RELIABILITY-BASED SELECTION OF STANDARD STEEL BEAMS

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Abstract

A reliability-based design approach using sensitivity analysis is presented for the selection of standard steel beams subjected to concentrated and or distributed loads, assuming lognormal probability density function. Serviceability criteria of bending stress, transverse deflection, and web maximum shear stress are considered. The load, strength, and geometric parameters are considered to be log normal in distribution. Equations are developed for the coefficients of variation for the mentioned serviceability criteria so as to determine design reliability. Deterministic design equations are transformed into probabilistic ones by replacing the traditional "safety factor" with a "reliability factor". The reliability factor is determined from a specified reliability goal and the coefficients of variation in design models and parameters.

Design examples are presented and results are compared with solutions based on the traditional ASD (Allowable Stress Design) method. Two of the four design examples have identical solutions with the ASD method. The third example result is practically the same as that of ASD method, except that the beam is slightly lighter. The fourth example has a similar result as the ASD method but the beam is deeper. The deeper beam solution stems from the use of both bending stress and deflection serviceability criteria in the new approach while only the bending stress criterion was used in the ASD method. All the chosen beams indicate a reliability of at least 99.73% for bending stress and at least 95.22% reliability for deflection. In all the chosen beams, the web maximum shear stress has a reliability factor of at least 4.91, making shear failure improbable. The agreement between the results of these different methods is amazingly interesting. All computations in the study were done on Microsoft Excel Spreadsheet, demonstrating that probabilistic design can be done with inexpensive computer resource and not too advanced analytical skills.

This study shows that acceptable design solutions can be achieved by modification of deterministic or traditional design method through probabilistic consideration of "design or safety factor". From the very favorable comparison of the results between the new approach and ASD method, it appears reasonable to conclude that the method proposed in this paper is satisfactory for the probabilistic design of standard steel beams. The covs for practical design applications should be based on data from material vendors and historical data on loads. Engineering companies providing design services would have such data and a simple statistical analysis will give realistic cov values for use in their design practice.

Keywords: *Beam, Structural, Standard, Reliability, Lognormal, Cov, Safety, Factor*

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1. INTRODUCTION

Interest in probabilistic design approaches is largely due to the fact that design parameters are more realistically modeled as random variables [1, 2]. Random variable analysis shows that mean values of functional relationships are obtained by substituting mean values of the variates [3]. Probabilistic design methods allow quantification of survival or failure probability and provide justification for safety assessment. When quality, reliability, and safety are paramount, probabilistic design is the preferred method desired for design [2, 4, 5]. Safety of structure is a fundamental criterion for design [6].

The main factors that affect uncertainties in design are variation in loads, material properties, geometry, and accuracy of analytical model. Variability in service load is usually the largest but most difficult to predict, especially at the design phase [7]. The user may not follow guidelines; service environment may be different from that assumed during design, and in fact, a host of variables completely outside the control of the designer come into play. Variations in material properties and component geometry are controlled by manufacturing practices. Engineering design models are approximations of reality and formulations for the same problem vary. However, simplified models in engineering design have been in practice for over two hundred years [8]. Statistically, variability is measured by variance or standard deviation. But the use of the coefficient of variation (ratio of standard deviation to the mean) in characterizing variability is particularly desirable since it can conveniently summarize a large class of materials and parts [9].

Several approaches are possible in probabilistic design. These include simulation [1, 4, 5, 7, 10], experimental [12, 13], and analytical [6, 14, 15, 16]. Simulation methods are varied but Monte Carlo techniques are perhaps the most popular and they can be used to study means, variances, range, distributions, etc. and offer a comprehensive and accurate approach to probabilistic design. Simulation methods are computationally intensive and therefore costly to implement in time and resources. Monte Carlo Simulation is not economical for simple component design. Experimental probabilistic design methods generally use design of experiments (DOE) techniques. Taguchi methods are typical examples of experimental probabilistic methods which can be used to determine optimal mean values of design parameters that minimize performance variation. Randomization techniques are used to select design points or parameter levels so as to minimize efforts. Because experiments can be costly, this is not always possible but can yield cheaper solutions than Monte Carlo simulation. DOE is recommended when distributions are unknown or when some parameters vary uniformly. Analytical methods generally use sensitivity analysis often of the Taylor's series expansion type. First order Taylor's series expansion is more common but second order expansion can be used to improve accuracy. The method is good for linear functions and less accurate for operating points in non-linear functions. However if variability is small, it may still yield good results for non-linear functions. Analytical models are the least expensive and generally more efficient than Monte Carlo simulation or DOE but less accurate when responses are not close to linear region of samples [1]. All these methods incorporate varying levels of optimization.

