PREDICTION OF SHEAR STRENGTH OF SLENDER RC BEAMS WITHOUT SHEAR REINFORCEMENT

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Abstract

This paper describes a study on shear strength of slender RC beams without shear reinforcement suggested by three shear evaluation methods viz. a proposed shear strength equation based on the fracture mechanics approach and two Indian codes of practice namely IS 456 (2000) and IRS 1997 (2003). Four hundred and fifty eight test beams selected from ACI–DAfStb database (2013) are considered for the study. The statistical analysis and demerit points classification indicate the proposed equation to show better estimate of shear strength of the test beams. Also the proposed equation captures well the influence of design parameters on shear strength of RC beams.

Keywords : *Shear Strength, Fracture Mechanics, ACI–DAfStb Database (2013), Demerits Points Classification.*

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1. INTRODUCTION

Shear strength of RC beams is a debate subject of the century. Understanding shear behaviour of RC beams is quite complicated. Many investigators through experimental observation have proposed numerous theories on shear mechanism of RC beams. The shear in RC beams without shear reinforcement is resisted by uncracked concrete in the compression zone, aggregate interlock across the cracks and the dowel action of longitudinal reinforcement. Percentage of longitudinal reinforcement, compressive strength of concrete and effective depth of beam are important design parameters affecting the shear capacity of RC beams. The expressions for shear strength suggested in various standard codes of practice are empirical or semi empirical and consider the above design parameters to predict the shear strength with suitable safety and strength reduction factors.

2. SHEAR STRENGTH PREDICTION BY THE SHEAR EVALUATION METHODS

Three shear evaluation methods viz. a proposed shear strength equation based on the fracture mechanics approach and the shear strength suggested by two Indian codes of practice namely IS 456 (2000) and IRS Code 1997 (2003) are considered in the present study. The expressions for shear strength suggested in these shear evaluation methods are as follows.

(i) Proposed Shear Strength Equation

Based on fracture mechanics approach, Chidananda and Raviraj (2016) proposed Eq. 1 for shear strength of RC beams without shear reinforcement

$$v_u = \frac{2}{3} f_{ctm} \left[3.09\,(p_t)^{0.31} (f_{ck})^{-0.40} (d)^{-0.34} \left(f_{syk}\right)^{0.22} \right] + \frac{0.23\,(p_t)^{8/9}\,(f_{ck}+4)^{1/3}}{(d)^{1/3}} \qquad (1)$$

where,

$$f_{ctm} = 0.3\, f_{ck}^{2/3} \qquad \text{if } f_{ck} \leq 50 \text{ MPa}$$

$$f_{ctm} = 2.12 \ln(1 + 0.1 f_{cm}) \qquad \text{if } f_{ck} > 50 \text{ MPa}$$

$p_t \ngtr 3\%$
$f_{ck} \ngtr 60$ MPa
$f_{syk} \ngtr 1000$ MPa

The first term in Eq. 1 represents the shear carried by concrete over the effective shear depth which includes the shear mechanisms of uncracked concrete and aggregate interlocking effect across the smeared crack. The second term represents the shear carried by the dowel action of longitudinal reinforcement as suggested by Reineck (1991). Further, Eq. 1 is modified to obtain the design shear strength equation $v_{ud} = \emptyset_v v_u$, where $\emptyset_v$= 0.75 is the shear strength reduction factor.
[Remarks : in S.I. units]

(ii) IS 456 (2000) and SP 24 (S&T) (1983) (Bureau of Indian Standards)

Clause 40.2 of IS 456 (2000) and Clause 39.2 of SP 24 (S&T) (1983) suggests the design shear strength τ_c of concrete in RC beams without shear reinforcement as

$$\tau_c = \frac{0.85\sqrt{0.8\, f_{ck,cu}}\,\left(\sqrt{1+5\beta}-1\right)}{6\,\beta} \leq \tau_{c,max} \qquad (2)$$

where,
$\beta = \frac{0.8 f_{ck,cu}}{6.89 P_t} \nless 1$
$f_{ck,cu} \ngtr 40$ MPa
$p_t \ngtr 3\%$
$\tau_{c,max} = 0.85\left[0.83\sqrt{0.8 f_{ck,cu}}\right]$

