

# ENHANCEMENT OF TORSIONAL RESISTANCE IN FIBROUS NORMAL STRENGTH CONCRETE BEAMS

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## Abstract

This paper highlights the influence of thickness of concrete cover on the torsional resistance in the fibrous rectangular solid concrete beams under pure torsion. In this investigation, thickness of concrete cover vary between 18 and 53 mm. To study the influence of concrete cover in under-reinforced fibrous normal strength concrete, four fibrous concrete beams were cast and test under pure torsion. The under-reinforced concrete beams was designed based on ACI-318-14. The transverse and longitudinal reinforcement indexes are kept constant in the beams. While the aspect ratio of the beam cross section and span to depth ratio of the beams are 1.22 and 5.75, respectively. The test results were claimed that the torsional resistance provided by concrete and reinforcements plus fibre were enhanced at crack and peak loads up to 76.4% and 64.4%, respectively. In contrast, twisting angle, shear strain in concrete, strain in transverse and longitudinal reinforcements were reduced up to 57.8%, 65%, 89.3% and 95.6%, respectively. The spacing between cracks and the angle of inclination of crack at failure was increased whereas the number of cracks reduced. The dimensional analysis was used to proposed equations to predict torsional resistance at crack and peak loads including the contribution of concrete cover. In addition, the Buckingham  $\Pi$  theorem was used for this purpose. Based on the 34 data from previous researches and the data in this research, nonlinear multiple regressions in Minitab software version 16 was used to predict the coefficients of the proposed  $\Pi$ -groups equations. The proposed models have shown a good agreement with test results.

**Keywords:** Fibrous Concrete, Concrete Cover, and Dimensional Analysis

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## 1. INTRODUCTION

The torsional resistance of fibrous concrete beams subjected to pure torsion is affected by the properties of concrete and geometric details of the section at pre-crack stage whereas the fibre and reinforcements are contributed to resist torsion at peak load[1].

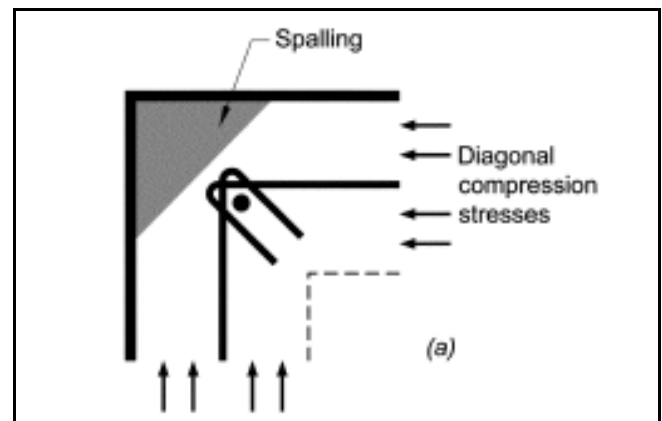
In fact, the contribution of reinforcement to resist torsion is reduced at peak load due to reduction in strain of transverse reinforcement from high bond strength between reinforcement and fibrous normal strength concrete[2]. In contrast, the contribution of fibre to resist torsion is issued at peak load either the fibre is pullout or rupture from high bond.

Moreover, the inclusion of fibre in concrete improved the tensile strength prior to cracking. Consequently, the torsional resistance of beam is enhanced. However, in post cracking stage, the corner of the non-fibrous concrete beams tend to spall off due to the inclined compressive stresses in the diagonal compression strut in space truss[3, 4] as shown in Figure 1, the fibrous concrete is no spall off at failure in torsion[5].

Indeed, the small thickness of concrete cover in non-fibrous concrete beams under pure torsion is improved the torsional resistance and vice versa[6].

## 2. RESEARCH SIGNIFICANCE

The research highlights the influence of extra thickness of concrete cover on the torsional resistance of fibrous normal strength concrete beams in prior to cracking and post-cracking stages. The fibre is eliminated the phenomenon of spall of concrete. Thus, the overall area of the section is contributed to resist torsional resistance, in particular, in post crack stage.



**Fig -1:** Spall off concrete cover in non-fibrous concrete beams under pure torsional loading[3]

## 2. EXPERIMENTAL PROGRAMME

The under-reinforced concrete (B-1-N) beam is a control beam and the other beams (C-1-N, C-2-N and C-3-N) with varying thickness of concrete cover were cast for experimental purpose. The concrete cover of the beams varied from 18 to 53 mm. The span to depth ratio and the aspect ratio of the beam section are kept as 5.75 and 1.22, respectively. The torsional moment was applied on the beams from two point loads act on the loading arms which are swapped to pure torsion on the tested beam.

### 2.1 Materials, Mix Proportions And Specimen Preparation

#### 2.1.1 Materials And Mix Proportion

The fibrous normal strength concrete beams were cast with cylinder compressive strength 25 MPa. An ordinary Portland cement (Tasik cement) was used. Crushed stone 10 mm maximum size of aggregate, silica sand, silica fume, tap water, HRWR super-plasticizer Sika VC2199, retard-admixture (Plastiment-R) with two size of micro steel fibre were used. The mix proportion of the materials used for producing fibrous normal strength concrete

**Table -1:** Mix proportion of fibrous normal strength concrete

Materials	Quality, kg/m <sup>3</sup>
Cement (Type)	275
Silica sand	926.7
Crushed stone	763.8
Silica fume	13.75
Water	228
Super-plasticizer VC2199	5.5
Retarder-admixture (Plastiment R)	1.375
Micro steel fibre A (21mm X 0.35 mmΦ)	18.055
Micro steel fibre B (12 mm X 0.2 mmΦ)	72.22
Slump, mm	90

Φ: diameter of fibre, mm

#### 2.1.2 Proportioning of Beam Specimen

The longitudinal reinforcement contained 4-12 mm diameter bars, two of them at the bottom and the rest at the top. Transverse reinforcement was provided in the form of two – legs rectangular stirrup with 135° standard hooks. The 6mm diameter bars were made stirrups with dimension 166 mm wide and 216 mm depth and the spacing between stirrups was 95 mm as shown in Figure 2[7]. The dimensions of the beams are tabulated in Table 2.