Structural beams primarily resist bending loads. They include girders, spandrels, joists, stringers, girts, lintels, rafters, and purlins. They may be categorized into primary and secondary beams. Generally primary beams in buildings such as girders carry their self-weight as distributed load and the transferred load from secondary beams as concentrated loads. Secondary beams such as joists usually carry distributed loads of self-weight, weight of supported building materials, and prescribed live loads depending on occupancy. The common practice in structural design is to examine the floor or roof framing plan and determine the most loaded rafter, joist, or girder for design analysis. Design analysis is conceived as consisting of design sizing and design verification tasks. The objective of design sizing is to determine the size of a component while the objective of design verification is to determine the adequacy of a sized component based on desired reliability value. For ordinary steel beams, sizing and verification are based on serviceability criteria of bending stress and deflection. For deep slender webbed beams such as plate girders, shear stress in the web and buckling of web and flange are possible failure modes that should be checked.

Traditional or deterministic engineering design methods using a "factor of safety" in simplified analytical models give results that are to all intent and purposes correct [8]. According to Shingley and Mischke [3] deterministic and familiar engineering computations are useful in stochastic problems if mean values are used. Several authors [3, 17, 18, 19, 20], have presented the "factor of reliability" approach in design in some various forms and titles. However, they have concentrated mainly at the design verification task in which the adequacy of a design is assessed using a factor of safety. In this study, the focus is on using probability-based "factor of reliability" for design sizing tasks, while design verification is based on desired reliability. The term "Reliability factor" is used to describe a probabilistically determined "factor of safety" based on the variability of design parameters and a target reliability. The study presents a systematic and generic approach to evaluating reliability factor using first order Taylor's series expansion. Expressions for the coefficients of variation required for calculating the reliability for bending stress, web shear stress, and transverse deflection of beams are presented. Some design examples are presented, demonstrating the applicability of the method. All symbols used in design equations are defined in Nomenclature section at the end of the paper.

2. A LOGNORMAL RELIABILITY MODEL

The concept of a safety or design factor is an old one but its value was and is experiential and somewhat accounted for uncertainties in design. A statistical equivalent called reliability factor is proposed [21, 22] based on the lognormal probability distribution of design parameters. The model was initially based on fatigue design, now it is being applied to standard steel beam design and other serviceability criteria in this study. The reliability factor n_z can be defined as:

$$n_z = \frac{\text{Average Failure Value}}{\text{Expected Model Value}} \tag{1}$$

In a stress-based design, the serviceability failure value may the yield strength in a static failure of ductile material as in most structural design situations. In a deformation-based design such as lateral deflection, axial deformation or torsion deformation, the serviceability failure value is the maximum allowable deformation. In each failure mode, a limit value can be established for the serviceable criterion, and a design model expected value can be evaluated.

The lognormal probability distribution model has inherent properties that recommend it for machine and structural design applications [3, 21]. A reliability model based on the lognormal distribution function proposed by [22] gives the reliability factor which is rendered as:

$$n_z = \exp\left[s_n\left(z + 0.5 s_n\right)\right] \tag{2}$$

$$s_n = \sqrt{\mathrm{In}\left\{\left(1+\vartheta_F^2\right)\left(1+\vartheta_M^2\right)\right\}} \tag{3}$$

In some design situations such as fatigue and as explained later in structural design, an approximation ϑ_{Mo} of ϑ_M is used to estimate n_z. This is because ϑ_M cannot be accurately evaluated in some design situations until dimensions of a component are known. In such cases, an approximate reliability factor, which is here called design factor n_o, is obtained from Eqn. 2 as:

$$n_o = \exp\left[s_{no}\left(z + 0.5 s_{no}\right)\right] \tag{4}$$

$$s_{no} = \mathrm{In}\left\{\left(1+\vartheta_F^2\right)\left(1+\vartheta_{Mo}^2\right)\right\} \tag{5}$$

In general, it will be assumed that the design sizing of a component is based on n_o instead of n_z since the later cannot always be determined with confidence at the beginning of a design task. However, n_z should be evaluated during design verification. For design verification, the unit normal variate and the corresponding reliability are evaluated. From Eq. 2, the unit normal variate is [22]:

$$z = \frac{\ln(n_z) - 0.5s_n^2}{s_n} \tag{6}$$

For 1.28 < z < 7.0, the reliability can be estimated as [24]:

$$R_z = 1 - 10^{-(0.2006z^2 + 0.2299z + 0.3818)} \tag{7}$$

Eq. 6 gives the unit normal variate z based on the reliability factor and design model parameters variability. Eq. 7 gives the reliability for the unit normal variate z. If a desired reliability or failure probability is specified, then z is known and the necessary reliability factor n_z for achieving this reliability can be obtained from Eq. 6.