The factor 0.8 in the formulae is for converting cylinder strength to cube strength and 0.85 is a reduction factor similar to partial safety factor γ_m for materials.
[Remarks : in S.I. units]

(iii) IRS 1997 (2003) (Indian Railway Standard)

Clause 15.4.3 of IRS 1997 (2003) discusses the ultimate shear stress v_c of concrete in RC beams without shear reinforcement and is given by

$$v_c = \frac{0.27}{\gamma_m}\left(\frac{100 A_s}{b_w d}\right)^{1/3}\left(f_{ck,cu}\right)^{1/3} \quad (3)$$

where,
$\frac{100 A_s}{b_w d} \ngtr 3\%$
$f_{ck,cu} \ngtr 40$ MPa
$\gamma_m = 1.25$
[Remarks : in S.I. units]

3. SELECTION OF TEST BEAMS CONSIDERED FOR THE STUDY

A total of 458 slender simply supported RC test beams without shear reinforcement are selected from ACI–DAfStb database (2013) to study the performance of the considered shear evaluation methods. The selected beams satisfy the following criteria.

1. Rectangular in cross section having reinforcement only at the tension side.
2. Percentage of reinforcement p_t upto 3%.
3. Characteristic cylinder compressive strength of concrete f_{ck} in between 12 and 60 MPa.
4. Characteristic yield strength of reinforcing steel f_{syk} upto 1000 MPa.

Table 1 shows the list of investigators of 458 test beams selected from ACI–DAfStb database (2013).

Table 1 : List of Investigators of selected 458 test beams

Sl. No.	Investigators	Sl. No.	Investigators
1	Ahmad et al. (1986) (2)	29	Leonhardt and Walther (1962) (27)
2	Angelakos et al. (2001) (5)	30	Marti et al. (1977) (2)
3	Aster and Koch (1974) (5)	31	Mathey and Watstein (1963) (9)
4	Lubell et al. (2004) (9)	32	Moody et al. (1954) (21)
5	Bernander (1957) (6)	33	Morrow and Viest (1957) (9)
6	Bhal (1968) (8)	34	Mphonde and Frantz (1984) (1)
7	Bresler and Scordelis (1963) (3)	35	Niwa et al. (1987) (3)
8	Cladera and Mari (2002), Cladera (2002) (3)	36	Podgorniak-Stanik (1998) (3)
9	Chana (1981) (23)	37	Rajagopalan and Ferguson (1968) (5)
10	Chang and Kesler (1958) (15)	38	Regan (1971) (4)
11	Collins and Kuchma (1999) (5)	39	Rehm et al. (1978) (1)
12	Diaz de Cossio and Siess (1960) (2)	40	Rosenbusch and Teutsch (2002) (3)
13	Elzanaty et al. (1986) (6)	41	Rusch et al. (1962) (3)
14	Ferguson (1956) (1)	42	Salandra and Ahmad (1989) (2)
15	Ghannoum (1998) (10)	43	Taylor (1968) (8)
16	Hallgren (1994) (8)	44	Taylor (1972) (5)
17	Hamadi (1976) (4)	45	Walraven (1978) (3)
18	Hanson (1958) (3)	46	Xie et al. (1994) (1)
19	Hanson (1961) (4)	47	Lubell (2006) (7)
20	Hedmann and Losberg (1978) (4)	48	Sherwood (2008) (8)
21	Kani (1967) (41)	49	Thiele (2010) (5)
22	Kani et al. (1979) (63)	50	Winkler (2011) (5)
23	Kawano and Watanabe (1998) (2)	51	Tureyen (2001), Tureyen and Frosch (2002) (3)
24	Kim and Park (1994) (14)	52	Bentz and Buckley (2005) (9)
25	Krefeld and Thurston (1966) (28)	53	Krefeld and Thurston (1966) (12)
26	Kung (1985) (5)	54	Leonhardt and Walther (1962) (6)
27	Kulkarni and Shah (1998) (4)	55	Shioya (1989) (3)
28	Laupa et al. (1953) (2)	56	Iguro et al. (1985) (5)

