TRANSIENT RESPONSE OF LAMINATED COMPOSITE SPHERICAL SHELL CAP

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Abstract

The transient behaviour of laminated composite spherical shell cap is investigated using the finite element method . An eight node degenerated isoparametric shell element is considered with five degrees of freedom at each node. The governing equations of forced vibration are solved using the Newmark's direct integration method. Results are presented for dynamic response of deflection and stresses of orthotropic and laminated spherical shell caps subjected to suddenly applied uniform normal pressure.

Keywords: *Transient response, Spherical shell cap, Forced vibration, Composite shell, Finite element method*

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1. INTRODUCTION

Fibre reinforced plastic laminated composite materials are being increasingly used in aerospace and other applications due to their superior properties such as high specific strength, high specific stiffness and low specific density . They, in the form of shells, find application in aerospace and other industries. Spherical shells are used for many applications such as aerospace vehicles, roof domes, pressure vessels and submarines. The study of forced vibration of composite spherical shell cap is an important engineering problem.

The forced vibration of laminated composite spherical shell cap has been considered by few researchers. Sun and Sun [1] investigated the dynamic response of laminated composite shells subjected to axially symmetric dynamic loadings using the classical method of separation of variables. A specific example of simply supported laminated shell subjected to suddenly applied dynamic pressure is solved. Sheinman and Greif [2] considered the free and forced vibration of thin walled shells of revolution using Hamilton's variational principle. The equations do not include the effect of transverse shear deformation and rotary inertia. Results are presented mostly for cylindrical shells. A result showing the dependence of fundamental frequency on fiber orientation is given for simply supported spherical dome. Wu et. al. [3] investigated the natural frequencies and the dynamic response of thin laminated composite shells of general form using a high-order curved shell finite element. Numerical results are presented for cylindrical shells, shells of positive and negative Gauss curvature and shells of translation. An example for transient response of spherical shell panel is given. Chao and Tung [4] studied the dynamic response of orthotropic hemispherical shells subjected to idealized step pressure , rectangular pressure pulse and realistic blast loading. Xu and Chia [5] investigated nonlinear vibration of thin laminated spherical caps with flexible supports using a Fouier-Bessel series solution . Numerical results for non-linear free vibration of laminated spherical caps are presented for various values of cap rise , number of layers, material properties and in-plane edge / rotational stiffnesses. Cheung and Fu [6] derived the non-linear axisymmetric dynamic governing equations for symmetric cross-ply shallow spherical caps. Numerical results for dynamic response and dynamic buckling of spherical caps with circular holes are presented. Sai Ram and Sreedhar Babu [7] investigated the free vibration of composite spherical shell cap with and without a cutout using finite element method based on higher-order shear deformation theory.

In this paper, the transient behaviour of laminated composite spherical shell cap is investigated using the finite element method based on first-order shear deformation theory. An eight node degenerated isoparametric shell element is considered with five degrees of freedom at each node. The governing equations of forced vibration are solved using the Newmark's direct integration method. Dynamic response of deflection and stresses are studied for orthotropic and symmetrically and anti-symmetrically laminated cross-ply spherical shell caps, with simply supported and clamped boundary conditions, subjected to suddenly applied uniform normal pressure. Dynamic load factors for deflection and stresses are presented.

2. ANALYSIS

Consider a laminated composite shell (Fig.1) of uniform thickness, consisting of a number of thin laminae, each of which may be arbitrarily oriented at an angle 'θ' with reference to the x-axis of the local co-ordinate system . The displacements along the local coordinate axes x, y and z at any point in the shell are assumed as

$$u = u_0 + z\theta_y \; ; \quad v = v_0 - z\theta_x \; ; \quad w = w_0. \tag{1}$$

The complete governing equations of the laminated composite are given in ref [7].

An eight noded degenerated isoparametric shell element [9,10] is considered in the present analysis. Five degrees of freedom are considered at each node. The stiffness matrix and nodal load vector of the element are derived using the principle of minimum potential energy. The geometry of the element is defined by the global coordinates X, Y and Z. The complete finite element formulation to obtain stiffness matrix [Ke], mass matrix [Me] and load vector {Fe(t)} of the element are given in ref [7].

Stiffness matrix [Ke], mass matrix [Me] and load vector {Fe(t)} of individual elements are assembled to obtain Global stiffness matrix [K], and Global mass matrix [M] and Global load vector{F(t)}. The governing equation of motion for forced vibration is given by

$$[K]\{U\} + [M]\{\ddot{U}\} = \{F(t)\} \tag{15}$$

where
{U}=Global displacement vector
{Ü}= Global acceleration vector

The above governing equation is solved using Newmark's direct integration method [11].

3. RESULTS AND DISCUSSION

The analysis described in the previous sections is applicable to the forced vibration of various types of laminated composite shells . In the present investigation, the transient behaviour of laminated composite spherical shell cap (Fig.2) is investigated. The local coordinate axes x and y are oriented along circumferential and meridional directions , respectively. Fibre orientation □ is measured with reference to circumferential direction, i.e. x-axis. Hence, fibre orientation 0^0 means that the fibres are along the circumferential direction, whereas fibre orientation 90^0 means that the fibres are along the meridional direction. It is assumed that fibre volume in a lamina is constant along the meridian. Results are presented for orthotropic and cross-ply laminated composite spherical shell cap with simply supported and clamped boundary conditions. U0i and V0i are restrained along the supported edge. The following lamina material properties are used throughout the investigation. E1= 172369MPa, E2=6895MPa, G12 = G13 = 3448 MPa, G23=1379MPa, ν12 =0.25 and ρk $=16\times10^{-10}$N.s^2/mm^4. For finite element analysis, the spherical shell cap is considered to have a circular cutout at the crown (Fig.2a) with □ 0 =0.001□ 1 and the value of □ □ □ is considered as 10^0. The radius of curvature and the thickness of the spherical shell cap considered are 1000mm and 5mm respectively. The intensity of suddenly applied uniform normal pressure (p0) is 0.1N/mm^2. Convergence studies were carried out in ref. [8] and the entire spherical shell cap is modeled with 112 elements, a quarter of which is shown in Fig.2b. The validity of the present finite element analysis is verified by means of the following problems.