The mean and standard deviation or cov of design models must be evaluated in order to apply Eqs. 2 to 7. Suppose a function χ, has the random variables $x_1,\ x_2,\ x_3 \ldots\ldots x_n$ as independent variables. Then:

$$\chi = f\left(x_1,\ x_2,\ x_3,\ \ldots\ x_n\right) \tag{8}$$

Taylor series expansion can be used to estimate expected values and variances (or standard deviation) for unskewed or lightly skewed distributions [3]. Based on Taylor series expansion, a first order estimate of the mean and standard deviation of χ may be obtained respectively:

$$\mu_\chi = f\left(\mu_{x1},\ \mu_{x2},\ \mu_{x3},\ \ldots\mu_{xn}\right) \tag{9a}$$

$$\varpi_\chi = \sqrt{\sum_{i=1}^{n} \frac{\partial \chi}{\partial x_1} \varpi_{xi}^2} \tag{9b}$$

The coefficient of variation is the ratio of standard deviation to the mean value:

$$\vartheta_\chi = \frac{\varpi_\chi}{\mu_\chi} \tag{10}$$

Eqn. 9a simply says that the mean value of a function is obtained by substituting mean values of its independent variables. Eqns. 9b and 10 provide means of quickly obtaining the standard deviation and cov of the function using partial differentiation.

3. DESIGN PROCEDURE OF STEEL BEAMS

1. Load Analysis
   a. Identify all load types acting on member.
   b. Draw load diagrams and determine support reactions.
   c. Draw the shear force and bending moment diagrams.
   d. Determine maximum shear force and maximum bending moment values.

   Sub steps b. c., and d. can be replaced with use of formula if load configuration is standard. Alternatively, use available software for load analysis.

2. Establish Serviceability criteria
   a. Determine principal failure modes of member.
      i. Maximum bending stress and shear stress can be limiting criteria.
      ii. Maximum lateral deflection and torsional buckling can be limiting criteria.
   b. Establish material property for each failure mode.
   c. Determine desired reliability target.

3. Material Selection
   a. Examine usage environment of member (temperature, humidity, corrosiveness, etc.).
   b. Use serviceability criteria to identify applicable material properties in environment.
   c. Establish failure value for each serviceability failure mode.
   d. Identify and select candidate material. Use software if available.
4. Design Sizing
   a. Establish or develop design model.
   b. Use proportional design as much as possible.
   c. Set up primary dimension of member as objective of design model.
   d. Calculate primary and other dimensions.

5. Design Verification
   a. Determine weight of member and add to loads.
   b. Refine maximum shear force and maximum bending moment values.
   c. Determine expected values of design model(s).
   d. Determine reliability
   e. Assess design adequacy.
6. If design is inadequate, resize and verify (Step 5) until it is adequate. Note that it may be necessary to select a different grade of a material or a different material altogether to achieve desired reliability target sometimes.

4. DESIGN BASIS

The principal failure modes of standard steel beams are excessive bending stress and excessive deflection. Such beams must possess enough section modulus (Z_x) so that the service induced maximum bending stress precludes failure. I-beams (W-Shapes and S-shapes) are the most popular standard steel beams. The maximum bending stress usually occurs at the flanges in I-beam. When shear force is high, stiffeners may be required or thicker webbed shapes are chosen. A check for the maximum web shear stress is often done. According to Onouye [23], flanges in I-beams resist

90% of the bending stress while the web resists approximately 90% of the total shear stress. Also, standard steel beams should have enough second moment of area (I_x) so that the induced maximum deflection is acceptable. The second moment of area is here called *area inertia*, so as to distinguish it from the second moment of mass (*mass inertia*) that is commonly used in dynamics, mechanics of machines, and machine design disciplines. This will help to avoid any possible confusion. Generally, deeper sections are often lighter so they are preferred because they are least expensive [24]; except some other factor(s) indicate otherwise. Standard steel beams are designed by selecting standard steel sections with predefined shapes that meet design requirements.

