ON THE NON HOMOGENEOUS HEPTIC DIOPHANTINE EQUATION $(x^2 - y^2)(8x^2 + 8y^2 - 14xy) = 19 (X^2 - Y^2) z^5$

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Abstract

Five different methods of the non-zero non-negative solutions of non- homogeneous Heptic Diophantine equation $(x^2 - y^2)(8x^2 + 8y^2 - 14xy) = 19(X^2 - Y^2) z^5$ are observed. . Introducing the linear transformations $x = u + v$, $y = u - v$, $X = 2u + v$, $Y = 2u - v$, $u \neq v \neq 0$ in $(x^2 - y^2)(8x^2 + 8y^2 - 14xy) = 19 (X^2 - Y^2) z^5$, it reduces to $u^2 + 15v^2 = 19z^5$. We are solved the above equation through various choices and the different methods of solutions which are satisfied it. Some interesting relations among the special numbers and the solutions are exposed.

Keywords: *The Diophantine Equation, Heptic Equation, Integral Solutions, Special Numbers, A Few Interesting Relation*

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2010 Mathematics Subject Classification: 11D09

Notations used:

P_n^m : Pyramid number of rank n with size m

$T_{n,m}$: Polygonal number of rank n with size m

G_a : Gnomonic number of rank a

P_a : Pronic number of rank a

$f_{4,3}^r, f_{5,3}^r$: Fourth and fifth dimensional figurate number r, whose generating polygon is a Triangle

$f_{4,4}^r, f_{5,4}^r$: Fourth and fifth dimensional figurate number of rank r, whose generating polygon is a Square

$f_{4,5}^r, f_{5,5}^r$: Fourth and fifth dimensional figurate number of rank r, whose generating polygon is a Pentagon

$f_{4,6}^r, f_{5,6}^r$: Fourth and fifth dimensional figurate number of rank r, whose generating polygon is a Hexagon

$f_{4,7}^r, f_{5,7}^r$: Fourth and fifth dimensional figurate number of rank r, whose generating polygon is a Heptagon

$f_{4,8}^r, f_{5,8}^r$: Fourth and fifth dimensional figurate number of rank r, whose generating polygon is a Octagon

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1. INTRODUCTION

The number theory is the queen of Mathematics. In particular, the Diophantine equations have a blend of attracted interesting problems. For an extensive review of variety of problems, one may refer to [1-21]. In 2014, Jayakumar.P, Sangeetha. K, [22] have published a paper in finding the integer solutions of the non- homogeneous Heptic Diophantine equation $(x^2 - y^2)(9x^2 + 9y^2 - 16xy) = 21(X^2 - Y^2)z^5$. In 2015, the same authors Jayakumar.P, Sangeetha. K [23] published a paper in finding integer solutions of then non- homogeneous Heptic Diophantine equation $(x^2 - y^2)(5x^2 + 5y^2 - 8xy) = 13(X^2 - Y^2) z^5$ Inspired by these, in this work, we are observed another interesting five different methods the non-zero integral solutions of the non- homogeneous Heptic Diophantine equation $(x^2 - y^2)(8x^2 + 8y^2 - 14xy) = 19(X^2 - Y^2) z^5$

2. DESCRIPTION OF METHOD

Consider the Heptic Diophantine equation

$$(x^2 - y^2)(8x^2 + 8y^2 - 14xy)19(X^2 - Y^2)z^5 \quad (1)$$

We introduce the linear transformation

$$x = u + v,\ y = u - v,\ X = 2u + v,\ Y = 2u - v \quad (2)$$

Using (2) in (1), gives to $u^2 + 15v^2 = 19z^5$ (3)

We solve (3) through various methods and hence obtain patterns of solutions to (1)

2.1 Method: I

Let us take $z = z(a, b) = a^2 + 15b^2$ (4)

where a and b are non - zero distinct integers

Take 19 as $19 = (2 + i\sqrt{15})(2 - i\sqrt{15})$ (5)

