4 | 1.3 | 1.8572 | 6.2443 | 1.5761 | 27.0761 | 0.8526 |
| 5 | 1.4 | 2.0922 | 5.6227 | 1.8393 | 25.1421 | 0.9788 |
| 6 | 1.5 | 2.3311 | 5.0865 | 2.1248 | 23.466 | 1.1116 |

## 4. RESULTS:

### 4.1: Effect of Pitch:

#### 4.1.1: Effect on load distribution:

A load of 100 N was applied and the load carried by each thread was plotted against the thread number for both tension and compression.

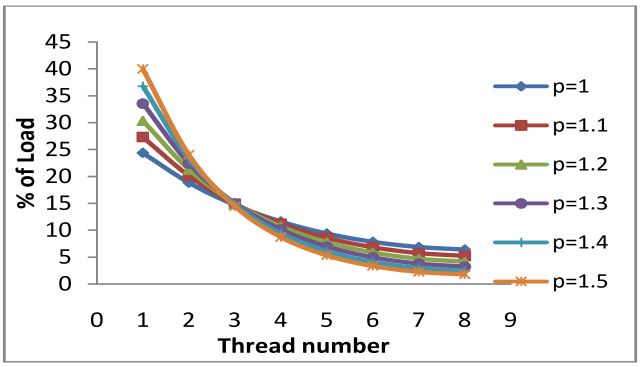


**Fig-4**: Effect of pitch on load distribution in compression Case (spring model)

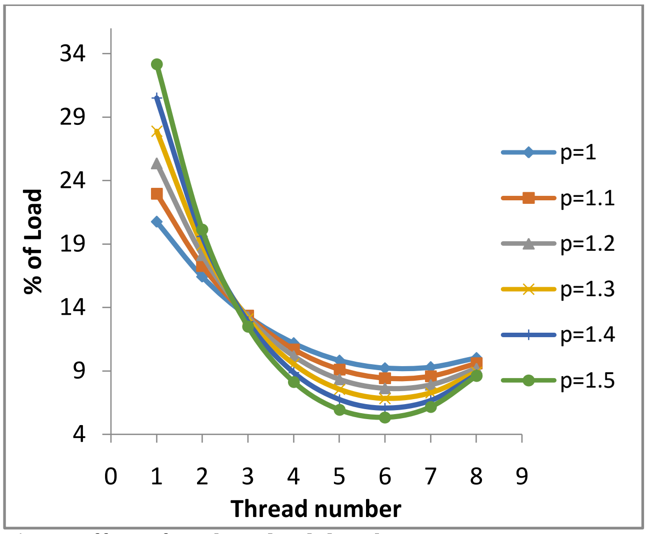


**Fig-5**: Effect of pitch on load distribution in tension case (Spring model)

For verifying these results in Finite Elements Methods, Ansys workbench (15.0) was used. The model was 2D axisymmetric, elements size was 0.05mm near the contact area. The results were plotted and summarized in the figure 6 & 7.

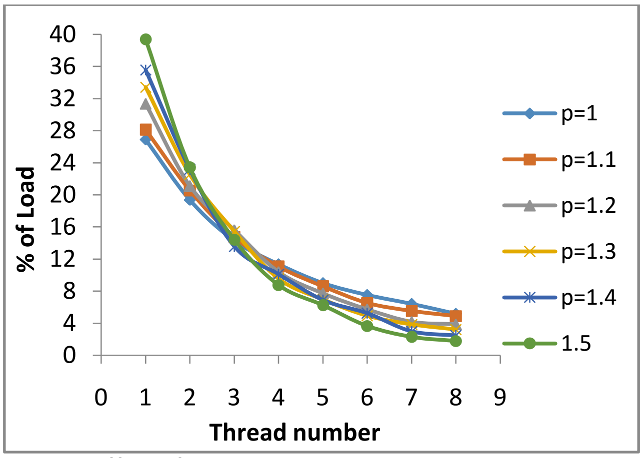


**Fig-6**: Effect of pitch on load distribution in compression Case (FEM)

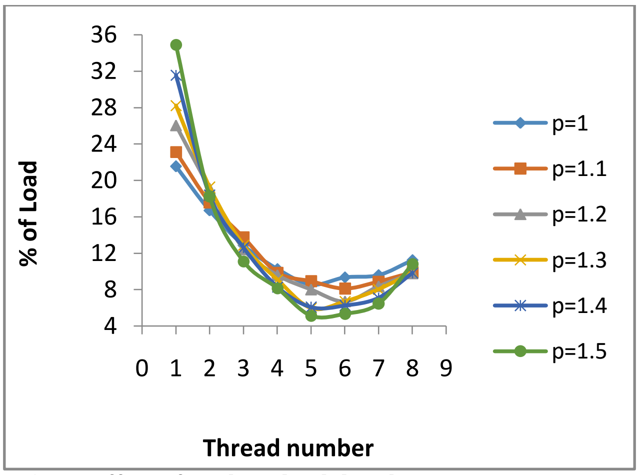


**Fig-7**: Effect of pitch on load distribution in tension case (FEM)

**4.1.2: Effect on Efficiency:**

The efficiency improves as pitch increases. The improvement in efficiency can be seen in the figure below.