The design of main structural members such as beams and columns are firmly established and formulas are available in textbooks and specifications of organization such as AISC (American Institute of Steel and Construction). Analysis of the principal failure modes of bending stress and transverse deflection follows. Equations developed are made for the purpose of providing means of evaluating covs of analytical design model and model parameters. Where commercial software is available for any of the steps in the design procedures above, it should be employed

4.1 Flange Bending Stress Design

For beams under only uniformly distributed load only, such as secondary structural beams like joists, the design stress model for beams is:

$$\sigma = \frac{M_x}{Z_x} \tag{11}$$

The bending moment is:

$$M_x = M_{dD} + M_{dL} \tag{12a}$$

$$M_{dD} = \beta_d q_D L^2 \tag{12b}$$

$$M_{dL} = \beta_d q_L L^2 \tag{12c}$$

If a uniformly distributed load q_D or q_L is constant over the beam span, β_d = 0.125 (1/8). This is the most common case in structural beam design. Otherwise it can be evaluated for any given configuration. However, if load analysis is performed with software, it is not necessary to know β_d since M_{dD} or M_{dL} will be directly available. Eq. (12) is used to estimate the contributions of these moments to design model variability.

Apply rules of Eq. (9b), and Eq. (10) to Eq. (12a) to obtain:

$$\vartheta_{Mx} = \frac{1}{M_x}\sqrt{\vartheta_{xdD}^2 M_{dD}^2 + \vartheta_{xdL}^2 M_{dL}^2} \tag{13a}$$

Apply rules of Eq. (9b), and Eq. (10) to Eq. (12b) and (12c) to obtain, respectively:

$$\vartheta_{xdD} = \sqrt{\vartheta_D^2 + 4\vartheta_l^2} \tag{13b}$$

$$\vartheta_{xdL} = \sqrt{\vartheta_L^2 + 4\vartheta_l^2} \tag{13c}$$

Note that the self-weight of a joist should be included in q_D , but in general, it will be unknown at the beginning of a design problem. An allowance may be made for it by introducing a beam weight factor greater than unity in either Eq. (11) or Eq. (12b). Therefore, ϑ_x can only be an approximation at the start of the design and hence the reliability factor based on it is only approximate then. Thus the use of design factor appears appropriate for design sizing since it is defined as an approximation of the reliability factor.

Apply rules of Eq. (9b), and Eq. (10) to Eq. (11) to obtain the cov for the design model as:

$$\vartheta_\sigma = \left[\vartheta_x^2 + \vartheta_Z^2\right]^{0.5} \tag{14a}$$

$$\vartheta_M = \left(\vartheta_m^2 + \vartheta_\sigma^2\right)^{0.5} \tag{14b}$$

The cov of the failure parameter is:

$$\vartheta_F = \vartheta_Y \tag{14c}$$

For design sizing:

$$\sigma \le \frac{S_Y}{n_o} \tag{15}$$

Use Eqs. (14), (4) and (5) to estimate n_o and determine the required section modulus for the beam. For standard steel shapes or sections, combine 11, 12a and 15 to obtain:

$$Z_x \ge \frac{n_o M_x}{S_Y} \tag{16}$$

Value from Eq. (16) is used to lookup section shape property tables and a size is chosen. The properties of the chosen section can then be used for design verification.

For design verification, evaluate σ (Eq. (11)) and then:

$$n_z = \frac{S_Y}{\sigma} \tag{17}$$

Next re-evaluate S_n (Eq. (3)) if necessary, and then determine the unit normal variate z (Eq. (6)) and the expected reliability, R_z (Eq. (7)).

The bending moment under combined uniformly distributed and concentrated loads is:

$$M_x = M_{xD} + M_{xL} \tag{18a}$$

$$M_{xD} = M_{dD} + M_{cD} \tag{18b}$$

$$M_{xL} = M_{dL} + M_{cL} \tag{18c}$$

For the purpose of variability evaluation:

$$M_{cD} = \beta_c F_{cDL} L \tag{19a}$$

$$M_{cL} = \beta_c F_{cLL} L \tag{19b}$$

If a concentrated load is at the midspan of a beam, then β_c = 0.25. For other configurations it can be evaluated. However, if load analysis is performed with software, it is not necessary to know β_c since M_{cD} or M_{cL} will be directly available.