Using (4) and (5) in (3) and applying the factorization process, define

$(u + i\sqrt{15}\ v) = (2 + i\sqrt{15})(a + i\sqrt{15}\ b)^5$

This gives us

$u = u(a, b) = 2a^5 - 75a^4b - 300a^3b^2 + 2280a^2b^3 + 2450ab^4 - 3375b^5$

$v = v(a, b) = a^5 + 10a^4b - 150a^3b^2 - 300a^2b^3 + 1225ab^4 + 450b^5$

In sight of (2) the relating solutions of (1) are given by

$x = x(a, b) = 3a^5 - 65a^4b - 450a^3b^2 + 1950a^2\boldsymbol{b^3} + 3675ab^4 - 2925b^5$

$y = y(a, b) = a^5 - 85a^4b - 150a^3b^2 + 2530a^2b^3 + 1225ab^4 - 3825b^5$

$X = X(a, b) = 5a^5 - 140a^4b - 750a^3b^2 + 4200a^2b^3 + 6125ab^4 - 6300b^5$

$Y = Y(a, b) = 3a^5 - 160a^4b - 450a^3b^2 + 4800a^2b^3 + 2775ab^4 - 7200b^5$

$z = z(a, b) = a^2 + 15b^2$

Observations:

1. z (1, 1) is a perfect number

2. $z(a, a) - 16\, t_{4,a} = 0$

3. $x(a, 1) - 3y(a, 1) - 190t_{4,a}^2 + 5700\, t_{4,a} \equiv 0 \pmod 2$

4. $x(a, 0) - 3y(a, 0) = 0$

5. $x(a, 1) - Y(a,1) - 95\, t_{4,a}^2 + 2850\, t_{4,a} - G_{45a} \equiv 0 \pmod 2$

6. $x(a, 1) - Y(a, 1) = 0$

7. $3X(a, 1) - 5Y(a, 1) - 480\, t_{4,a}^2 + 11400\, t_{4,a} - G_{2250a} \equiv 0 \pmod 7$

8. $3X(a, 1) - 5Y(a, 1) = 0$

Each of the following is a nasty number

9. $2z(0, 1), \frac{3}{2} z(1, 1), 2x(1,0), 6X(1, 0), 2Y(1,0)$

2.2 Method: II

Also take (3) as $u^2 + 15v^2 = 19z^5 * 1$ (6)

Take 1 as $1 = \frac{1}{16}(1 + i\sqrt{15})(1 - i\sqrt{15})$ (7)

The following techniques is same as Pattern-I, the relating non-zero dissimilar integral values of (1) are given by

$x = x(a, b) = \frac{1}{4}[-10a^5 - 290a^4b - 1550a^3b^2 + 8700a^2b^3 - 12250ab^4 - 138050b^5]$ (8)

$y = y(a, b) = \frac{1}{4}[-16a^5 - 160a^4b + 2400a^3b^2 - 4800a^2b^3 - 19600ab^4 - 7200b^5]$ (9)

$X = X(a, b) = \frac{1}{4}[-23a^5 - 515\, a^4b + 2450a^3b^2 + 15450a^2b^3 - 28175ab^4 - 30375b^5]$ (10)

$Y = Y(a, b) = \frac{1}{4}[-26a^5 - 385\, a^4b + 4350a^3b^2 - 45525a^2b^3 - 35255a^4b - 17325b^5]$ (11)

Since our intension is to find integer solutions, taking a as 2a and b as 2b in (4), (8) (9),(10)and (14), the relating two parametric integer solutions of (1) are found as

$x = x(a, b) = -80a^5 - 2320a^4b - 12400a^3b^2 + 5600a^2b^3 - 196000ab^4 - 208800b^5$

$y = y(a, b) = -128a^5 - 1280\, a^4b + 19200a^3b^2 - 38400a^2b^3 - 156800ab^4 - 57600b^5$

$X(a, b) = -184a^5 - 4120\, a^4b + 19600a^3b^2 + 123600a^2b^3 - 225400ab^4 - 243000b^5$

$Y = Y(a, b) = -208a^5 - 3080a^4b + 34800a^3b^2 + 108000a^2b^3 - 284200ab^4 - 138600b^5$

$z = z(a, b) = 4a^2 + 60b^2$

Observations:

1. z (a, a) is a perfect square

2. $x(a,1) + 1920\, f_{5,7}^{a} + 8832\, f_{4,7}^{a} + 11776P_a^5 - 14544P_a + G_{105635a} \equiv 0 \pmod 2$

3. $y(a, 1) + 384\,0\, f_{5,6}^{a} + 2304\, f_{4,7}^{a} - 44288P_a^5 + 58752P_a + G_{49054a} \equiv 1 \pmod 2$