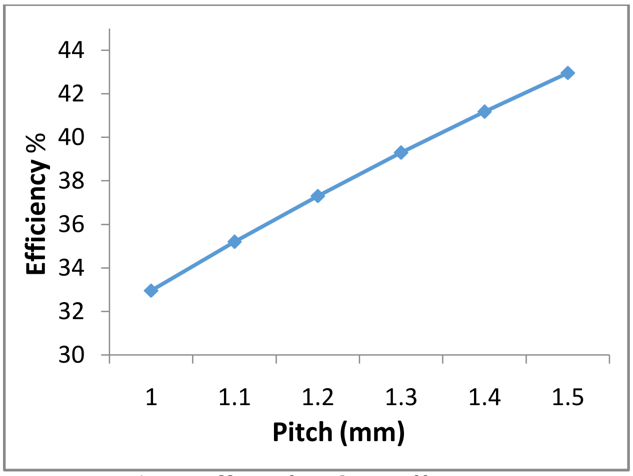


**Fig-8: Effect** of pitch on Efficiency.

Actually, the improvement in efficiency with pitch continues with a pitch value of 13mm, where efficiency reaches maximum of 73.43% and then it reduces. But for a pitch value of 1 mm to 1.5 mm, improvement in efficiency is linear. This can be seen in figure 9 below.

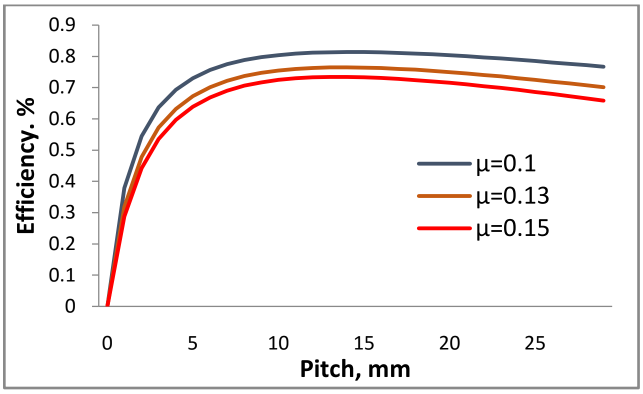


**Fig-9:** Effect of pitch on Efficiency for different coefficient Of friction

To have more life of threads, the load carried by single thread should be as less as possible. This can be achieved by even load distribution by keeping the pitch minimum which is evident from figures 4 & 5. But the reduction in pitch hampers the efficiency which can be seen in figure 8.

Thus to have best efficiency along with minimum load on a thread, the ratio (Efficiency /Load) should be as maximum as possible. This is compared in the table 3.

**Table 3**: Variation of Efficiency & Load with pitch

| **p(mm)** | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
|---|---|---|---|---|---|---|
| **η (%)** | 32.96 | 35.2 | 37.31 | 39.3 | 41.18 | 42.96 |
| **P(N)** | 24.37 | 27.31 | 30.39 | 33.56 | 36.78 | 39.99 |
| **η/P** | 1.352 | 1.289 | 1.228 | 1.171 | 1.119 | 1.074 |

## 4.2: Effect Of Nominal Diameter ( $D_0$ ) :

**4.2.1: Effect on Load distribution:**
The nominal diameter was varied from 5 mm to 6 mm with a step of 0.2 mm for a constant pitch of 1 mm. The effect of this on load distribution is summarized in the figures 9 & 10.

It is evident from these graphs that the increase in diameter does not have much effect on load distribution. There is very little improvement in load distribution with increase in diameter. But the increase reduces the efficiency. This can be seen from the figure 11.

Since effect of diameter is not significant, the results from FEM model are not included here.