#### 2.1.3 Specimen Preparation

The fibrous normal strength concrete was mixed in the two pan mixer with 0.05 m<sup>3</sup> capacity using the following procedure: Crushed stone and silica sand were blended for 1.5 minutes. Next, the cement was added to the blended materials. Then, the whole water and super-plasticizer were added to the mix for another 0.5 minute. Afterwards, silica fume and retarder-admixture were added to the mix for 0.5 minutes. After that, micro steel fibre was added to the mixed materials, which was passed through the steel wire screen during 3 minutes. The blending materials was continued for an additional two minutes to confirm that the fibre was distributed in the concrete uniformly.

Fibrous normal strength concrete was cast in the plywood mold with 4 layers and each layer was vibrated for 45 seconds in four points in the length of the beam outside of the mold. The process of casting was associated with casting of three cubes, three cylinders, three prisms and six cubes for testing bond and they were vibrated for 45 seconds on vibrating table[8-11].

#### 2.1.4 Testing of Beams

Under-reinforced fibrous normal strength concrete beams were tested under pure torsional moment. The Universal Testing Machine of 500 kN capacity in the Structural Laboratory in the School of Civil Engineering was used for the test.

The specimens were tested in pure test arrangement as shown in Figure 2 and 3. The twisting angle was measured during loading of the beam by using U shape steel frame and two LVDTs on the arms of U shape as shown in Figure 4.

The shear strain in concrete and axial strain in reinforcements were measured by LVDTs and strain gauges. The load was applied manually until the concrete beam was failed under pure torsion.

**Table -2:** Details of the beams

Beam denotation	Concrete cover, mm	Width, mm	Height, mm	Span length, mm
B-1-N	29	230	280	1587
C-1-N	18	208	258	1464
C-2-N	28	228	278	1624
C-3-N	53	278	328	1878

## 3. TEST RESULTS AND DISCUSSION

The properties of fibrous normal strength concrete for each beam were measured and tabulated in Table 3. The torsional resistance and twisting angles at crack and ultimate loads were measured. Besides, the inclusion angle of crack at failure was measured as shown in Table 4.

### 3.1 Cracking Torsional Moment

The cracking torsional resistance was improved up to 76.4% due to thickening of concrete cover while the cylinder compressive strength of fibrous normal strength concrete beams was slightly changed as shown in Figure 6.

**Table -3:** Measured properties of fibrous normal strength concrete beams

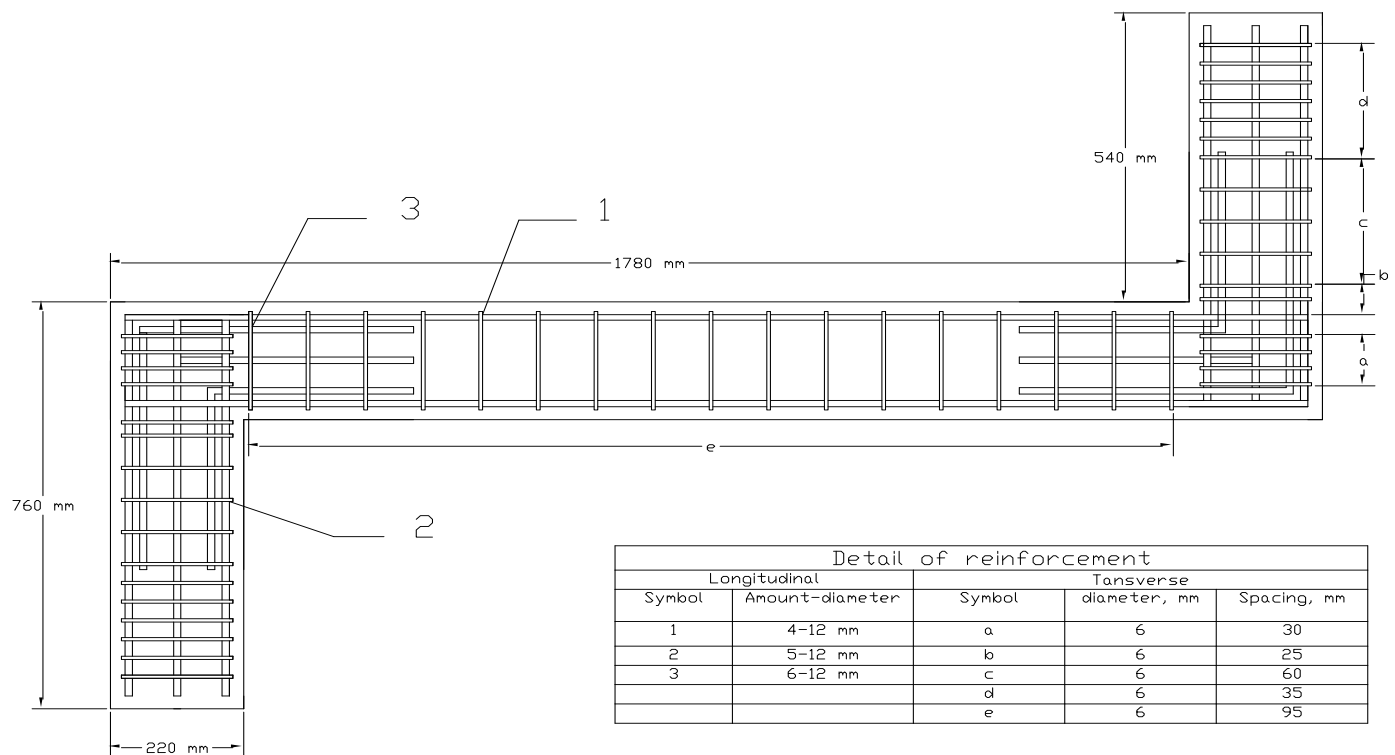
Beam denotation	$f_{c'}$ , MPa	$f_{sp}$ , MPa	$f_r$ , MPa	Bond strength, MPa	
				$f_{bt}$	$f_{bL}$
B-1-N	29.5	4.473	7.571	1.314	6.644
C-1-N	25.8	3.898	5.474	2.924	8.387
C-2-N	28.66	3.574	5.894	2.285	6.553
C-3-N	26.27	3.489	4.710	3.529	9.428