An isotropic simply supported square plate of size 25cm x 25cm, thickness = 5cm, ν_{12} = 0.25, E = 2.1 x 10^6 N/cm^2 and □ □ = 8 x 10^{-6} N sec^2/cm^4 is analysed subjected to a uniformly distributed pulse load. A quarter of the plate is discretised with 64 elements. The results obtained are compared with those in [12] and are shown in Table 1. Next, a simply supported [$0^0/90^0$] spherical shell panel subjected suddenly applied uniform normal pressure is considered. A quarter of the panel is analysed with 4 elements. The central deflection is plotted with time (Fig.3) and is compared with that available in reference [13]. R = 100cm, a = b =50cm, h= 1cm, E1=25x10^6 N/cm^2, E2 = 10^6 N/cm^2, G12 = G13 = 0.5x10^6 N/cm^2 , G23 = 0.2x10^6 N/cm^2 , ν12 = 0.25 and ρ= 1N.s^2/cm^4. From results presented in Table 1 and Fig.3, it is evident that the present finite element analysis is accurate and reliable.

The response of deflection of the crown and the stress □ x at the top of the crown of simply supported and clamped orthotropic and laminated spherical shell caps are 6 shown in Figs. 4-8. The dynamic load factors for deflection and stresses are shown in Table 2. From the results presented, the following observations may be made.

In the case of [0^0] spherical shell cap (Fig.4), the shell cap deflects more downwards than upward during vibration. The normal stress at the top of the crown is predominantly compressive in nature. The responses are similar for simply supported and clamped boundary conditions. The variations of deflection and normal stress at top of the crown with time are almost repeating without appreciable change. The dynamic load factors for normal stress at the top are very high ,viz., 47.49 and 41.03 for simply supported and clamped boundary conditions , respectively.

In the case of [90^0] spherical shell cap (Fig.5), the shell cap deflects more downwards than upward during vibration. The normal stress at the top of the crown is predominantly compressive in nature. The responses are dissimilar for simply supported and clamped boundary conditions. The variations of deflection and normal stress at the top of the crown are repeating with the time with appreciable change.

In the case of [$0^0/90^0$] spherical shell cap (Fig.6), the shell cap deflects more downwards than upward during vibration. The normal stress at top of the crown is predominantly tensile in nature. The responses are quite different for simply supported and clamped boundary conditions. The variations of deflection and normal stress at the top of the crown are not repeating with time and not even one cycle of vibration is completed in 25 seconds.

In the case of [$90^0/0^0$] spherical shell cap (Fig.7), the shell cap deflects only downwards during vibration. The normal stress at the top of the crown is predominantly compressive in nature. The responses are similar for simply supported and clamped boundary conditions. The variations of deflection and normal stress at the top of the crown with time are almost repeating without appreciable change.

In the case of [$0^0/90^0/0^0$] spherical shell cap (Fig.8), the shell cap deflects more downwards than upward during vibration. The normal stress at the top at the crown is predominantly compressive in nature. The responses are similar for simply supported and clamped boundary conditions. The variations of deflection and normal stress at the top of the crown with time are almost repeating without appreciable change.

4. CONCLUSION

The transient behavior of laminated composite spherical shell cap is analysed using the finite element method based on first-order shear deformation theory. An eight-node degenerated isoparametric shell element is considered with five degrees of freedom at each node. The governing equation of forced vibration is solved using by direct integration using Newmark‘s numerical integration method. Responses of deflection and stresses are presented for orthotropic and laminated symmetric/ anti-symmetric spherical shell caps with simply supported and clamped boundary conditions subjected to transient uniform normal pressure. From the results presented, the following conclusions may be made.

1. During vibration, the spherical shell caps considered deflected downward more than upwards.
2. Higher amplitude of vibration is observed in the case of orthotropic spherical shell caps with fibres along circumferential direction(0^0). The stresses developed in these shell caps are less. This is due to little stiffness in the meridonial direction.
3. The time taken to complete one cycle of vibration is only a fraction of a second for [0^0] and [90^0] spherical shell caps whereas it is about 28 seconds in the case of [$0^0/90^0/0^0$] spherical shell cap and it is still higher for [$0^0/90^0$] and [$90^0/0^0$] spherical shell caps.

NOTATION

E1,E2 Young’s moduli along 1 and 2 axes of a lamina
FEM Finite element method
G12, G13, G23 Shear moduli in 1-2, 1-3 and 2-3 planes of a lamina, respectively
n Number of layers
p0 Suddenly applied uniform normal pressure
R Radius of curvature of spherical shell cap
t Thickness of a shell
u,v,w Displacement components along x, y and z axes, respectively
u0 , v0 ,w0 Displacements of the mid-surface along x , y and z axes, respectively
W Central deflection along Z-axis
U0i , V0i ,W0i Displacements of the mid-surface along X ,Y and Z axes, respectively at a node i
x,y,z Local Cartesian coordinate axes at any point on the mid-surface of a shell, x and y axes being tangential to the mid-surface whereas z-axis is normal the mid-surface
X,Y,Z Global Cartesian coordinate axes
Ө Fibre orientation in a lamina with reference to x-axis
ν Poisson’s ratio of isotropic material
ν_{12} Poisson’s ratio with respect 1 and 2 axes of a lamina
σx Normal stress along x-axis
ρ Mass per unit volume of isotropic material
ρ_k Mass of a lamina per unit volume

