

SENSITIVITY ANALYSIS OF RAINFALL RECHARGE MODEL FOR A NOISY DATA

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Abstract

Groundwater resource estimation with particular focus on the estimation of annual replenishable recharge is an important task. For the accurate assessment of groundwater resource, the recharge from rainfall is to be estimated correctly. In the present study, a rainfall recharge model is presented considering both instantaneous recharge and delayed recharge contribution to the groundwater. The model estimates these parameters at zonal level on a regional scale considering three different antecedent moisture conditions. The parameter estimation model is developed using output error criterion. The model uses Gauss-Newton algorithm for minimization; where the objective is to minimize the least square error between observed and computed recharge or water level change. The model is applied on one hypothetical system and on one real field problem. Sensitivity analysis of the parameters are also studied using parameter perturbation method. The results indicate that the proposed rainfall recharge model can be conveniently used for estimation of rainfall recharge. This is highly useful in hard rock areas where the major contribution to groundwater is from rainfall during only in monsoon season.

Keywords: Sensitivity analysis, noisy data, parameter perturbation method, annual replenishable recharge, hypothetical system.

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1. INTRODUCTION

Groundwater constitutes a vital resource in India and is extensively used for drinking and agricultural needs. The management of groundwater largely depends on proper assessment of annual replenishable recharge. The contribution to the groundwater is mainly due to rainfall. Recharge from the rainfall may be either instantaneous or there may be some delay and in many situation a combination of instantaneous and delayed recharge contribution takes place. It is the usual practice that these two parameters are lumped and recharge parameter is generally estimated as a specified percentage of rainfall. This is fairly acceptable for the estimation of annual recharge. However, in hard rock areas, where the recharge from rainfall is mainly during monsoon season, the lumped estimation may not give realistic values. In this study, an attempt is made to estimate instantaneous and delayed recharge parameters.

2. MODEL DEVELOPMENT

The present study is aimed at the development of a rainfall recharge model considering Antecedent Moisture Conditions (AMC). A rainfall recharge equation of the following type is used in the present study.

$$R_c = C_{1j} + C_{2j} R_f \quad \dots \text{(Eqn. 1)}$$

Where, R_c = Recharge in mm/day

R_f = Rainfall in mm/day

C_1 = Delayed recharge factor in mm

C_2 = Instantaneous recharge factor in %

j = Antecedent moisture index

In the present study, three antecedent moisture index is considered and thus in total the model has 6 parameters to be estimated i.e., C_{11} , C_{12} , C_{13} and C_{21} , C_{22} , C_{23} .

The recharge equation (Eqn. 1) is used in a conventional two dimensional anisotropic, non-homogeneous groundwater flow equation for the purpose of parameter estimation. In the present study, parameter estimation model for recharge estimation is developed using output error criterion and weighted least square method is used for minimization of the objective function. In the present study, Gauss – Newton method is used for the purpose of parameter estimation. The governing equation for a two dimensional unsteady groundwater flow in an isotropic and non-homogeneous, unconfined aquifer along with suitable initial and boundary condition is solved using finite element method. The regional model is coupled with an optimization algorithm using an output error criterion, where a set of parameters is iteratively improved until the model response is close to that of the measured output. In the present study, output is taken as seasonal water level fluctuation at select space-time domain. Sensitivity coefficients are determined by parameter perturbation technique. Since parameter perturbation technique is used, the analysis model has to run N+1 times, where N is the number of parameters to be estimated. Three different antecedent moisture conditions namely dry,

moderate and wet is used, considering 5 day cumulative rainfall. The model uses rainfall, draft, water level fluctuations and aquifer parameters such as transmissivity (T) and specific yield (S_y) as input data on a zonal basis and six parameters are estimated for each zone. A posteriori analysis is also carried out and the parameter uncertainty is estimated using parameter covariance matrix.

3. MODEL APPLICATION AND DISCUSSION OF RESULTS

The model developed is applied on two regional aquifer systems.

- i. Hypothetical Aquifer system
- ii. Real field problem

3.1 Hypothetical Aquifer System

The model is applied on a hypothetical aquifer system. The study area is about 200 Km² and is divided into 5 zones. One year data is used and 4 seasonal (2 monsoon, 2 non-monsoon) measurements are considered. Table 1 shows the parameters used in the study.

Table 1. Parameter for Hypothetical Aquifer System

Z	T _x	T _y	S _y	C ₁₁	C ₁₂	C ₁₃	C ₂₁	C ₂₂	C ₂₃
1	100	25	0.02	0.5	0.7	1.0	0.20	0.15	0.12
2	150	50	0.03	0.4	0.6	0.9	0.18	0.16	0.10
3	150	75	0.04	0.3	0.5	0.8	0.16	0.13	0.08
4	40	80	0.01	0.2	0.6	0.7	0.17	0.16	0.09
5	250	250	0.10	0.6	0.8	1.2	0.25	0.20	0.18

In this case, in total 30 parameters are simultaneously estimated using 10 spatial and four temporal measurements of water level changes. Thus 30 parameters are estimated using only 40 measurements. For the purpose of parameter estimation, random noise (zero mean, unit variance) is introduced with 20 % noise in the measured data. The model is tested for its robustness using different initial guess values of parameters and the parameters are converged in all cases. Table 2 shows the results of parameter estimation for 20% noise. Table 3 shows the estimation error and uncertainty for a noise level of 30%.