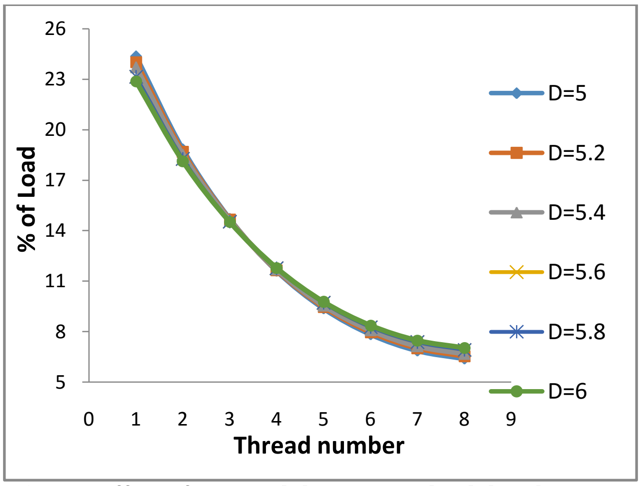


**Fig-10**: Effect of Nominal diameter on load distribution in Compression case (spring model)

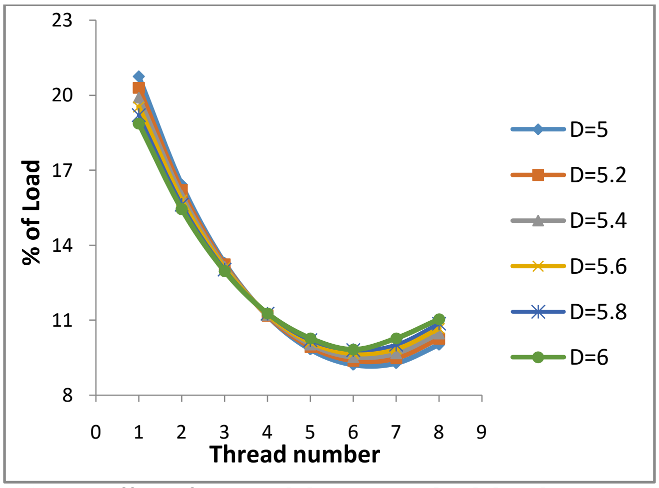


**Fig-11**: Effect of Nominal diameter on load distribution in Tension case (spring model)

**4.2.2: Effect on Efficiency:**
Efficiency reduces with increase in nominal diameter. This is due to the increase in frictional torque. The variation can be seen in figure 10.

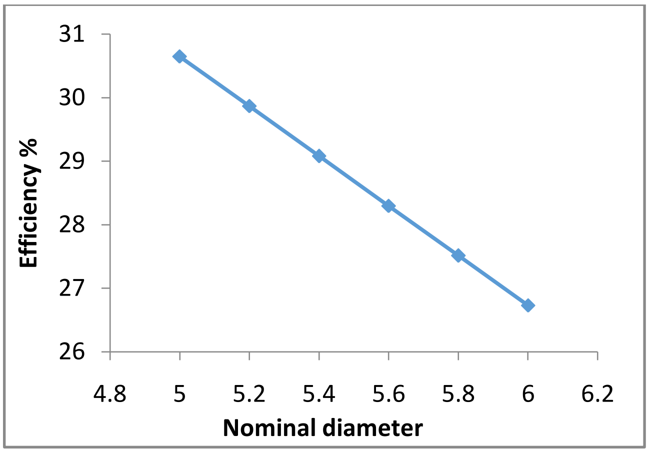


**Fig-12**: Effect of Nominal diameter on Efficiency

## 5. CONCLUSION

- The load distribution amongst thread was found to be very uneven with the first thread carrying maximum load.
- Increase in pitch causes more unevenness in load distribution and load on first thread increases.
- Change in nominal diameter did not have much effect on the load distribution.
- The spring model presented in [1]& [4] was used for calculation of load distribution amongst threads and it gave satisfactory results compared with Finite Element Methods.
- Among tension and compression case, the load on first thread was maximum in compression case.
- Form table 3, optimum pitch can be decided depending on whether life of thread is important or efficiency is important for a particular application.

## NOMENCLATURE

p = pitch of thread
$D_0$ =Nominal diameter of thread
P = Load carries by each thread
F = external applied load
$K_s^i$= axial stiffness of the screw section i between the Consecutive threads
$K_n^i$= axial stiffness of the nut section i between the Consecutive threads
$K_{sb}$ = stiffness of thread of screw in bending
$K_{nb}$ = stiffness of thread of nut in bending
$K_T$ = equivalent spring stiffness in bending of thread pair
$L_i$ = load on body section i
n = no of active threads
$S_i$ = load on nut section i
$u_n^i$ = deflection of nut thread ring i
$\delta_s^i$= axial deflection of corresponding springs having Spring constant $K_s^i$
$\delta_n^i$ = axial deflection of corresponding springs having Spring constant $K_n^i$
$\delta_T^i$ = axial deflection of corresponding springs having Spring constant $K_T^i$
α, β & γ = coefficient used in finite difference equations

## REFERENCES

[1]. W. Wang, K. M. Marshek, "Determination of load distribution in a threaded connector with yielding threads", Mech. Mach. Theory Vol. 31, No.2, pp. 229-224 (1996).
[2]. W. Wang, K.M. Marshek, "Determination of the load distribution in a threaded connector having dissimilar materials and varying thread stiffness", ASME, Journal of Engineering for Industry Vol 117 (1995), pp 1-8.
[3]. Hua Zhao, "A numerical method for load distribution in threaded connections", Transactions of ASME Vol. 118(1996) pp 274-279.
[4]. D. Miller, K. M. Marshek, Mohammad Nazi, "Determination of load distribution in a threaded connection", Mechanism and Machine Theory Vol. 18, no. 6, pp. 421-430.
[5]. David Murphy, "Load distribution on lead screw threads wearing under varying operating conditions", ASME Journal of Tribology, Vol. 130 (2008).
[6]. https://www.idomaths.com
[7]. Machinery's Handbook 27th Edition, pp 1825-1846