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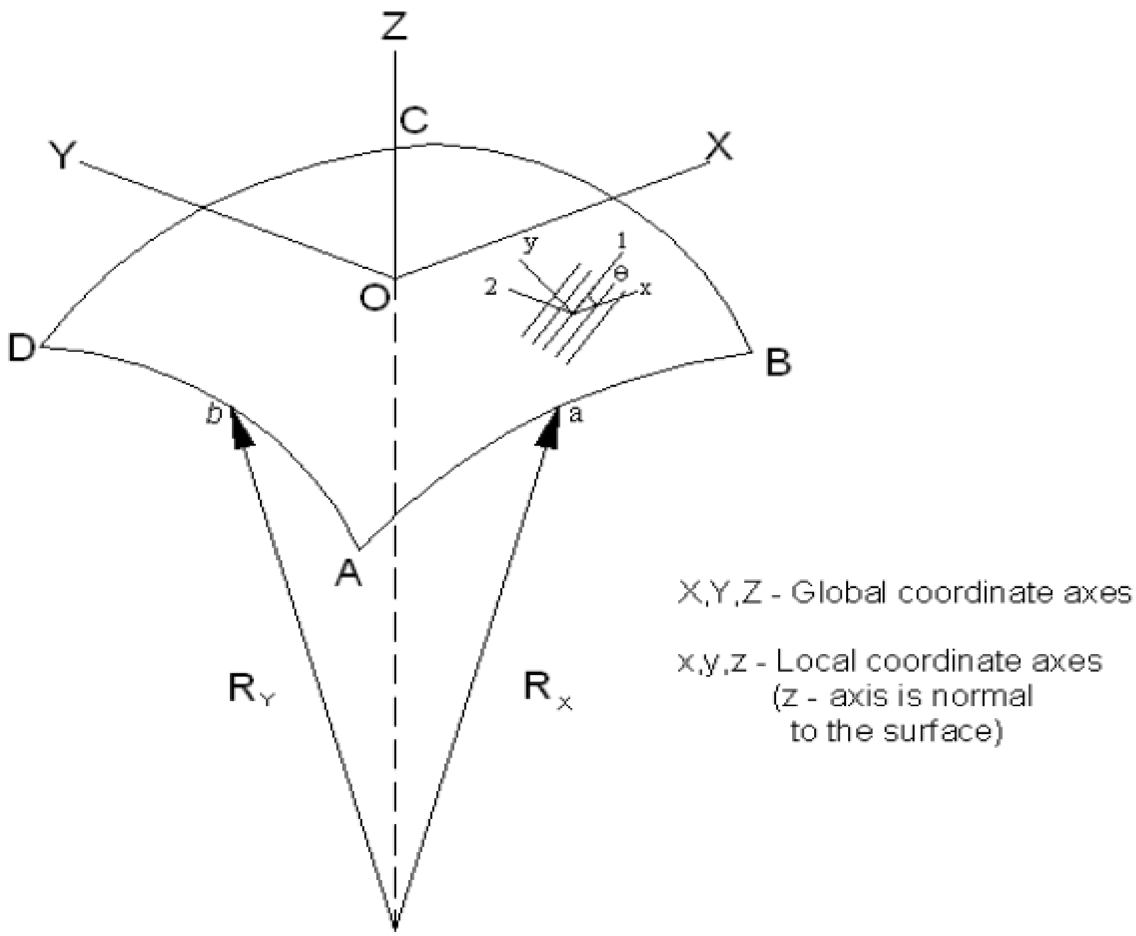
Table 1 Comparison of results of a simply supported isotropic square plate subjected to uniformly distributed pulse load of intensity 10 N/cm^2

Time (μs)	Central deflection cm (*10^3)	Normal stress at centre N/cm^2
40	0.1372 (0.1375)	5.630 (5.799)
80	0.7372 (0.7388)	60.87 (61.49)
120	1.3804(1.3844)	120.19(120.2)
160	1.6207(1.6232)	137.01(137.8)
200	1.3626(1.3671)	120.24(120.6)

Values in ( ) are from ref. [12]

Table 2 Dynamic load factors of deflection and stress

Laminate configuration	Simply supported		Clamped	
	Deflection at the crown	σ_x at the top of the crown	Deflection at the crown	σ_x at the top of the crown
[0^0]	2.23	47.50	2.19	41.50
[90^0]	2.75	2.35	2.73	2.46
[0^0/90^0]	1.54	2.16	2.85	2.27
[90^0/0^0]	2.39	2.37	2.41	2.42
[0^0/90^0/0^0]	2.57	1.53	2.63	1.67