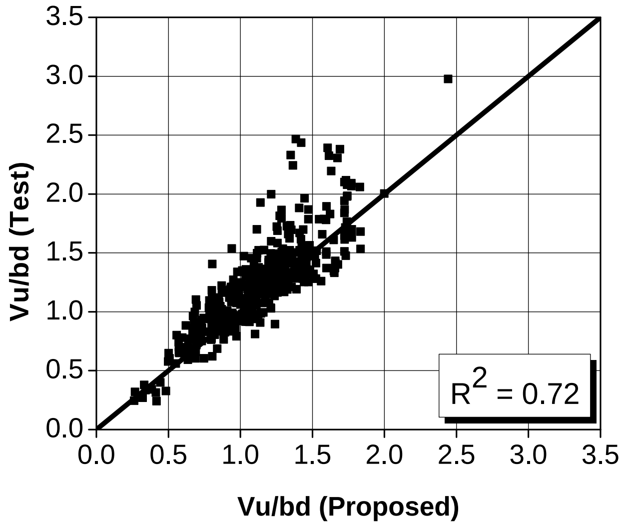
In Table 1, the values in the first and the second parentheses indicate the year of testing and the number of selected beams of the investigators respectively. Among the selected 458 test beams, 432 beams are subjected to either mid point or two point loadings and the remaining 26 beams, tested by the last four investigators (Sl. No. 53 to 56), are subjected to uniformly distributed loading. Table 2 shows the consolidated limits for various parameters of selected 458 test beams.

Table 2 : Consolidated limits for the parameters of selected 458 test beams

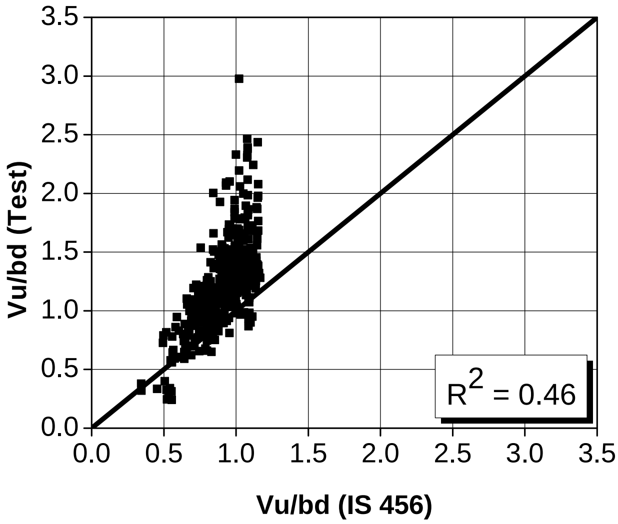
Sl. No.	Parameter	Unit	Minimum	Maximum
1	b	mm	50	3005
2	d	mm	65	3000
3	a/d	–	2.4	8.1
4	p_t	(%)	0.139	2.890
5	f_{ck}	MPa	12.27	59.45
6	f_{syk}	MPa	228.18	908.18

4. STATISTICAL ANALYSIS OF THE SHEAR EVALUATION METHODS

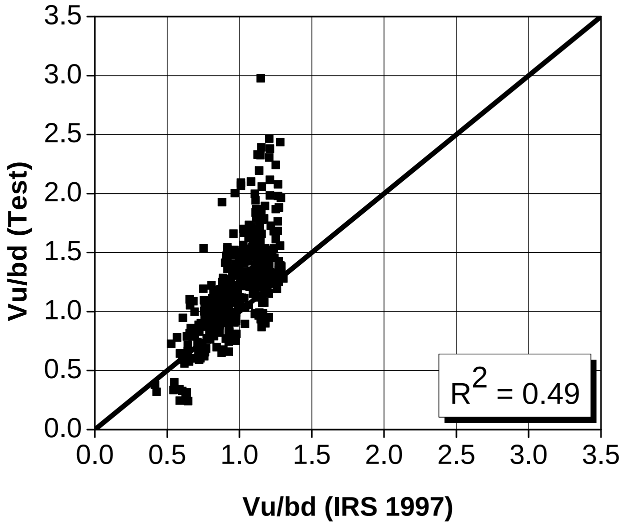
Unit partial safety factors, unit reduction factors and suitable conversion factors for characteristic concrete compressive strength and characteristic yield strength of reinforcing steel given in Appendix A are applied to the expressions suggested in the three considered shear evaluation methods to predict the shear strength (V_{pre}) of selected 458 test beams. The predicted shear strengths are compared with the corresponding experimental shear strength (V_{test}) results. The statistical results are summarized in Fig. 1 and Table 3.