Apply rules of Eq. (9b), and Eq. (10) to Eq. (18) to obtain the cov for the bending moment as:

$$\vartheta_{Mx} = \frac{1}{M_x}\sqrt{\vartheta_{xD}^2 M_{xD}^2 + \vartheta_{xL}^2 M_{xL}^2} \tag{20a}$$

$$\vartheta_{xD} = \frac{1}{M_{xD}}\sqrt{\vartheta_{xcD}^2 M_{cD}^2 + \vartheta_{xdD}^2 M_{dD}^2} \tag{20b}$$

$$\vartheta_{xL} = \frac{1}{M_{xL}}\sqrt{\vartheta_{xcL}^2 M_{cL}^2 + \vartheta_{xdL}^2 M_{dL}^2} \tag{20c}$$

$$\vartheta_{xcD} = \sqrt{\vartheta_D^2 + \vartheta_l^2} \tag{20d}$$

$$\vartheta_{xcL} = \sqrt{\vartheta_L^2 + \vartheta_l^2} \tag{20e}$$

The design stress model for beams under combined uniformly distributed and concentrated loads is given by:

$$\sigma = \frac{M_x}{Z_x} \le \frac{S_Y}{n_o} \tag{21}$$

Apply rules of Eq. (9b), and Eq. (10) to Eqs. (21) and the cov for the design model is:

$$\vartheta_\sigma = \left[\vartheta_{xM}^2 + \vartheta_Z^2\right]^{\frac{1}{2}} \tag{22a}$$

$$\vartheta_M = \left(\vartheta_m^2 + \vartheta_\sigma^2\right)^{0.5} = \left(\vartheta_m^2 + \vartheta_{Mx}^2 + \vartheta_Z^2\right)^{0.5} \tag{22b}$$

The cov of the failure parameter is given by Eq. (14c).

For design sizing, use Eq. 16. Then use Eqs. (3, 20, 17, 6) and (7) for design verification.

4.2 Beam Transverse Deflection Design

The maximum transverse deflection of a beam under uniformly distributed dead load is:

$$\delta = \delta_{dD} + \delta_{dL} \tag{23a}$$

$$\delta_{dL} = \frac{q_L L^4}{K_d E I_x} \tag{23b}$$

$$\delta_{dD} = \frac{q_D L^4}{K_d E I_x} \tag{23c}$$

If a uniformly distributed load q_D or q_L is constant over the beam span, K_d = 76.8 (384/5). This is the most common case in structural beam design. Otherwise it can be evaluated for any given configuration.

Apply rules of Eq. (9b), and Eq. (10) to Eq. (23), and the cov of the beam deflection is:

$$\vartheta_{\delta dD} = \left[\vartheta_D^2 + \vartheta_E^2 + 16\vartheta_l^2 + \vartheta_I^2\right]^{0.5} \tag{24a}$$

$$\vartheta_{\delta dL} = \left[\vartheta_L^2 + \vartheta_E^2 + 16\vartheta_l^2 + \vartheta_I^2\right]^{0.5} \tag{24b}$$

$$\vartheta_\delta = \frac{1}{\delta}\left[\vartheta_{\delta dD}^2 \delta_{dD}^2 + \vartheta_{\delta dL}^2 \delta_{dL}^2\right]^{0.5} \tag{24c}$$

$$\vartheta_M = \left(\vartheta_m^2 + \vartheta_\delta^2\right)^{0.5} \tag{24d}$$

The cov of the failure parameter is:

$$\vartheta_F = \vartheta_{\delta A} \tag{24e}$$

For design sizing:

$$\delta \le \frac{\delta_A}{n_o} \tag{25a}$$

$$\delta_A = \frac{L}{\eta} \tag{25b}$$

To size standard steel shapes or sections, combine 25a, 25b, and 23 to obtain:

$$I_x \geq \frac{n_o \eta L^3}{K_d E}\left[q_D + q_L\right] \tag{26}$$

Value from Eq. (26) is used to lookup section shape property tables and a size is chosen. The properties of the chosen section can then be used for design verification.

For design verification, evaluate δ using Eq. (23) and then:

$$n_z = \frac{\delta_A}{\delta} \tag{27}$$

Next re-evaluate s_n (Eq. (3)) if necessary, and then determine the unit normal variate z (Eq. (6)) and the expected reliability, R_z (Eq. (7)).

The design model for beam deflection under combined uniformly distributed and concentrated loads is:

$$\delta = \delta_L + \delta_D \tag{28a}$$

$$\delta_D = \delta_{dD} + \delta_{cD} \tag{28b}$$

$$\delta_L = \delta_{dL} + \delta_{cL} \tag{28c}$$

$$\delta_{cD} = \frac{F_{cDL} L^3}{K_c E I_x} \tag{29a}$$

$$\delta_{cL} = \frac{F_{cLL} L^3}{K_c E I_x} \tag{29b}$$

If a concentrated load F_{cDL} or F_{cLL} is at the midspan of the beam span, K_c = 48. It is 28.2 for two equal concentrated loads located symmetrically on a beam span, and 20.1 for three equal concentrated loads located symmetrically on a beam span [23]. It can be evaluated for any other given configuration.