(a) Shell geometry

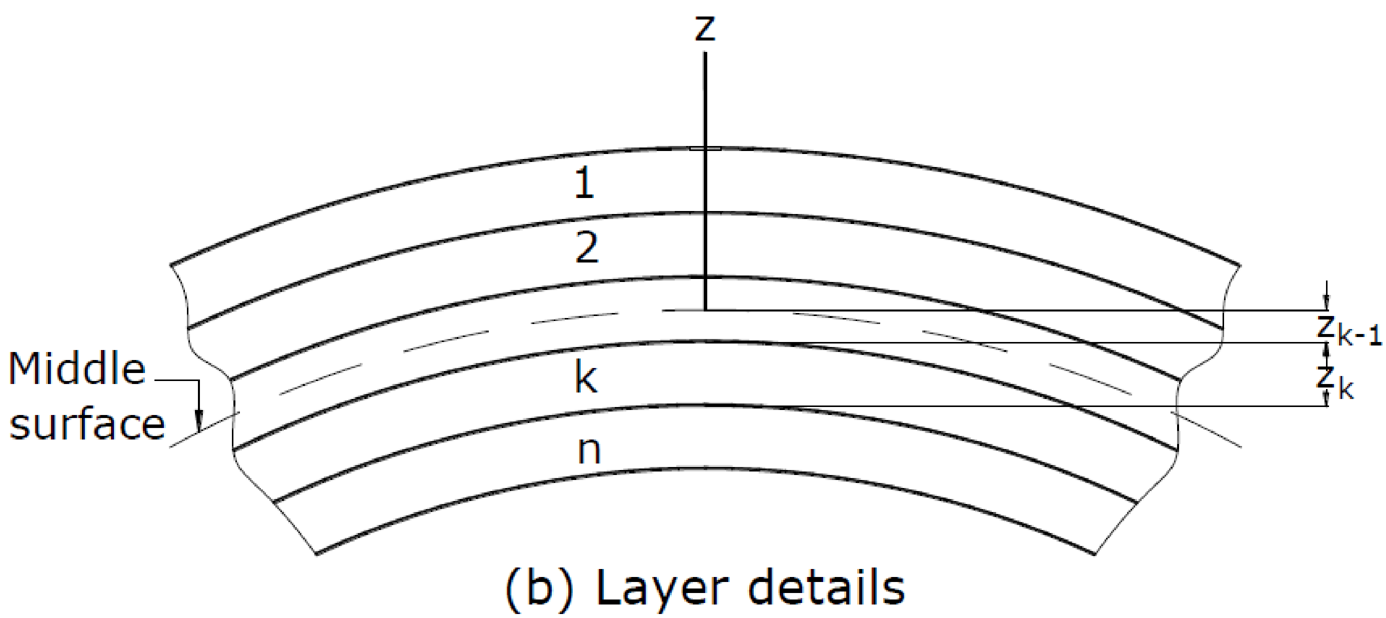
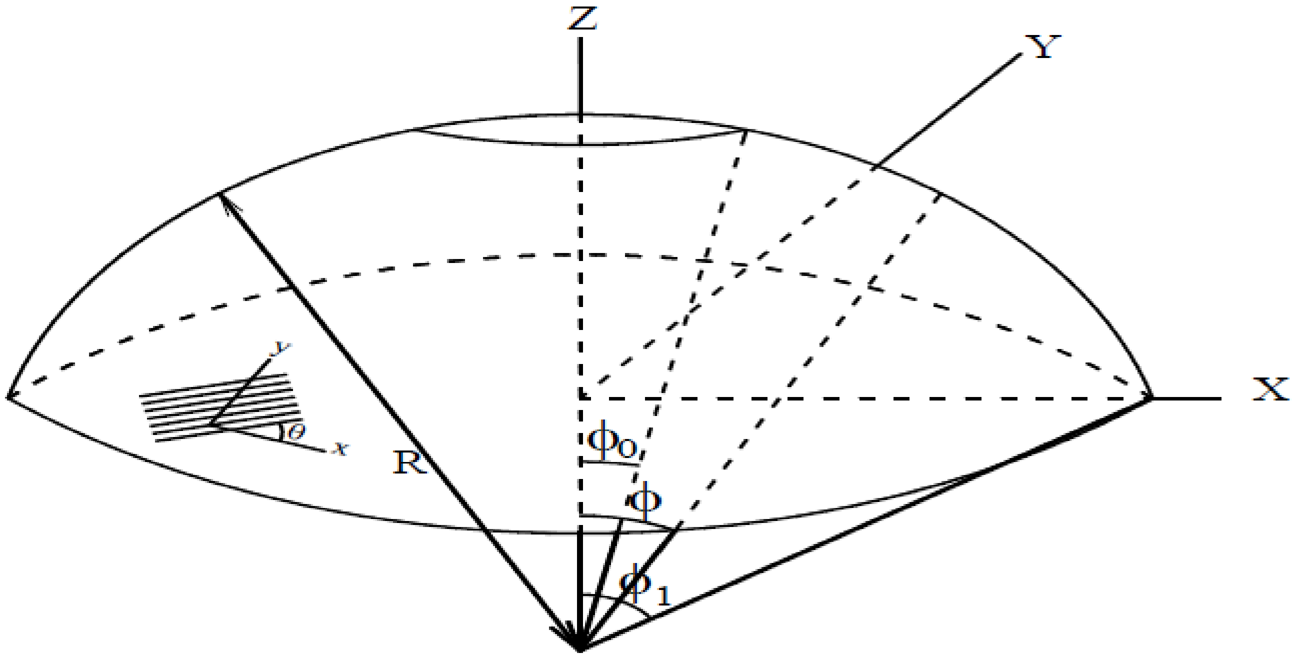
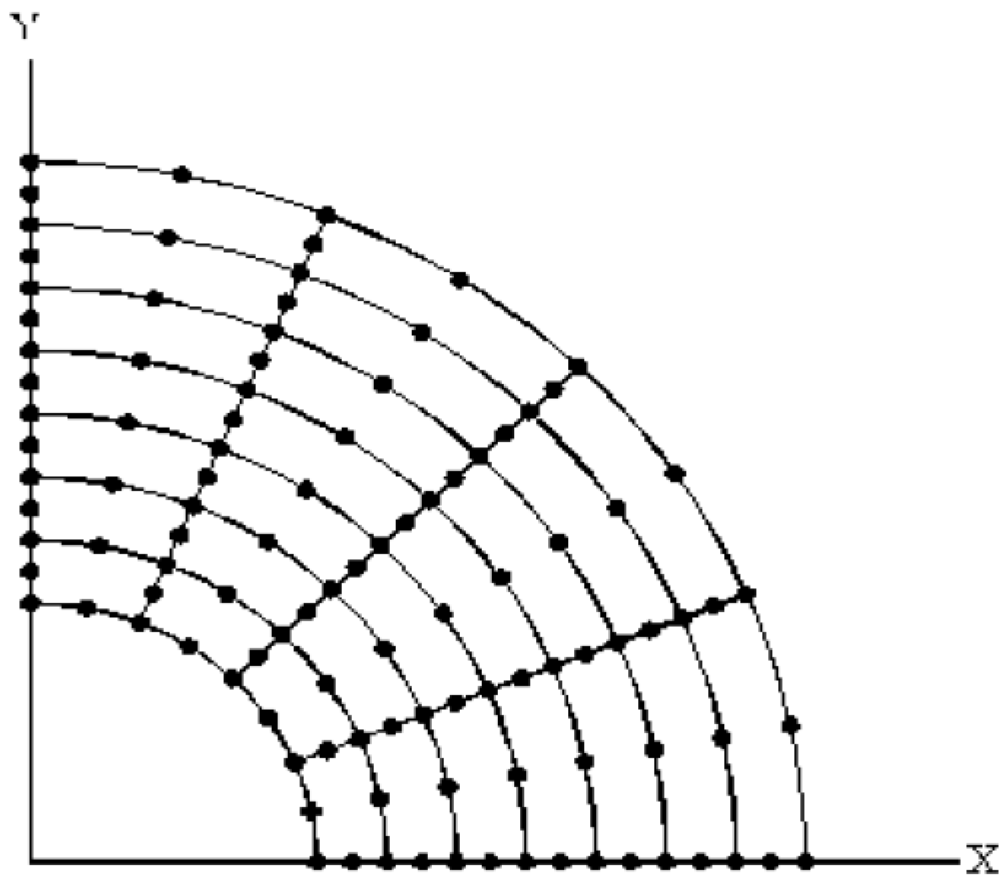