**Table -4:** Results of pure torsion test in fibrous normal strength concrete beams

Beam denotation	$T_{cr}$ , kN.m	$\Phi_{cr}$ , rad/m	$T_u$ , kN.m	$\Phi_u$ , rad/m	$\theta$ , degree
B-1-N	12.147	0.619	17.846	15.165	45
C-1-N	10.714	0.338	13.765	17.537	45
C-2-N	13.201	0.390	17.040	14.543	52
C-3-N	18.901	2.812	22.654	7.392	48

### 3.2 Torsional Resistance Provided By Reinforcement and Fibre

The torsional resistance provided by fibre and reinforcement was improved up to 94.66% due to thickening of concrete cover which improved by the contribution of fibre in the concrete cover as shown in Figure 7.

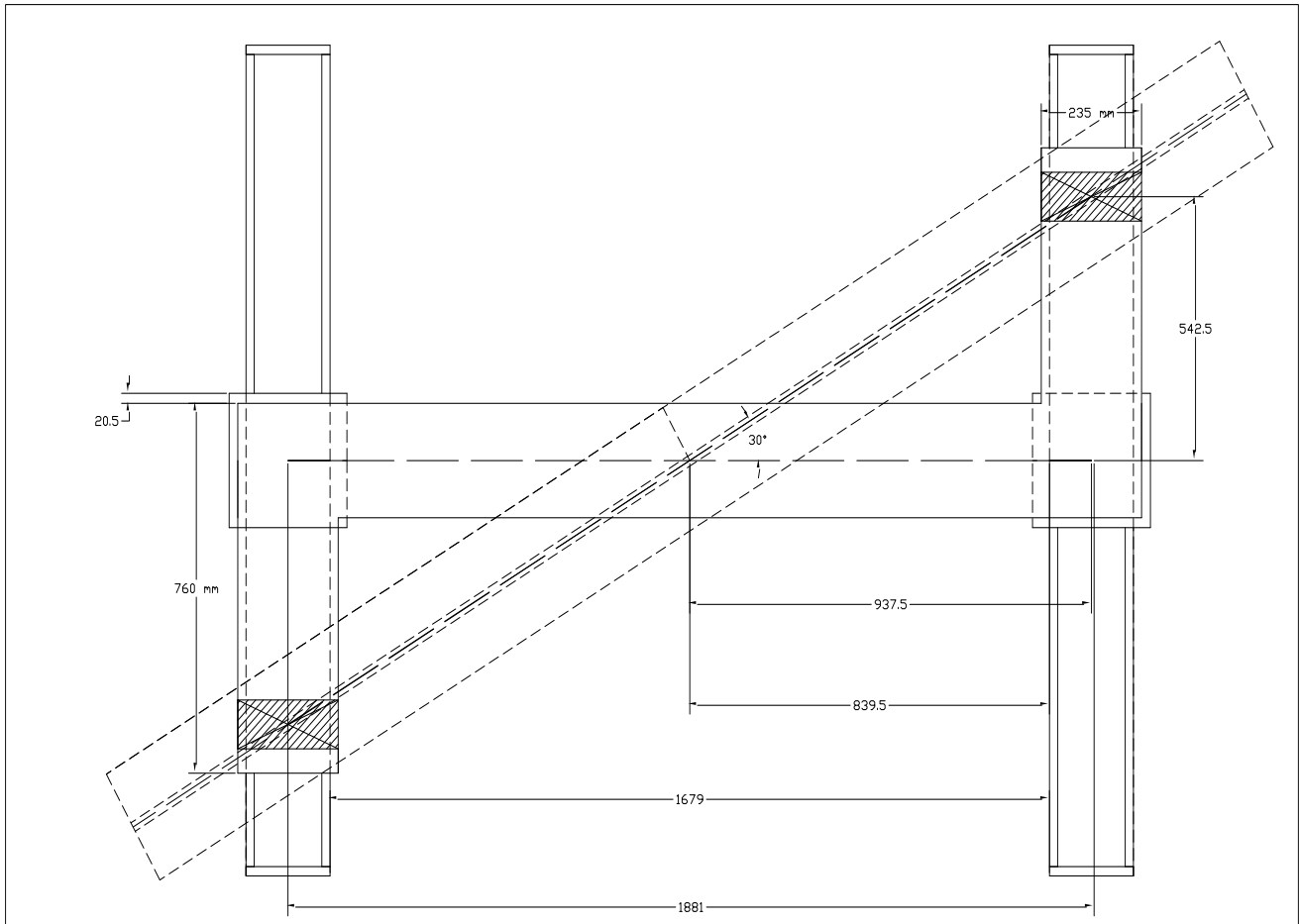


**Fig -2:** Detail of reinforcements in the beams

### 3.3 Torsional Resistance and Twisting Angle

The twisting angle prior to cracking was influenced by the thickness of concrete cover which improved the stiffness of the section. It was reduced up to 57.8%. In contrast, the

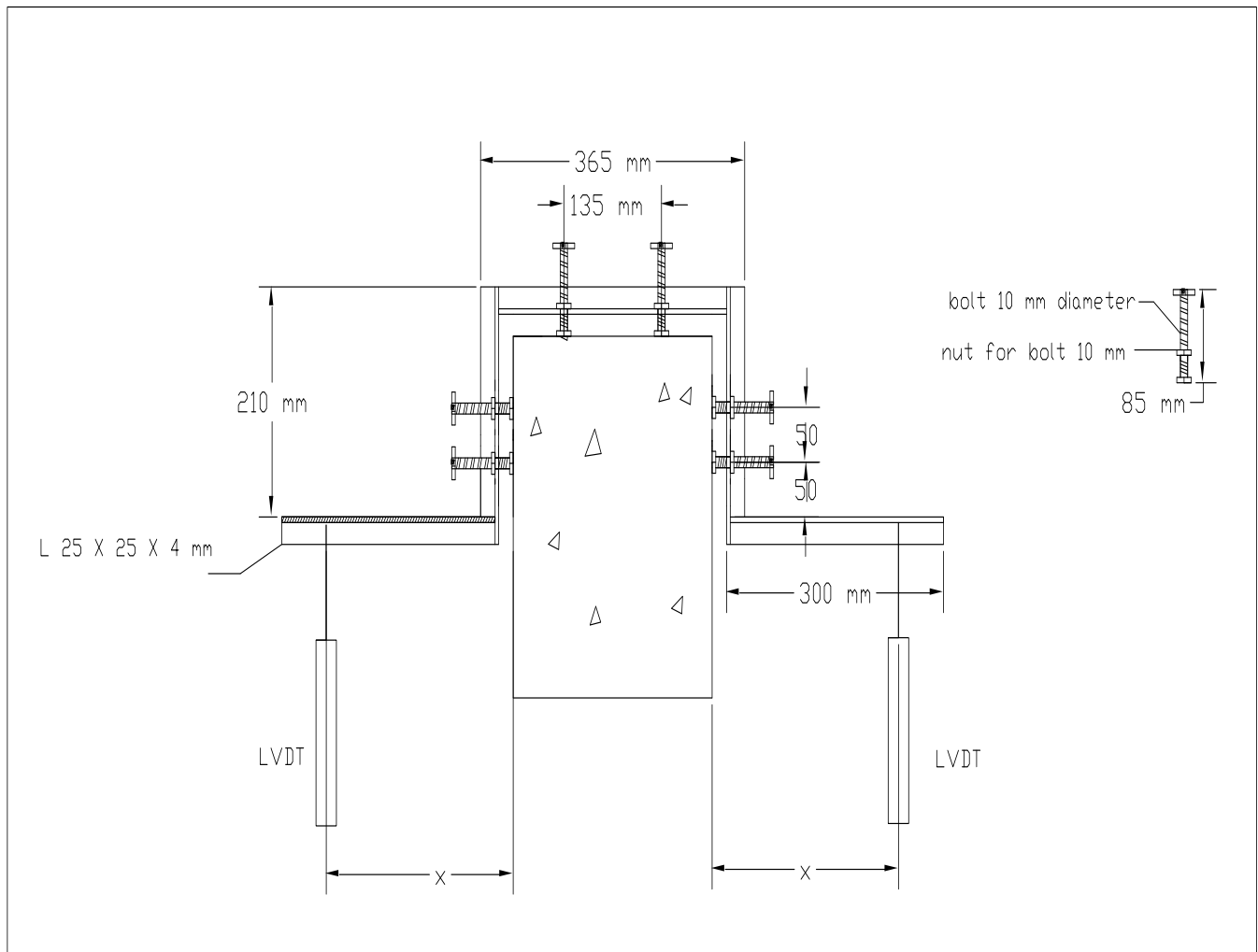
twisting angle of the section at peak load was improved up to 7 times due to improvement in the thickness of concrete cover from 18 to 53 mm as shown in Figure 8.