Apply rules of Eq. (9b), and Eq. (10) to Eqs. (28, 29), and the cov of the beam deflection is:

$$\vartheta_\delta = \frac{1}{\delta}\left[\delta_D^2 \vartheta_{\delta D}^2 + \delta_L^2 \vartheta_{\delta L}^2\right]^{\frac{1}{2}} \tag{30a}$$

$$\vartheta_{\delta D} = \frac{1}{\delta_D}\left[\vartheta_{\delta dD}^2 \delta_{dD}^2 + \vartheta_{\delta cD}^2 \delta_{cD}^2\right]^{\frac{1}{2}} \tag{30b}$$

$$\vartheta_{\delta L} = \frac{1}{\delta_L}\left[\vartheta_{\delta dL}^2 \delta_{dL}^2 + \vartheta_{\delta cL}^2 \delta_{cL}^2\right]^{\frac{1}{2}} \tag{30c}$$

$$\vartheta_{\delta cD} = \left[\left(\vartheta_D^2 + 9\vartheta_l^2 + \vartheta_E^2 + \vartheta_I^2\right)\right]^{\frac{1}{2}} \tag{30d}$$

$$\vartheta_{\delta cL} = \left[\left(\vartheta_L^2 + 9\vartheta_l^2 + \vartheta_E^2 + \vartheta_I^2\right)\right]^{\frac{1}{2}} \tag{30e}$$

$$\vartheta_M = \left(\vartheta_m^2 + \vartheta_\delta^2\right)^{0.5} \tag{30f}$$

The cov of the failure parameter is given by Eq. (25c).

To size standard steel shapes or sections, combine (26a), (26b) and (29) to obtain:

$$I_x \geq \left(\frac{n_o \eta L^2}{E}\right)\left[\frac{F_c}{K_c} + \frac{qL}{K_d}\right] \tag{31a}$$

$$q = q_D + q_L \tag{31b}$$

$$F_c = F_{cDL} + F_{cLL} \tag{31c}$$

$$F_d = F_{dDL} + F_{dLL} \tag{31d}$$

Note that F_{cDL} and F_{cLL} are assumed to be acting at the same point on a beam in Eq. (31a). Value from Eq. (31a) is used to lookup section shape properties tables and a size is chosen. The properties of the chosen section can then be used for design verification.

For design verification, use selected shape properties and loads to evaluate δ using Eq. (26) and n_z using Eq. 25. Then use Eqs. (3, 6, and 7) to evaluate the reliability.

4.3 Web Shear Stress Check

Most of the resistance to shear in thin-webbed I-beams is provided by the web and the average shear stress on it is slightly smaller than the maximum value. So a simplified method that divides the maximum shear force by the "web area" is often used to estimate the maximum shear stress on the web. If the "web area" is estimated as the product of the total beam depth and web thickness (depth area), the shear stress obtained is about 15% to 20% [24, p.284; 25, p. 436-7] less than the maximum value. If the "web area" is estimated as the product of the web depth and web thickness (web area), the shear stress obtained is at most about 10% [25] less than the maximum value. Hence a conservative estimate of the maximum shear stress should be obtained by using the web area and multiplying the value obtained by 1.1. Structural steel materials are ductile so the distortion energy failure rule of ductile materials can be used to estimate their shear yield strength.

$$\tau_w = \frac{F_\tau}{A_w} \tag{32}$$

$$F_\tau = F_{\tau D} + F_{\tau L} \tag{33a}$$

$$F_{\tau D} = F_{\tau dD} + F_{\tau cD} \tag{33b}$$

$$F_{\tau L} = F_{\tau dL} + F_{\tau cL} \tag{33c}$$

$$F_{dD} = \beta_d^/ q_D L \tag{34a}$$

$$F_{dL} = \beta_d^/ q_L L \tag{34b}$$

$$F_{cD} = \beta_c^/ F_{cDL} \tag{34c}$$

$$F_{cL} = \beta_c^/ F_{cLL} \tag{34d}$$

Eqs. 33 and 34 may seem intimidating, but there may be no need to evaluate them manually if software is used for load analysis. In that case, $F_{\tau dD}$, $F_{\tau dL}$, $F_{\tau cD}$, and $F_{\tau cL}$ are obtained directly from the shear force diagrams if each load type is treated separately. However, if a uniformly distributed load q_D or q_L is constant over the beam span, $\beta_d^/$ = 0.5 (1/2). Also $\beta_c^/$ = 0.5 if concentrated load is at midspan of beam or if multiple concentrated loads are symmetrically located on beam span. The equations help in estimating design model variability contributions.