(a)



(b)



(c)

Fig. 1 [(a) to (c)]: Correlation between the prediction from shear evaluation methods and the test results of selected 458 test beams

Table 3 : Statistical results of the shear evaluation methods

Sl. No.	Shear evaluation method	Statistical results		
		$\left(\frac{V_{test}}{V_{pre}}\right)_{Average}$	Standard deviation	Coefficient of Variation (CV) (%)
1	Proposed equation	1.10	0.17	15.45
2	IS 456 (2000)	1.34	0.30	22.39
3	IRS 1997 (2003)	1.23	0.27	21.95

From Fig. 1 and Table 3, it is inferred that the shear predicted by the proposed shear strength equation shows a better correlation with a correlation coefficient R^2 of 0.72, and an average (V_{test}/V_{pre}) ratio of 1.10 and a low CV of 15.45% in predicting the shear strength of selected 458 test beams than the two Indian codes of practice.

5. DEMERIT POINTS CLASSIFICATION

The demerit points classification suggested by Collins (2001) measures agreement between V_{test} and V_{pre}. In this classification, the ratio $\frac{V_{test}}{V_{pre}}$ is calculated for each of the beam in the database. A demerit point value as given in Table 4 is assigned to each beam which depends on $\frac{V_{test}}{V_{pre}}$ ratio. The summation of the demerit points of all the beams shows the overall performance of the shear evaluation method. A smaller summation indicates the shear evaluation method to be more reliable in predicting the shear strength.

Table 4 : Collins (2001) demerit points classification

Sl. No.	Classification	$\frac{V_{test}}{V_{pre}}$	Demerit points
1	Extremely dangerous	<0.50	10
2	Dangerous	0.50 – 0.65	5
3	Low safety	0.65 – 0.85	2
4	Appropriate safety	0.85 – 1.30	0
5	Conservative	1.30 – 2.00	1
6	Extremely conservative	>2.00	2

The demerit points classification is applied to evaluate the performance of the three shear evaluation methods in predicting the shear strength of selected 458 test beams. The demerit points value of the shear evaluation methods for each classification are summarized in Table 5. A low value of 'Total demerit points' of the proposed shear strength equation indicates that it performs well in predicting the shear strength than the two Indian codes of practice.

Table 5 : Demerit points value of the shear evaluation methods

Sl. No.	Shear evaluation method	$\frac{V_{test}}{V_{pre}}$						Total demerit points
		<0.50	0.50 to 0.65	0.65 to 0.85	0.85 to 1.30	1.30 to 2.00	>2.00	
1	Proposed equation	0	1	24	381	52	0	105
2	IS 456 (2000)	2	4	9	218	210	15	298
3	IRS 1997 (2003)	4	3	21	289	131	10	248

6. PARAMETRIC STUDIES

Parametric studies are carried out to study the influence of the design parameters viz. p_t, f_{ck} and d on shear strength of RC beams predicted by the shear evaluation methods considering respectively a few beams tested of Krefeld and Thurston (1966) and Kani et al. (1979); Moody et al. (1954) and Sherwood (2008); and Bhal (1968) and Walraven (1978). The details of the RC beams considered for the parametric study are tabulated in Appendix B. Comparison of shear predicted by the shear evaluation methods with the test results, for the three design parameters, are shown in Fig. 2. It is inferred that the proposed shear strength equation shows better agreement with the test results than the two Indian codes of practice.