**Fig -3: Schematic Test set-up**



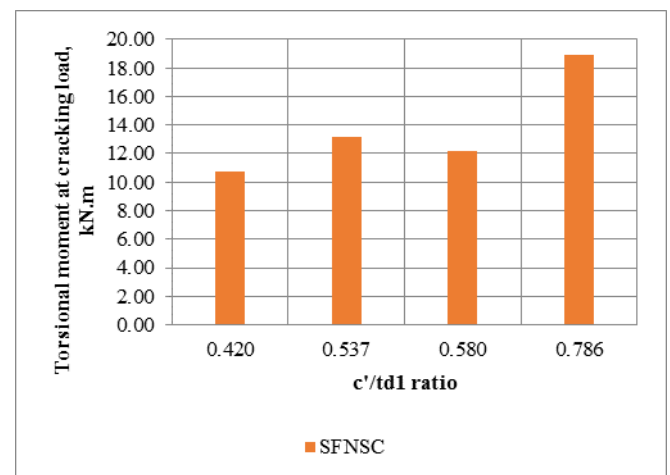
**Fig -4:** Experimental set-up of the beams



**Fig -5:** Schematic measuring of twisting angle

**Table -5:** Detail of spiral crack of fibrous normal strength concrete beams test under pure torsion

Beam denotation	No. of spiral cracks	$\Theta$ , degree	Average spacing between spiral cracks, mm
B-1-N	6	45	205
C-1-N	5	45	279
C-2-N	6	52	323
C-3-N	3	48	412



**Fig -6:** Cracking torsional moment versus  $c'/t_{d1}$

### 3.4 Shear Strain In Concrete

The amount of shear strain in concrete during torsion test was influenced by the thickness of concrete cover as shown in Figure 9. The thickening of concrete cover was reduced the value of shear strain up to 92.3%.

### 3.5 Strains In Longitudinal and Transverse Reinforcements

Due to thickening of concrete cover, the strain in longitudinal and transverse reinforcements are influenced and reduced at peak load up to 95.6% and 89.27%, respectively as shown in Figure 10 and 11.

### 3.6 Detail and Crack Patterns

The thickening of concrete cover is influenced on the detail of spiral cracks. For instance, number and spacing between cracks as well as inclination angle of crack at failure as tabulated in Table 5. The pattern of cracks of the tested beams which were tested and failed under pure torsional moment as shown in Figures 12-15.