Apply rules of Eq. (9b), and Eq. (10) to Eqs. (33, 34), and the cov of the maximum shear force model is obtained as:

$$\vartheta_{F\tau} = \frac{1}{F_\tau}\left[F_{\tau L}^2 \vartheta_{FL}^2 + F_{\tau D}^2 \vartheta_{FD}^2\right]^{0.5} \tag{35a}$$

$$\vartheta_{FL} = \frac{1}{F_{\tau L}}\left[\left(\vartheta_L^2 + \vartheta_l^2\right) F_{\tau dL}^2 + F_{\tau cL}^2 \vartheta_L^2\right]^{0.5} \tag{35b}$$

$$\vartheta_{FD} = \frac{1}{F_{\tau D}}\left[\left(\vartheta_D^2 + \vartheta_l^2\right) F_{\tau dD}^2 + F_{\tau cD}^2 \vartheta_D^2\right]^{0.5} \tag{35c}$$

Neglecting the slight taper of inner faces of flange and small fillets at flange-web joints for standard shapes, the web height is approximated as:

$$h_w = h - 2t_f \tag{36a}$$

The web area is estimated as:

$$A_w = t_w h_w \tag{36b}$$

The maximum shear stress is estimated as:

$$\tau_{\max} = 1.1\tau_w \tag{37}$$

The cov of $\tau_{\max}$ is:

$$\vartheta_{\tau\max} = \left(\vartheta_{F\tau}^2 + \vartheta_A^2\right)^{0.5} \tag{38a}$$

The cov of the maximum web shear stress design model is:

$$\vartheta_M = \left(\vartheta_m^2 + \vartheta_{\tau\max}^2\right)^{0.5} = \left(\vartheta_m^2 + \vartheta_{F\tau}^2 + \vartheta_A^2\right)^{0.5} \tag{38b}$$

The cov of the failure parameter is given by Eq. (14c).

Based on the distortion energy failure rule of ductile materials, the reliability factor of the web for shear stress failure is evaluated as:

$$n_z = \frac{S_Y}{\sqrt{3}\tau_{\max}} \tag{39}$$

Then use Eqs. (3, 6, and 7) to evaluate the reliability.

5. DESIGN PARAMETERS' VARIABILITY

The application of the reliability model assumes that the covs of design model parameters are known so that the design model variability can be estimated. The Young's modulus for many materials has a cov of 3 to 5% [2]. The cov of Young's modulus will be taken as 5%. Typical values of cov for analytical model uncertainties are 3 – 15% and the cov for dead loads is 5 – 10% [26]. A cov value of 10% will be assumed for analytical model. The cov value for dead load will be taken as 10% and the cov for live loads is taken as 25% as used in [27]. The cov of the yield strength of ordinary structural steel is taken as 10% [27]. The cov of allowable deflection is assumed to be of the same order as the yield strength; that is 10%.

The covs of beam length, depth, area, section modulus, and area inertia (geometric attributes) are also needed in the reliability model for beam design. The cov on section modulus of standard structural steel sections is stated as 5% [27]. The covs on beam depth, cross-sectional area, and area inertia can be estimated from that of the section modulus by assuming proportional design. That is: $Z = kh^3$; where Z = section modulus, k = proportionality factor, h = beam depth. Applying rules of Eq. (9b), and Eq. (10) to *Z*: $\vartheta_Z = 3\vartheta_h$. Hence $3\vartheta_h = 5$ and $\vartheta_h = 1.67$. Similarly: $\vartheta_A = 2\vartheta_h = 3.34$ (3.5) and $\vartheta_I = 4\vartheta_h = 6.68$ (7). The covs for cross-sectional area and area inertia will be taken as 3.5% and 7%, respectively. The cov for beam length is assumed as 0.2%. Table 1 summarizes the covs of the design parameters.