Fig.1 Laminated composite shell



(a) Geometry



(b) Discretisational details for a quarter

Fig.2 Laminated composite spherical shell cap

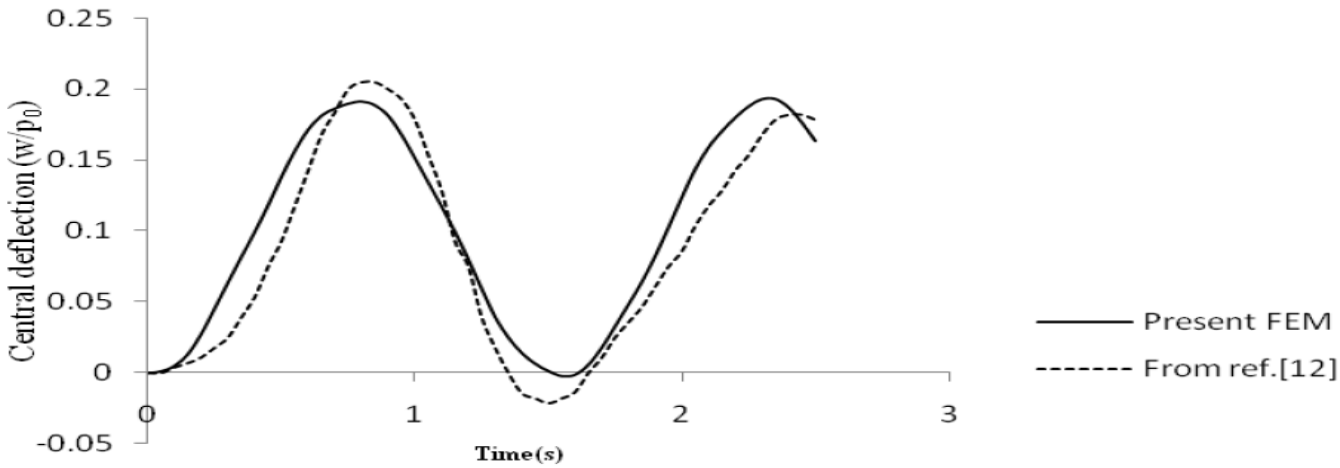
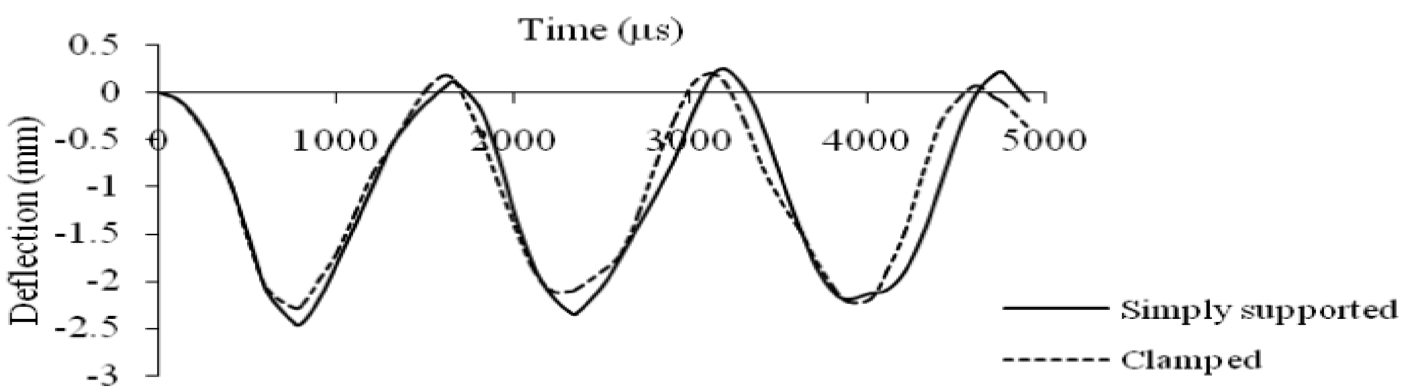
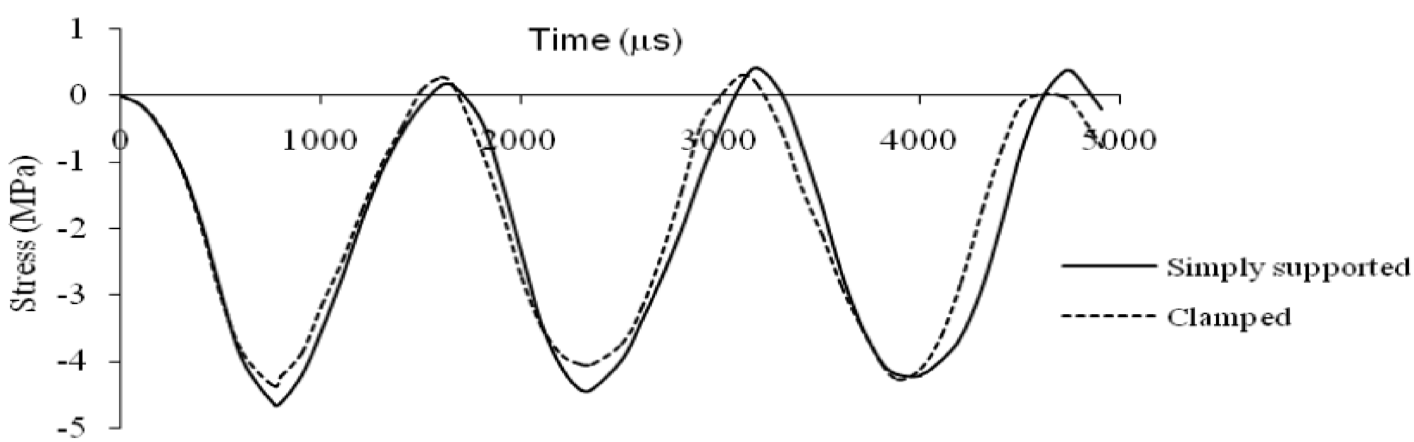