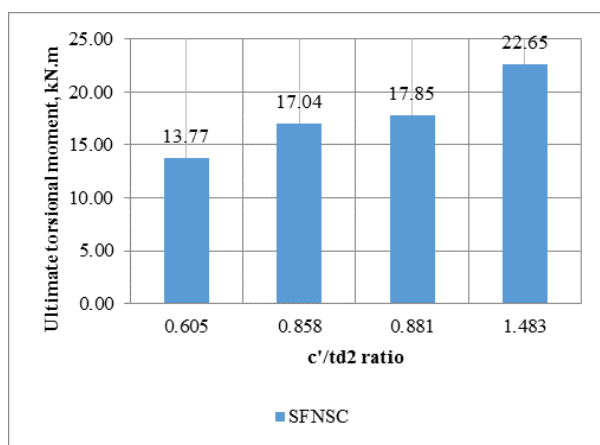


Fig -7: Cracking torsional moment versus  $c'/t_{d2}$

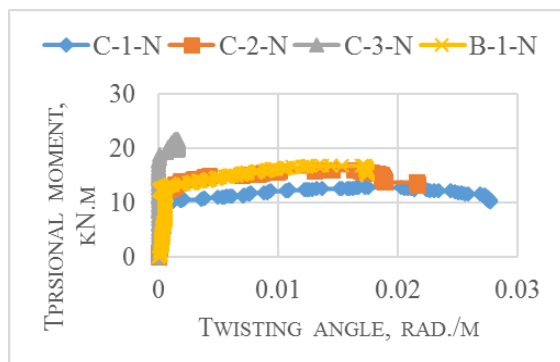


Fig -8: Torsional moment versus twisting angle

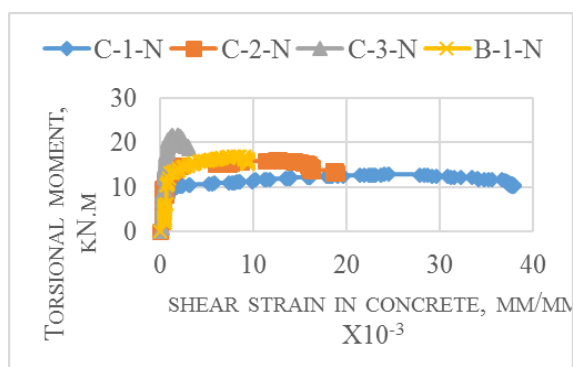


Fig -9: Torsional moment versus shear strain in concrete

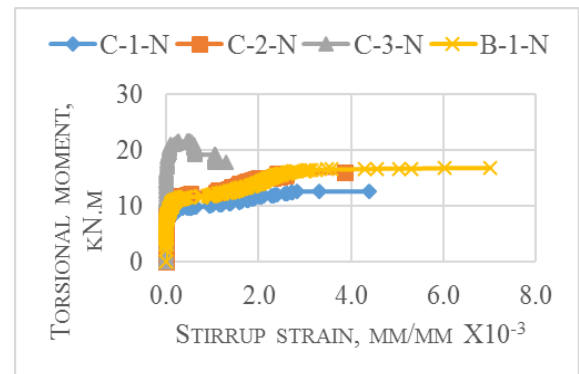


Fig -10: Torsional moment versus strain in transverse reinforcement

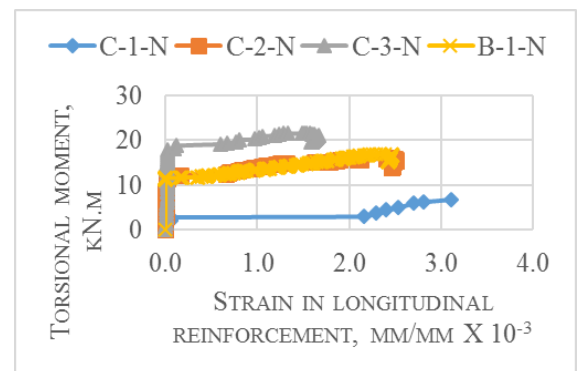


Fig -11: Torsional moment versus strain in longitudinal reinforcement

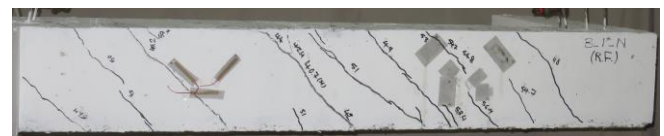


Fig -12: Pattern of cracks in beam B-1-N

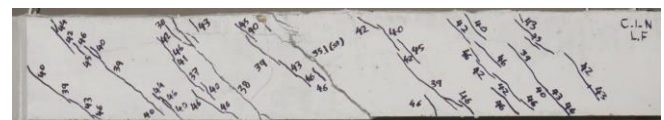


Fig -13: Pattern of cracks in beam C-1-N

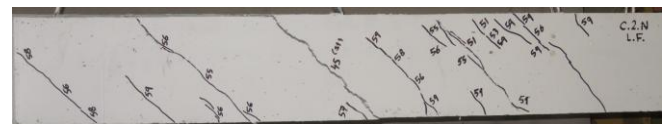


Fig -14: Pattern of cracks in beam C-2-N



Fig -15: Pattern of cracks in beam C-3-N

## 4. THEORETICAL MODEL

The dimensional analysis [12] was used as a tool to proposed equations to predict torsional resistance at crack and peak loads for under-reinforced fibrous normal strength concrete beams based on the data in this study and 33 data

from previous researches [5, 13-19]. The Buckingham Pi technique is used for evaluating the power of dimensionless parameters [20, 21].

#### 4.1 Cracking Torsional Resistance In Fibrous Normal Strength Concrete Beams

The variables could be affected on the cracking torsional resistance are effective tensile strength  $f_t^*$ , area to perimeter ratio of the cross-section  $A_{cp}/P_{cp}$  and thickness of concrete cover plus the diameter of the stirrup  $c'$ . The set of variables including dependent variable is [5, 13-19] and the total number of variables inside of the set is  $n$  and equal to 4. The primary dimensions are listed in Table (6).

$$\{V\} = \left\{ T_{cr}, f_t^*, \frac{A_{cp}}{P_{cp}}, c' \right\}$$

Where,  
 $f_t^* = (f_{sp} \cdot f_{c'}) / (f_{sp} + f_{c'})$