4. $X(1,b) + 5832000\, f_{5,8}^{b} + 863040\, f_{4,7}^{b} + 2953600P_b^5 + 2488960P_b - G_{438420b} \equiv 1 \pmod 2$

5. $Y(a, 1) - 6240\, f_{5,6}^{a} + 21024\, f_{4,7}^{b} - 8892P_a^5 - 69348P_a + G_{126054a} \equiv 1 \pmod 2$

Each of the following is a nasty number

6. $\frac{3}{2} z(1,0), \frac{1}{2} z(0, 1), \frac{-3}{8} x(1, 0), -\frac{3}{16} y(1,0)$

2.3: Method: III

In place of (5) take 19 as

$$19 = \left(13 + i3\sqrt{15}\right)\left(13 + i3\sqrt{15}\right) \quad (11)$$

The following technique is same as in Pattern-I, and applying a few calculations, the relating non-zero dissimilar integral solutions of (1) are given as

x = x (a, b) $= 128a^5 - 1280a^4b - 19200a^3b^2 + 38400a^2b^3 + 156800ab^4 - 57600b^5$

y = y (a, b) = $80a^5 - 2320a^4b - 12000a^3b^2 + 69600a^2b^3 + 69600ab^4 + 98000b^5$

X= X (a, b) $= 232a^5 - 3080a^4b + 27680a^3b^2 + 92400a^2b^3 + 204200ab^4 + 72400b^5$

Y = Y (a, b) $= 184a^5 - 4120a^4b - 34880a^3b^2 + 123600a^2b^3 + 225400ab^4 - 138600b^5$

z = z (a, b) = $4a^2 + 60b^2$

Observations:

1. z (1,1) is a perfect square

2. z (a, a) -64$t_{4,a}$ = 0

3. 10x (a, 1) –16 y (a, 1) -24320 $t_{4,a}^2$+ 107520 $t_{4,a}$ – $G_{727200a}$≡1(mod2)

4. 10x (a, 1) – 16 y (a, 1) = 0

5. y (a, 1) -1920 $f_{5,7}^{a}$ +13440 $f_{4,7}^{a}$ +11040 P_a^5- 78992P_a≡ 1(mod2)

6. X (a, 1) - 6960 $f_{5,6}^{a}$ +18120 $f_{4,8}^{a}$ - 644600 P_a^5– 62670 P_a≡ 1(mod2)

7. Y (a, 1) -5520 $f_{5,6}^{a}$ +25292 $f_{4,7}^{a}$ - 94672 P_a^5- 82032 P_a≡ 1(mod2)

Each of the following is a nasty number

8. 6 z(1, 0), $\frac{1}{2}$ z(0, 1), $\frac{3}{16}$ x(1, 0), $\frac{3}{10}$ y(1, 0), $\frac{3}{29}$ X(1,0)

2.4: Method: IV

Instead of (5) write 19 as

$$19 = \frac{1}{16}\left(17 + i3\sqrt{15}\right)\left(17 - i3\sqrt{15}\right) \quad (12)$$

The following techniques is same as in Pattern-I, and doing some calculations, the relating non-zero dissimilar integral solutions of (1) are found as

x = x (a, b) $= 144a^5 + 80a^4b - 2160a^3b^2 + 2400\,a^2b^3 + 1476400ab^4 + 3600b^5$

y = y (a, b) = $128a^5 + 1280a^4b - 19200a^3b^2 + 38400\,a^2b^3 + 140800ab^4 - 57600b^5$

X = X (a, b) = $280a^5 - 520a^4b - 42000a^3b^2 + 15600\,a^2b^3 + 343000ab^4 - 23400\,b^5$

Y= Y (a, b) = $264a^5 - 1880a^4b - 39600a^3b^2 + 56400a^2b^3 + 323400a^3b^2 - 84600b^5$

z = z (a, b) = $4a^2 + 60b^2$

Observations:

1. z (2, 2) is a perfect square

2. z (a, a) -64$t_{4,a}$ = 0

3. x (1, 0) is a perfect square

4. x (0, 1) is a perfect square

5. 8 x (a, 1) -9y (a, 1) -1216 0 $t_{4,a}^2$+326400 $t_{4,a}$ – G_{72000a}≡ 1 (mod 2)