Table 2. Estimation error and uncertainty for Hypothetical Aquifer System (Noise 20%)

Parameter	Zone	C ₁₁	C ₁₂	C ₁₃	C ₂₁	C ₂₂	C ₂₃
Estimation error (%)	1	20	15	16	8	9	4
	2	18	12	10	9	10	6
	3	16	13	12	10	8	3
	4	19	14	13	8	7	4
	5	22	16	14	9	5	5
	6	21	14	13	9	8	5
Uncertainty (%)	1	15	11	10	9	6	5
	2	12	10	15	10	5	3
	3	13	12	13	11	8	3
	4	16	14	14	10	8	4
	5	18	15	18	8	3	6
	6	15	12	14	10	6	4

Table 3. Estimation error and uncertainty for Hypothetical Aquifer System (Noise 30%)

Parameter	Zone	C ₁₁	C ₁₂	C ₁₃	C ₂₁	C ₂₂	C ₂₃
Estimation error (%)	1	22	18	17	10	10	6
	2	20	15	12	10	12	8
	3	18	15	13	11	11	5
	4	20	15	15	10	8	5
	5	22	18	15	9	6	6
	6	22	18	14	9	9	8
Uncertainty (%)	1	20	14	15	12	8	8
	2	18	15	20	12	10	6
	3	17	16	17	14	12	8
	4	14	18	18	12	10	6
	5	22	20	22	10	6	8
	6	21	20	18	12	8	10

Note:

- i. Estimation error is the difference between the true value and estimated value with respect to true value (expressed as a percentage).
- ii. Uncertainty is expressed as the ratio of individual parameter uncertainty (obtained through parameter covariance matrix) to the parameter estimated expressed as a percentage.

From the Table 2, it is observed that the average estimation error for C₁₁ (delayed recharge, AMC 1) is found as 21 %, while the corresponding values for AMC 2 & 3 are found as 14 % and 13 % respectively. The average estimation error for instantaneous recharge factor for three AMC conditions are found as 9 %, 8% and 5 % respectively. It is observed that parameter uncertainty is significantly more for delayed recharge compared to instantaneous recharge.

From the table 3, it is observed that when the noise level in the data is 30%, estimation error is marginally more while error due to uncertainty sharply increases.

3.2 Real Field Problem

The parameter estimation model is applied on a real field problem. The site is small water shed in the state of Madhya Pradesh which comprises an area of 100 Km². Heavy draft is observed in this area. The region is divided into 3 zones and 4 seasonal measurements are considered (i.e., one monsoon & one non-monsoon for two years, 4 spatial measurements are considered in each zone), thus in this case 18 parameters are estimated simultaneously using 48 measurements. In Real Field Problem, estimation error is not determined since true values of parameters are not known. Table 4 shows the results of parameter estimation.

Table 4 Parameter estimation for Real Field Problem

Parameter	Zon	C ₁₁	C ₁₂	C ₁₃	C ₂₁	C ₂₂	C ₂₃
Parameter value	1	0.2	0.2	0.3	0.1	0.1	0.2
	2	0.2	0.2	0.3	0.1	0.1	0.2
	3	0.2	0.3	0.3	0.1	0.2	0.3
Uncertainty (%)	1	25	22	18	18	16	10
	2	24	23	16	18	16	10
	3	30	27	15	23	20	12

From the table 4, it is observed that for delayed recharge as well as for instantaneous recharge, uncertainty is relatively more for dry and further improves for moderate and wet condition. It must be noted that compared to the various uncertainties involved in the field data (rainfall, draft, aquifer parameter etc.) the values of parameter uncertainty is well within acceptable limits.

4. CONCLUSION

A parameter estimation model is developed considering three antecedent moisture condition. Two parameter i.e., instantaneous and delayed recharge are estimated simultaneously for each zone and each moisture condition. The parameter estimation model is applied on one hypothetical system and one real field problem. Based on the study following conclusions are made.

The parameters estimation model is robust and can be used for estimation of recharge parameter on a regional scale. Delayed recharge parameter shows considerably more uncertainty when compared to instantaneous recharge parameter. Thus it can be concluded that delayed recharge parameter is relatively more sensitive compared to instantaneous recharge parameter.

Parameters in wet condition are relatively better estimated when compared to the dry condition. This is a significant conclusion and is highly useful in hard rock areas. However, a comparative study of this method with the conventional method (estimation of recharge as a percentage of rainfall) will help in drawing more quantitative conclusion. The model can be conveniently used in any system since only water level fluctuation over a specified time period (2-4 seasons) is required. This reduces the cost of data collection significantly. As the noise level increases, uncertainty sharply increases, while estimation error shows marginal increase.

REFERENCES

- [1]. Nagaraj, M.K. and Gouse Piera (2000). "Estimation of specific yield and rainfall recharge factor in a regional groundwater system". Proceedings of National Seminar, HYDRO 2000, REC, Kurukshetra.
- [2]. Yeh (1986). "Review of parameter identification procedures in groundwater hydrology inverse problem" . WRR, 22(2), 95-108.
- [3]. Yunjughyan .K and Lee (1998). "Model identification for inverse estimation of hydraulic parameter ". Groundwater, 36(2), 230-239.