The number of primary dimensions is equal to 2 based on the FLT system and the number of  $\Pi$ -dimensionless groups is equal to  $k = n - m = 4 - 2 = 2$ . Consequently, the number of repeating variables is equal to two and selecting the common variable which includes all possible primary dimension [M], [L], and [T]. Therefore,  $f_t^*$ , and  $c'$  are selected as a repeating variables.

The  $\Pi$ -dimensionless groups are

$$\Pi_1 = (T_{cr}) (f_t^*)^a (c')^b$$

$$\Pi_2 = \left( \frac{A_{cp}}{P_{cp}} \right) (f_t^*)^a (c')^b$$

The dimensional form for Eq.(Pi-1) can be stated as

$$[M^0 L^0 T^0] = [M^1 L^2 T^{-2}] [M^1 L^{-1} T^{-2}]^a [M^0 L^1 T^0]^b$$

Then, equate the powers for primary dimensions

Equating exponents of mass M:

$$0 = (1).1 + (1).a + (0).b \Rightarrow a = -1$$

Equating exponents of length L:

$$0 = (2).1 + (-1).a + (1).b \Rightarrow b = -3$$

**Table 6:** Primary dimensions in MLT and FLT systems for variables in {V}

Variable No.	Parameters	Primary dimensions in MLT system	Primary dimensions in FLT system
1	$T_{cr}$	$[M^1 L^2 T^{-2}]$	$[F^1 L^1 T^0]$
2	$f_t^*$	$[M^1 L^{-1} T^{-2}]$	$[F^1 L^{-2} T^0]$
3	$A_{cp}/P_{cp}$	$[M^0 L^1 T^0]$	$[F^0 L^1 T^0]$
4	$c'$	$[M^0 L^1 T^0]$	$[F^0 L^1 T^0]$
	$n=4$	$m=3$	$m=2$

Equating exponents of time T:

$$0 = (-2).1 + (-2).-1 + (0).-3 = 0 \text{ satisfy}$$

So the first  $\Pi$ -dimensionless group is

$$\Pi_1 = \frac{T_{cr}}{f_t^* \cdot c'^3}$$

The dimensional form for Eq.(Pi-2) can be written as

$$[M^0 L^0 T^0] = [M^0 L^1 T^0] [M^1 L^{-1} T^{-2}]^a [M^0 L^1 T^0]^b$$

Then, equate the powers for primary dimensions

Equating exponents of mass M:

$$0 = (0).1 + (1).a + (0).b \Rightarrow a = 0$$

Equating exponents of length L:

$$0 = (1).1 + (-1).a + (1).b \Rightarrow b = -1$$

Equating exponents of time T:

$$0 = (0).1 + (-2).0 + (0).-1 = 0 \text{ satisfy}$$

So the second  $\Pi$ -dimensionless group is

$$\Pi_2 = \frac{\frac{A_{cp}}{P_{cp}}}{c'}$$

The final form of  $\Pi$ -dimensionless groups is

$$\Pi_1 = \beta_1 \cdot \Pi_2^{\beta_2}$$

$$\frac{T_{cr}}{f_t^* \cdot c'^3} = \beta_1 \cdot \left( \frac{A_{cp}/P_{cp}}{c'} \right)^{\beta_2}$$