6. 8 x(a, 0) – 9 y (a, 0) = 0

7. 33X (a, 1) – 35Y (a, 1) – 48640 $t_{4,a}^2$ +1459200 $t_{4,a}$ ≡ 0 (mod 5)

8. 33X (a, 1) – 35Y (a, 1) = 0

Each of the following is a nasty number

9. $\frac{1}{600}$ x (0, 1), $\frac{3}{10}$ y (1, 0), $\frac{3}{35}$ X (1, 0), $\frac{1}{11}$ Y (1, 0)

2.3: Method: V

Instead of (5) write 1 as

$$1 = \frac{1}{64}\,(7 + i\sqrt{15}))(7 - i7\sqrt{15}) \quad (13)$$

The following techniques is same as in Pattern-III, and doing some calculations, the relating non- zero dissimilar integral solutions of (1) are given by

x = x (a, b) $= 32a^5 - 3080a^4b - 4800a^3b^2 + 6000a^2b^3 + 39200ab^4 - 122400b^5$

y = y (a, b) = $-40a^5 - 2720a^4b + 4800a^3b^2 + 4800a^2b^3 - 4900ab^4 - 12060b^5$

X = X (a, b) = $28a^5 - 5380a^4b - 4200a^3b^2 + 11400a^2b^3 + 34300ab^4 - 243900b^5$

Y = Y(a, b) $= -144a^5 - 5420a^4b + 6600a^3b^2 + 10200a^2b^3 - 53900ab^4 - 242140b^5$

z = z (a, b) = $4a^2 + 100b^2$

Observations:

1. – y(1, 0) is a perfect square

2. z (a, a) - 104$t_{4,a}$ = 0

3. 5x (a, 1) + 4 y (a, 1) + 26280 $t_{4,a}^2$+9600P_a^5-15600 $t_{4,a}$ ≡ 0 (mod 5)

4. 5 x (a, 1) + 4 y(a, 1) = 0

5. 7y (a, 1) + 10X (a, 1) + 72840 $t_{4,a}^2$+16800P_a^5–139200 $t_{4,a}$ ≡ 0 (mod 5)

6. 7y (a, 1) +10Y (a, 1) = 0

7. z (a, 0) – 4 $t_{4,a}$ = 0 8. 11X (a, 1) +7 Y (a, 1) +97120 t_4 a-2196500 $t_{4,a}$ ≡ 0 (mod 5) 9. 11X (a, 1) +7 Y (a, 1) = 0

Each of the following is a nasty number

10. $\frac{3}{4}$ x (1, 0), $-\frac{3}{5}$ y (1,0), $\frac{6}{7}$ X (1,0), -$\frac{6}{11}$ Y (1, 0)

3. CONCLUSION

In this work, we have observed various process of determining infinitely a lot of non-zero different integer values to the non-homogeneous Heptic Diophantine equation $(x^2 - y^2)(8x^2 + 8y^2 - 14xy) = 19(X^2 - Y^2)z^5$. One may try to find non-negative integer solutions of the above equations together with their similar observations.

4. REFERENCES

[1] Dickson,L.E.History of theory of number Vol.11, Chelsea publishing company, New –York (1952).

[2] Mordell, L.J. Diophantine equation, Academic press, London (1969) Journal of Science and Research,Vol (3)Issue 12, 20-22 (December-2014)

[3] Jayakumar. P, Sangeetha,K "Lattice points on the one $x^2 + 9y^2 = 50z^2$" International Journal of Science and Research, Vol (3), Issue 12, 20-22 December(2014)

[4] JayakumarP, Kanaga Dhurga, C,"On Quadratic Diophantine equation $x^2 + 16y^2 = 20z^2$" Galois J. Maths, 1(1) (2014),17-23.

[5] Jayakumar. P, Kanaga Dhurga. C "Lattice points on the cone $x^2+9y^2=50z^2$DiophantusJ.Math,3(2) (2014), 61-71

[6] Jayakumar. P, Prabha.S " On Ternary Quadratic Diophantine equation $x^2 + 15y^2 = 14z^2$" Archimedes J. Math 4(3) (2014), 159-164.