Fig.3 Central deflection verses time for [0°/90°] spherical shell panel

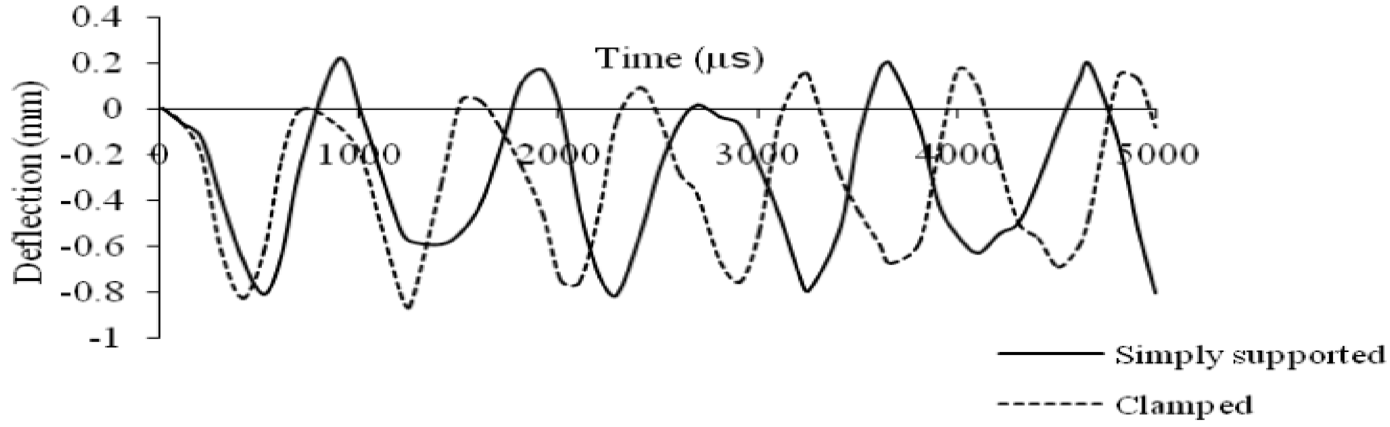


(a) Deflection at the crown

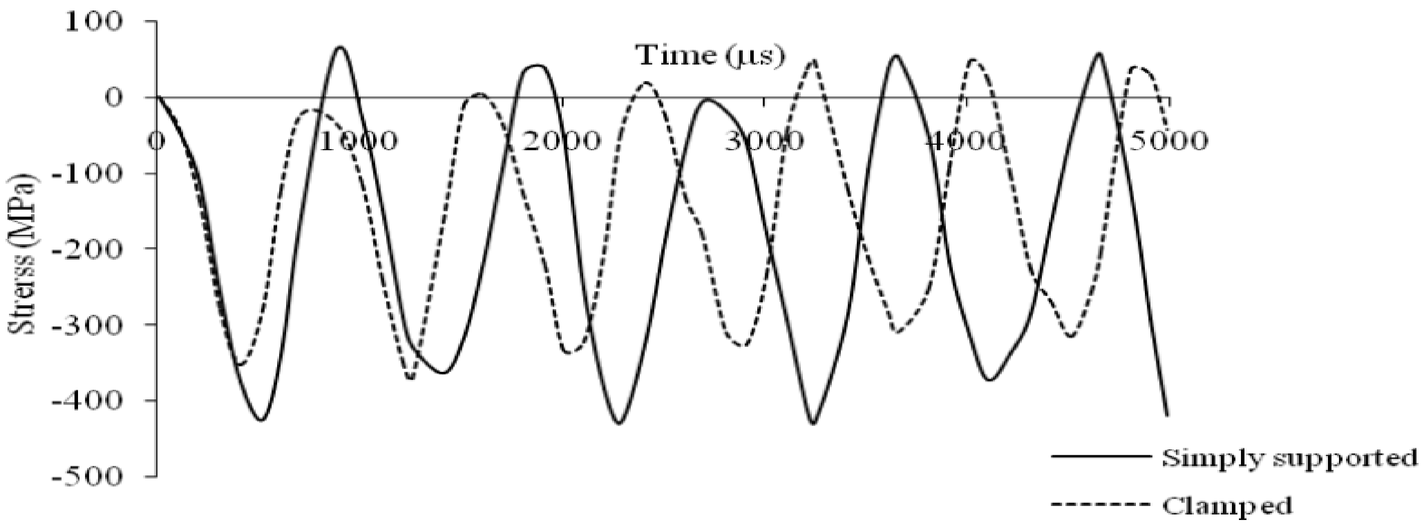


(b) Stress σ_x at the top of the crown

Fig. 4 Dynamic response of [0^0] spherical shell cap

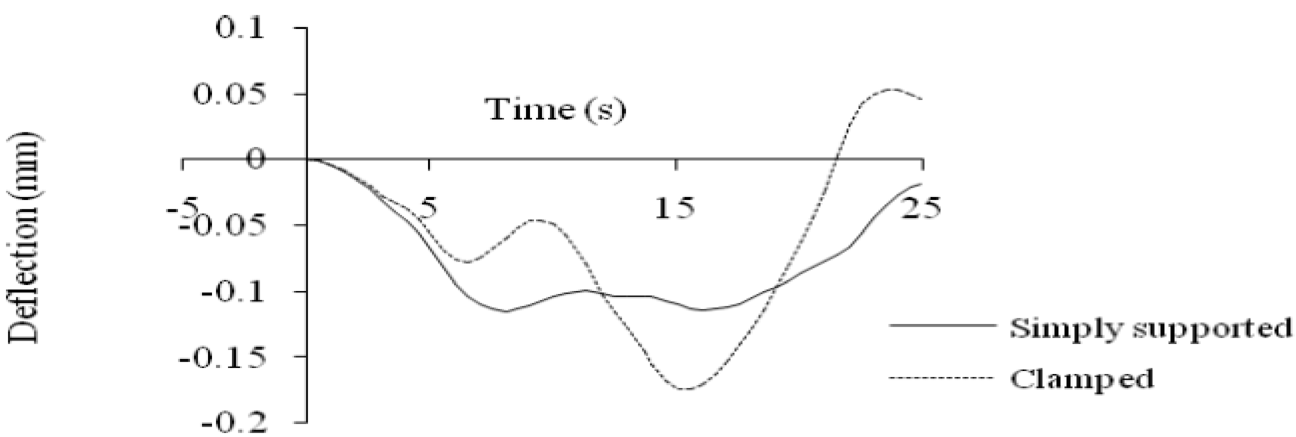


(a) Deflection at the crown