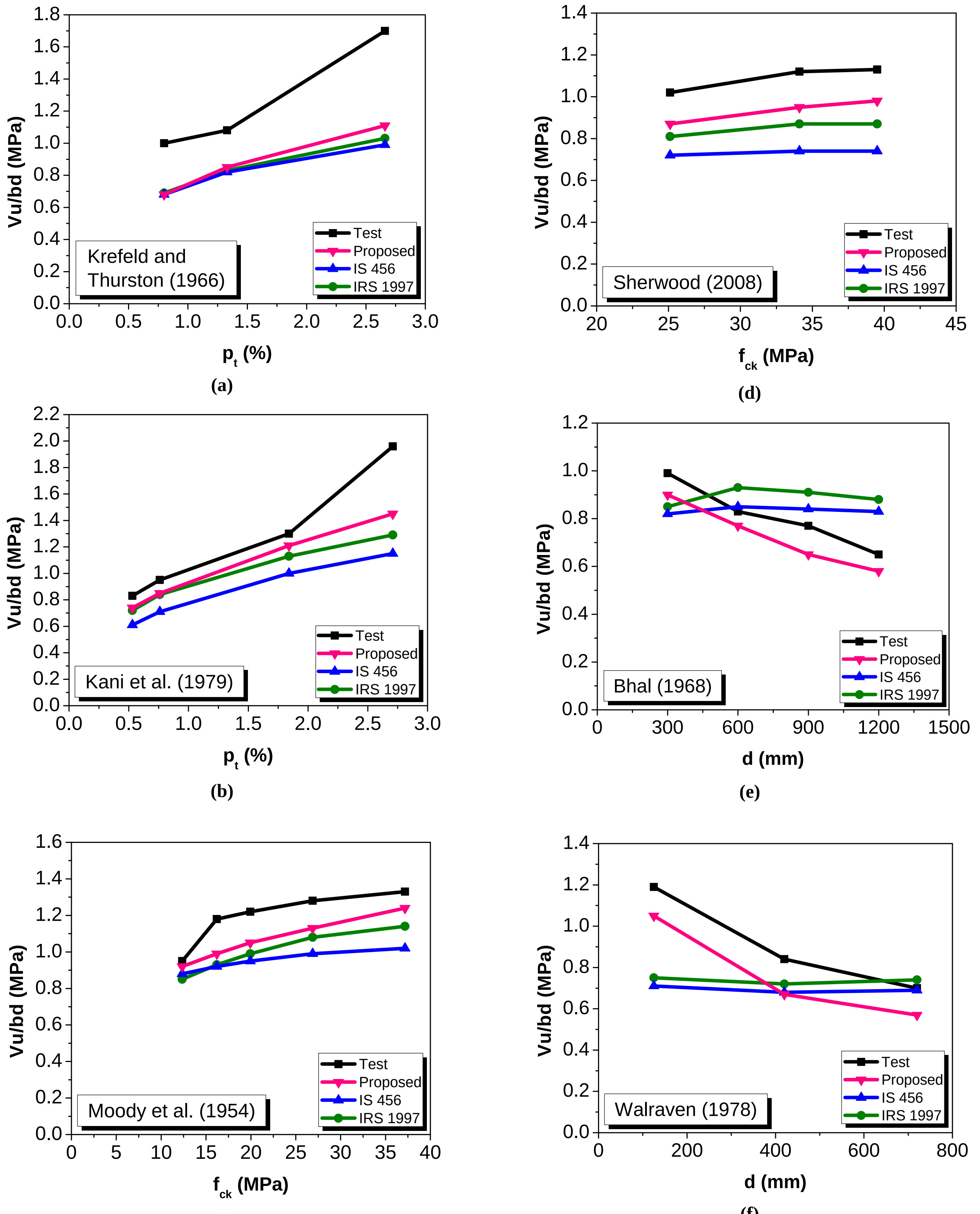


Fig. 2 [(a) to (f)]: Comparison of the shear predicted by shear evaluation methods with the test results of Krefeld and Thurston (1966), Kani et al. (1979), Moody et al. (1954), Sherwood (2008), Bhal (1968) and Walraven (1978)

7. CONCLUSIONS

The study presents the prediction of shear strength of selected 458 slender RC beams without shear reinforcement by three shear evaluation methods viz. a proposed shear strength equation based on the fracture mechanics approach and two Indian codes of practice namely IS 456 (2000) and IRS 1997 (2003). The following conclusions are drawn.

1. The statistical analysis and demerit points classification indicate that the shear strength predicted by the proposed equation agrees fairly well with the test results, whereas IS 456 (2000) and IRS 1997 (2003) predict the shear strength conservatively.
2. The comparison with the test results of Krefeld and Thurston (1966) and Kani et al. (1979); Moody et al. (1954) and Sherwood (2008); and Bhal (1968) and Walraven (1978) shows that the influence of design parameters viz. p_t, f_{ck} and d on shear carrying capacity is well captured by the proposed equation than the two Indian codes of practice.
3. It is suggested to consider the proposed equation for evaluating the shear strength of RC beams without shear reinforcement for practical design than the two Indian codes of practice.