[7] Jayakumar, P, Meena, J "Integral solutions of the Ternary Quadratic Diophantine equation: $x^2 + 7y^2 = 16z^2$International Journal of Science and Technology, Vol.4, Issue 4, 1-4, Dec 2014.

[8] Jayakumar.P, Shankarakalidoss,G "Lattice points on Homogenous cone: $x^2 + 9y^2 = 50z^2$ International Journal of Science and Research, Vol (4), Issue 1 2053-2055, January -2015.

[9] Jayakumar.P,Shankarakalidoss.G "Integral point on the Homogenous cone $x^2+ y^2 =10z^2$ International Journal for Scienctific Research and Development, Vol (2), Issue 11, 234-235, January -2015

[10] Jayakumar. P, Prabha. S "Integra points on theCone $x^2 + 25y^2 = 17z^2$" International Journal of Science and Research , Vol(4), Issue 1, 2050- 2052 ,January-2015.

[11] Jayakumar. P, Prabha. S, "Lattice points on The Cone $x^2 + 9y^2 = 26z^2$ " International Journal of science and Research, Vol (4), Issue 1, 2050- 2052,January -2015

[12] Jayakumar. P, Sangeetha.K, "Integral solution of the Homogeneous Biquadratic Diophantine equation with six unknowns: $(x^3 - y^3)\ z = (W^2 - P^2)\ R^4$ International Journal of Science and Research, Vol(3), Issue 12, December-2014

[13] Jayakumar. P, Meena. J " Ternary Quadratic Diophantine equation: $8x^2+8y^2-15xy=40z^2$ International Journal of Science and Research, Vol.4, Issue 12, 654–655,December-2015

[14] Jayakumar. P, Meena.J 'On the Homogeneous Biquadratic Diophantine equation with 5 Unknown $x^4 - y^4 = 26(z^2 - w^2)\ R^2$ International Journal of Science and Rearch Vol.4, Issue 12, 656–658, December - 2015.

[15] Jayakumar. P, Meena. J 'On the Homogeneous Biquadratic Diophantine equation with 5 unknown $x^4 - y^4 = 40(z^2 - w^2)\ R^2$ International Journal of Scientific Research and Development, Vol.3 Issue10 ,204 – 206, 2015.

[16] Jayakumar.P, Meena.J Integer Solution of Non–Homogoneous Ternary Cubic Diophantine equation: $x^2 + y^2 - xy = 103z^3$ International Journal of Science and Research,Vol.5, Issue 3, 1777-1779, March -2016

[17]Jayakumar. P, Meena. J 'On Ternary Quadratic Diophantine equation: $4x^2+4y^2-7xy=96z^2$ International Journal of Scientific Research and Development, Vol.4, Issue 01, 876 - 877, 2016.

[18] Jayakumar.P, Meena.J 'On Cubic Diophantine Equation "$x^2 + y^2 - xy = 39z^3$" International Journal of Research and Engineering and Technology, Vol.05, Issue03, 499-501,March-16.

[19] Jayakumar. P, Venkatraman. R" On Homogeneous Biquadratic Diophantine: equation $x^4 - y^4 = 17(z^2 - w^2)\ R^2$ International Journal of Research and Engineering and Technology, Vol.05, Issue03,502-505, March- 2016

[20] Jayakumar .P, Venkatraman. R " Lattice Points on the Homogoneous cone $x^2 + y^2 = 26z^2$ International Journal of Science and Research, Vol.5, Issue 3, 1774 - 1776, March - 2016

[21] Jayakumar. P, Venkatraman. R " On the Homogeneous BiquadraticDiophantin eequation With 5 unknown $x^4 - y^4 = 65(z^2 - w^2)\ R^2$ International Journal of Science and Research, Vol.5, Issue 3, 1863 - 1866, March - 2016

[22] Jayakumar. P, Sangeetha. K, "On the Non-Homogeneous Heptic equation with five unknown $(x^2-y^2)(9x^2+9y^2-16xy)=21(X^2-Y^2)z^5$ International Journal of Science and Research, Vol (3), Issue 11, 3163-3165 (November 2014)

[23] Jayakumar.P, Sangeetha. K,: On the non Homogeneneous Heptic equation with five 5 unknowns $(x^2-y^2)(5x^2+5y^2-8xy)=13(X^2-Y^2)z^5$, The PMU Journal of Humanities and Sciences 55-59, January- June -2015

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