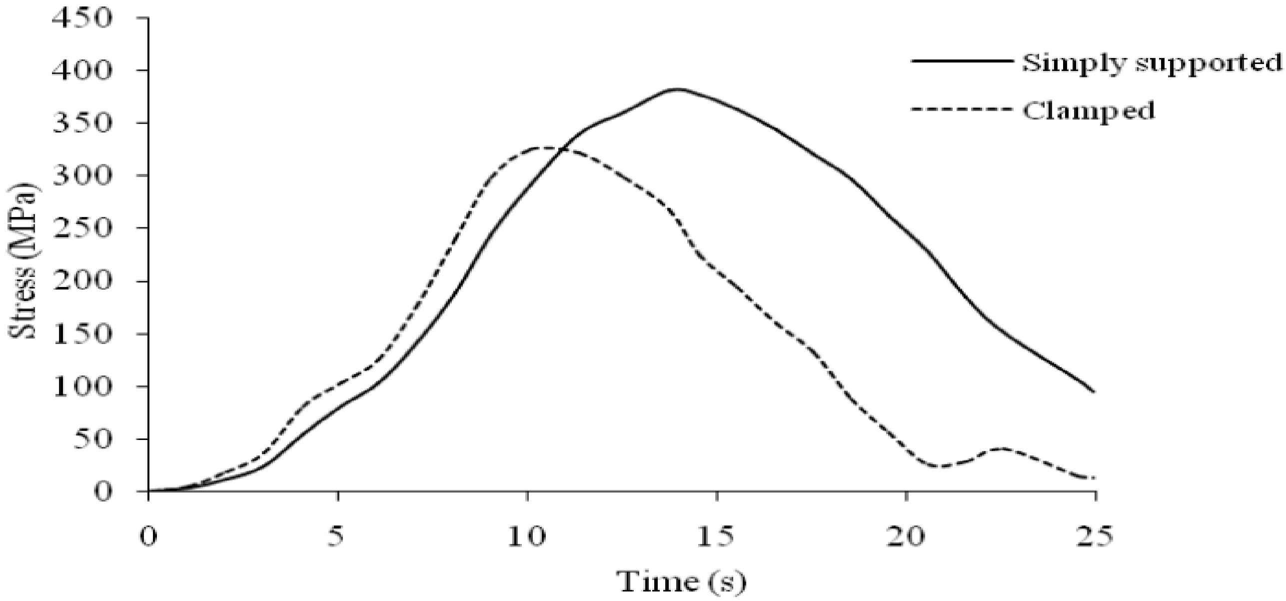


(b) Stress σ_x at the top of the crown crown

Fig. 5 Dynamic response of [90^0] spherical shell cap

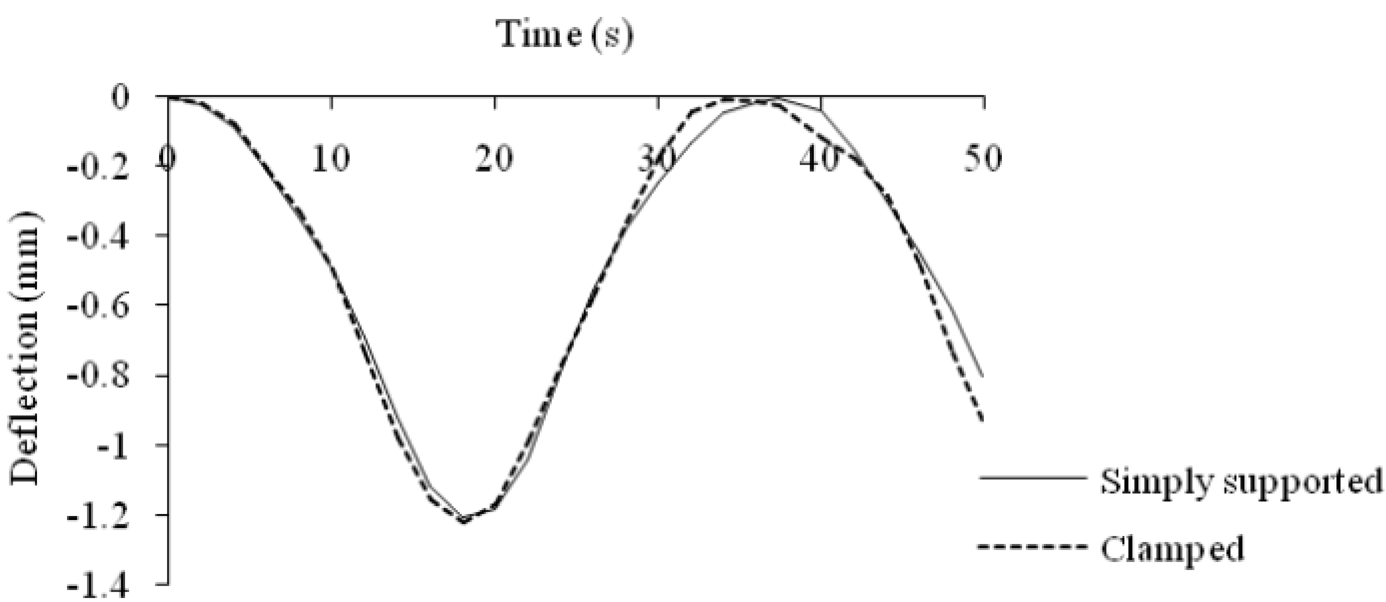


(a) Deflection at the crown

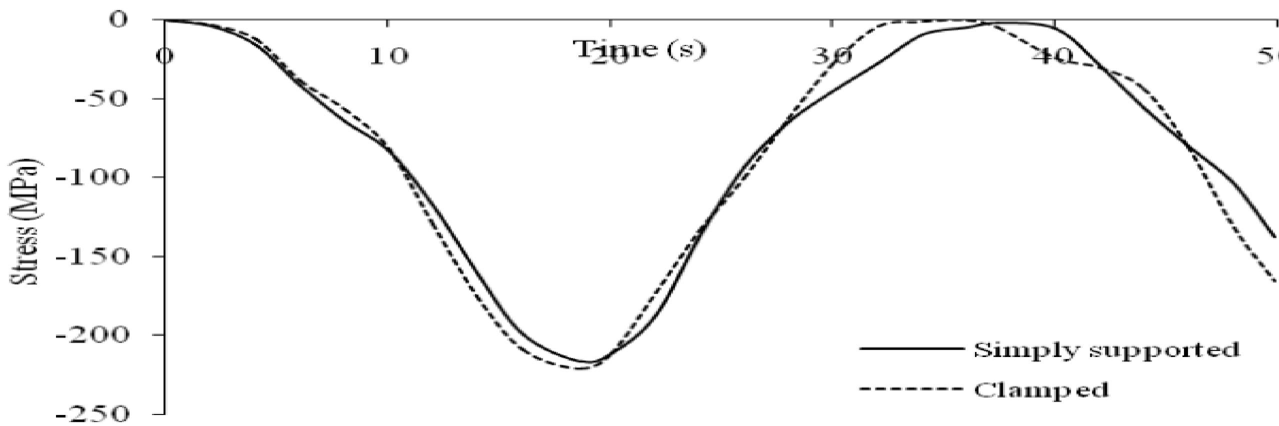


(b) Stress σ_x at the top of the crown

Fig.6 Dynamic response of [$0^0/90^0$] spherical shell cap