Based on 33 data from previous researches and 4 data in this study on fibrous normal strength concrete beams which are tabulated in Appendix G, nonlinear multiple regression is used to predict the value of  $\beta_1$  and  $\beta_2$ . The proposed equation (Pi-3) for predicting cracking torsional resistance in fibrous normal strength concrete beams becomes in the form (Pi-4) with coefficient of determination 0.959

$$T_{cr} = 14.6 \left( \frac{A_{cp}/P_{cp}}{c'} \right)^{2.953} f_t^* \cdot c'^3$$

#### 4.2 Prediction of Torsional Resistance In Fibrous Normal Strength Concrete Beams After Cracking Provided By Reinforcement And Fibres.

The possible parameters which are expected to effect on the torsional resistance of fibrous normal strength concrete beams after cracking are listed in Table (7) including



primary dimensions in MLT and FLT system. The total number of parameters in the set V is 6.

$$\therefore \{V\} = \left\{ T_{sf}, f_{ty}, \frac{A_t}{s}, c', A_L \cdot f_{Ly}, \Psi \right\}$$

The number of  $\Pi$ -dimensionless groups is k which equals to 4 and the number of repeating variables is equal to minimum number of primary dimensions which is 2. Consequently, it was selected  $f_{ty}$  and  $A_t/s$  as a repeating variables.

There are four  $\Pi$ -dimensionless groups, which are

$$\Pi_1 = (T_{sf}) \left( \frac{A_t}{s} \right)^a (f_{ty})^b$$

$$\Pi_2 = (A_L \cdot f_{Ly}) \left( \frac{A_t}{s} \right)^a (f_{ty})^b$$

$$\Pi_3 = (c') \left( \frac{A_t}{s} \right)^a (f_{ty})^b$$

$$\Pi_4 = (\Psi) \left( \frac{A_t}{s} \right)^a (f_{ty})^b$$

The dimensional form for Eq.(Pi-1) can be stated as

$$[M^0 L^0 T^0] = [M^1 L^2 T^{-2}] [M^0 L^1 T^0]^a [M^1 L^{-1} T^{-2}]^b$$

Then, equate the powers for primary dimensions

$$\begin{array}{l} \text{Equating exponents of mass M:} \\ 0 = (1).1 + (0).a + (1).b \Rightarrow b = -1 \end{array}$$

$$\begin{array}{l} \text{Equating exponents of length L:} \\ 0 = (2).1 + (1).a + (-1).b \Rightarrow a = -3 \end{array}$$

**Table 7:** Primary dimensions in MLT and FLT systems for variables in {V}

Variable No.	Parameters	Primary dimensions in MLT system	Primary dimensions in FLT system
1	$T_{sf}$	$[M^1 L^2 T^{-2}]$	$[F^1 L^1 T^0]$
2	$f_{ty}$	$[M^1 L^{-1} T^{-2}]$	$[F^1 L^{-2} T^0]$
3	$A_t/s$	$[M^0 L^1 T^0]$	$[F^0 L^1 T^0]$
4	$c'$	$[M^0 L^1 T^0]$	$[F^0 L^1 T^0]$
5	$A_L \cdot f_{Ly}$	$[M^1 L^1 T^{-2}]$	$[F^1 L^0 T^0]$
6	$\Psi = \lambda \cdot V_f$	[1]	[1]
	n=6	m=3	m=2

$$\begin{array}{l} \text{Equating exponents of time T:} \\ 0 = (-2).1 + (0).-3 + (-2).-1 = 0 \text{ satisfy} \end{array}$$

So the first  $\Pi$ -dimensionless group is

$$\Pi_1 = \frac{T_{sf}}{f_{ty} \cdot \left( \frac{A_t}{s} \right)^3}$$

The dimensional form for Eq.(Pi-2) can be written as

$$[M^0 L^0 T^0] = [M^1 L^1 T^{-2}] [M^0 L^1 T^0]^a [M^1 L^{-1} T^{-2}]^b$$

Then, equate the powers for primary dimensions

$$\begin{array}{l} \text{Equating exponents of mass M:} \\ 0 = (1).1 + (0).a + (1).b \Rightarrow b = -1 \end{array}$$

$$\begin{array}{l} \text{Equating exponents of length L:} \\ 0 = (1).1 + (1).a + (-1).b \Rightarrow a = -2 \end{array}$$

$$\begin{array}{l} \text{Equating exponents of time T:} \\ 0 = (-2).1 + (0).-2 + (-2).-1 = 0 \text{ satisfy} \end{array}$$