NOTATION

a	Shear span
b	Width of beam
d	Effective depth of beam
a/d	Shear span to effective depth ratio
$f_{c,cu}$	Mean cube (150 mm) compressive strength of concrete
f_{ck}	Characteristic cylinder (150x300 mm) compressive strength of concrete
$f_{ck,cu}$	Characteristic cube (150 mm) compressive strength of concrete
f_{cm}	Mean cylinder (150x300 mm) compressive strength of concrete
f_{ctm}	Mean axial tensile strength of concrete
$f_{lc,cu}$	Uniaxial compressive strength of concrete derived from $f_{c,cu}$
$f_{lc,cyl}$	Uniaxial compressive strength of concrete derived from f_{cm}
f_{sy}	Yield strength of reinforcing steel
f_{syk}	Characteristic yield strength of reinforcing steel (i.e. Grade of steel)
p_t	Percentage of reinforcement
D	Overall depth of beam
V_{pre}	Predicted shear strength
V_{test}	Experimental shear strength

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Appendix A

Conversion factors for characteristic concrete compressive strength and characteristic yield strength of reinforcing steel

1. Concrete :

a. Cylinder compressive strength [Reineck et al. (2010)]
$f_{ck} = f_{cm} - \Delta f$, where $\Delta f = 4$ MPa (for laboratory conditions)
$f_{lc,cyl} = 0.95\, f_{cm}$

b. Cube compressive strength [Reineck et al. (2010)]
$f_{lc,cu} = 0.79\, f_{c,cu}$
if $f_{cm} \leq 54$ MPa
$f_{lc,cu} = 0.95\, f_{c,cu} - 10.5$
if $f_{cm} > 54$ MPa

c. Relation between cylinder and cube compressive strengths are obtained by equating uniaxial compressive strengths.

d. $f_{ck,cu} = f_{c,cu} - 3$ [from Clause 16.1 of IS 456 (2000) for compliance requirement]

2. Reinforcing steel :

$f_{syk} = \frac{f_{sy}}{1.1}$ [Reineck et al. (2010)]

Appendix B

Details of RC beams for the study of design parameters

Sl. No.	Specimen Nomenclature	b (mm)	d (mm)	a/d	p_t (%)	f_{sy} (MPa)	$f_{lc,cyl}$ (MPa)	V_{test} (kN)
Krefeld and Thurston (1966) – Influence of p_t								
1	IV-13A2	152.4	319	2.67	0.798	366	18.93	48.48
2	15B2	152.4	316	2.69	1.332	386	19.66	52.04
3	18E2	152.4	316	2.69	2.664	386	18.80	81.84
Kani et al. (1979) – Influence of p_t								
4	179	153.2	264.2	2.57	0.526	400	30.73	33.58
5	163	156.0	272.5	2.49	0.756	378	33.61	40.48
6	197	150.4	273.6	2.48	1.835	376	34.20	53.38
7	214	153.4	271.8	2.50	2.708	412	34.20	81.84
Moody et al. (1954) – Influence of f_{ck}								
8	16	152.4	268.2	3.41	1.898	310	15.53	38.83
9	12	152.4	268.2	3.41	1.898	310	19.20	48.17
10	10	152.4	268.2	3.41	1.898	310	22.73	49.95
11	7	152.4	268.2	3.41	1.898	310	29.35	52.17
12	9	152.4	268.2	3.41	1.898	310	39.11	54.40
Sherwood (2008) – Influence of f_{ck}								
13	S-40 N2	122	280	2.78	0.835	494	27.65	34.86
14	S-20 N1	122	280	2.78	0.835	494	37.24	39.16
15	S-50 N2	122	280	2.78	0.835	494	41.33	40.58
Bhal (1968) – Influence of d								
16	B1	240.0	300.0	2.94	1.257	426	22.02	70.99
17	B2	240.0	600.0	2.94	1.257	426	28.12	119.48
18	B3	240.0	900.0	2.94	1.257	426	26.11	166.38
19	B4	240.0	1200.0	2.94	1.257	426	23.95	187.10
Walraven (1978) – Influence of d								
20	A1	200	125	3	0.829	440	22.90	29.80
21	A2	200	420	3	0.741	440	22.90	70.60
22	A3	200	720	3	0.792	440	23.20	100.80