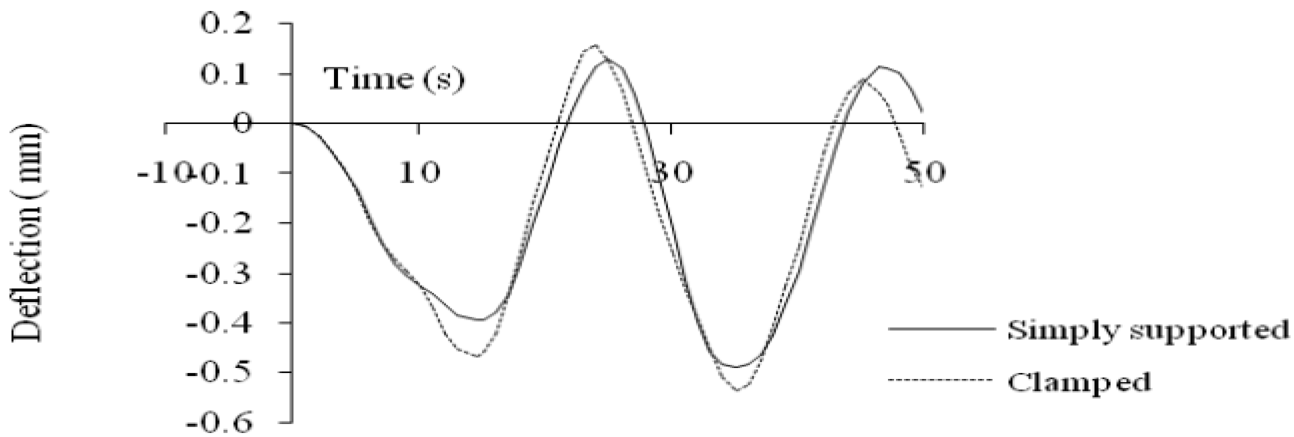


(a) Deflection at the crown

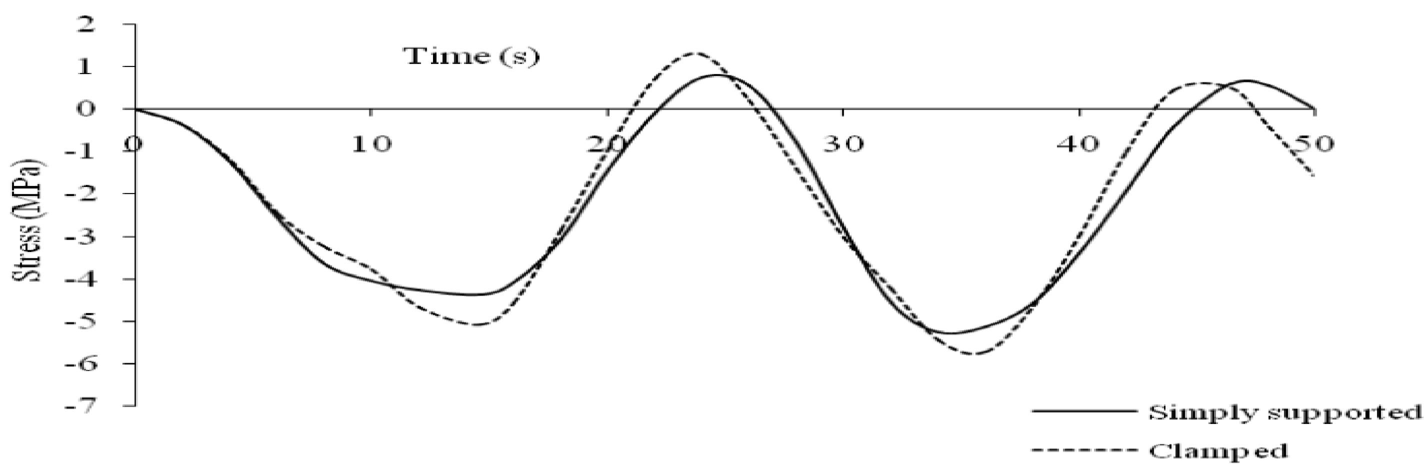


(b) Stress σ_x at the top of the crown

Fig.7 Dynamic response of [$90^0/0^0$] spherical shell cap



(a) Deflection at the crown



(b) Stress σ_x at the top of the crown

Fig.8 Dynamic response of [$0^0/90^0/0^0$] spherical shell cap