So the second  $\Pi$ -dimensionless group is

$$\Pi_2 = \frac{A_L \cdot f_{Ly}}{f_{ty} \cdot \left( \frac{A_t}{s} \right)^2}$$

The dimensional form for Eq.(Pi-3) can be written as

$$[M^0 L^0 T^0] = [M^0 L^1 T^0] [M^0 L^1 T^0]^a [M^1 L^{-1} T^{-2}]^b$$

Then, equate the powers for primary dimensions

$$\begin{array}{l} \text{Equating exponents of mass M:} \\ 0 = (0).1 + (0).a + (1).b \Rightarrow b = 0 \end{array}$$

$$\begin{array}{l} \text{Equating exponents of length L:} \\ 0 = (1).1 + (1).a + (-1).b \Rightarrow a = -1 \end{array}$$

$$\begin{array}{l} \text{Equating exponents of time T:} \\ 0 = (0).1 + (0).-1 + (-2).0 = 0 \text{ satisfy} \end{array}$$

So the third  $\Pi$ -dimensionless group is

$$\Pi_3 = \frac{c'}{\frac{A_t}{s}}$$

The dimensional form for Eq.(Pi-4) can be written as

$$[M^0 L^0 T^0] = [1] [M^0 L^1 T^0]^a [M^1 L^{-1} T^{-2}]^b$$

Then, equate the powers for primary dimensions

$$\begin{array}{l} \text{Equating exponents of mass M:} \\ 0 = (0).a + (1).b \Rightarrow b = 0 \end{array}$$

$$\begin{array}{l} \text{Equating exponents of length L:} \\ 0 = (1).a + (-1).b \Rightarrow a = 0 \end{array}$$

$$\begin{array}{l} \text{Equating exponents of time T:} \\ 0 = (0).0 + (-2).0 = 0 \text{ satisfy} \end{array}$$



So the fourth  $\Pi$ -dimensionless group is

$$\Pi_4 = \Psi = \lambda \cdot V_f$$

The final form of  $\Pi$ -dimensionless groups is

$$\Pi_1 = \beta_1 \cdot \Pi_2^{\beta_2} \cdot \Pi_3^{\beta_3} \cdot \Pi_4^{\beta_4}$$

$$\frac{T_{sf}}{f_{ty} \cdot \left(\frac{A_t}{s}\right)^3} = \beta_1 \cdot \left(\frac{A_L \cdot f_{Ly}}{f_{ty} \cdot \left(\frac{A_t}{s}\right)^2}\right)^{\beta_2} \cdot \left(\frac{c'}{\frac{A_t}{s}}\right)^{\beta_3} \cdot (\lambda \cdot V_f)^{\beta_4}$$

To evaluate the values of  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ , and  $\beta_4$ , the nonlinear multiple regression was used based on 4 data in this study and 33 data from the previous researches. The predicted values of  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ , and  $\beta_4$  are 45.174, 0.901, 0.365 and 0.270, respectively. The coefficient of determination of the proposed equation was 0.825 for predicting torsional resistance provided by reinforcements and fibres in fibrous normal strength concrete beams and the proposed equation becomes

$$T_{sf} = 45.174 \cdot \left(\frac{A_L \cdot f_{Ly}}{f_{ty} \cdot \left(\frac{A_t}{s}\right)^2}\right)^{0.901} \cdot \left(\frac{c'}{\frac{A_t}{s}}\right)^{0.365} \cdot (\lambda \cdot V_f)^{0.270} \cdot f_{ty} \cdot \left(\frac{A_t}{s}\right)^3$$

### 3. CONCLUSIONS

The following conclusions could be carried out from the tested results of under-reinforced fibrous normal strength concrete beams subjected to pure equilibrium torsion.

1. The torsional resistance of fibrous normal strength concrete beams was improved up to 76.4% at crack load and 94.66% at peak load due to thickening of concrete cover.
2. The value of twisting angle was reduced up to 57.8% prior to cracking whereas it was improved more than 7 times at peak load.
3. Shear strain in concrete beams at peak load was reduced up to 92.3% due to thickening of cover for the beam.
4. The axial strain in longitudinal and transverse reinforcements were reduced up to 95.6% and 89.27%, respectively due to widening of concrete cover. Thus, the contribution of reinforcement was reduced to resist torsion after cracking of the section.
5. The average spacing between spiral cracks was increased due to extra thickness of concrete cover. Consequently, the number of cracks was reduced.
6. The proposed equations by dimensional analysis to predict torsional resistance at crack and peak load reflecting the influence of concrete cover have been confirmed to show a good agreement with test results.

### ACKNOWLEDGEMENT

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### NOTATIONS

Symbol	Description
$A_t$	Area of one leg of stirrup, mm <sup>2</sup>
$A_{cp}$	Area of outside dimensions of cross-section, mm <sup>2</sup>
$A_L$	Total area of longitudinal reinforcement, mm <sup>2</sup>
$c'$	Concrete cover plus diameter of stirrup, mm
$f_c'$	Cylinder compressive strength, MPa
$f_{ty}$	Yield strength of transverse reinforcement in tension test, MPa
$f_{Ly}$	Yield strength of longitudinal reinforcement in tension test, MPa
$f_{sp}$	Split tensile strength, MPa
$f_{t*}$	Effective tensile strength, MPa
$P_{cp}$	Perimeter of concrete cross section, mm
$S$	Spacing between stirrups, mm
$V_f$	Volume fraction of fibre
$t_{d1}$	Thickness of idealized shear flow zone prior to cracking, mm
$t_{d2}$	Thickness of idealized shear flow zone after cracking, mm
$T_{cr}$	Torsional resistance provided by concrete, kN.m
$T_{sf}$	Torsional resistance provided by fibres and reinforcements, kN.m
$\lambda$	Aspect ratio of fibre
$\Theta$	Inclination angle of crack at failure, degree

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