Table 1: Summary of Design Parameter Covs

Design Parameter	COV (%)
Live load	25.0
Dead load	10.0
Analytical model	10.0
Yield strength	10.0
Allowable deflection	10.0
Elastic modulus	5.0
Shape area	3.5
Section modulus	5.0
Area inertia	7.0
Beam length	0.20

6. DESIGN EXAMPLES

Design examples are provided in this section as demonstration of applications of the reliability model above. Structural steel beam is designed currently either by Allowable Stress Design (ASD) or Load and Resistance Factor Design (LRFD) and both methods yield similar results. The examples considered here were previously designed using ASD method. Example 1 is joist design while Example 2 is a girder design. These two examples form the main structural members of floor assembly. Example 3 is similar to Example 2, except that the concentrated loads are not all equal. Example 4 is added because a different material grade is used.

6.1 Examples 1 & 2: Floor Assembly

The floor assembly of Fig. 1 consists of light weight concrete on a steel deck. The joists are spaced 3.05 m on center while the girders are spaced 8.54 m on center. The floor supports a live load pressure of 2.873 kPa and a dead load pressure of 1.915 kPa. The dead load includes allowance for the concrete slab and other structural members. The deflection due to the live load is limited to L/360. Select the lightest A36 W-shapes for the joists and girders [23, p. 352 - 356]. Data units' conversion to SI is done by author.

From Fig. 1, the span of a joist is 8.54 m while that of a girder is 12.2 m. The three middle joists (one of them is labeled J) are the most loaded while the middle girder (labeled G) is the most loaded. These will be used in the design, assuming that the other joists and girders are of the same size, respectively.

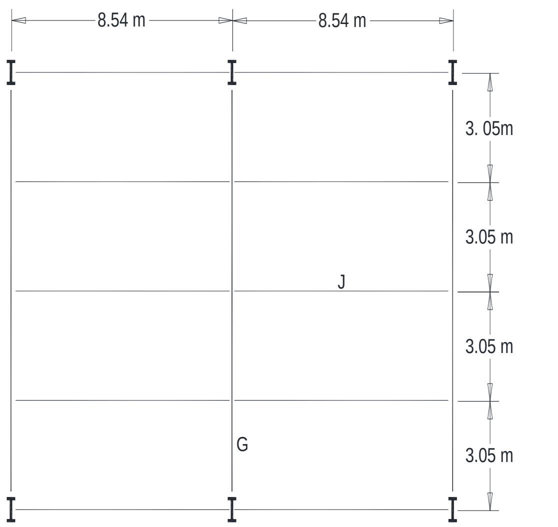

Fig. 1: Floor framing plan (After [23])

Fig. 2 shows the loading on the joist while Fig. 3 shows the loading on the girder. The A36 material specified in the design problem has minimum yield strength of 250 MPa [23]. According to Hess et al [28] the mean yield strength of ordinary structural steel is about 1.3 times the minimum. Hence the mean yield strength of A36 will be taken as 325 MPa.

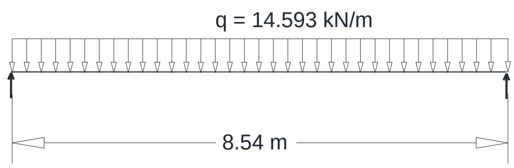

Fig. 2: Floor joist loading (After [24])

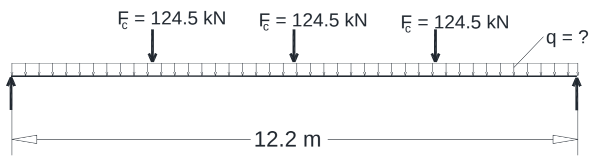

Fig. 3: Floor girder loading (After [24])

6.2 Example 3

Select the lightest steel wide-flange section (W-shape) for the beam in Fig. 4. Consider moment and shear. The allowable bending stress is 165 MPa, and the allowable shear stress is 100 MPa. [29, p. 406 – 408]. From the stipulated allowable stresses, ASTM A36 material is assumed for the beam material.

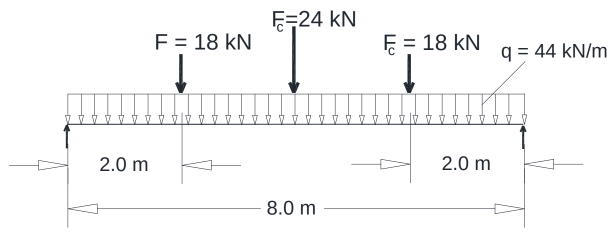

Fig. 4: Beam1 loading (After [29])

6.3 Example 4

Select the lightest W-beam that will support a point load of 178 kN as shown in Fig. 5. Assume that the steel material has minimum yield strength of 345 MPa (50 ksi) [30]. According to Hess et al [28] the mean yield strength of high strength structural steel is about 1.19 times the minimum. Hence the mean yield strength of the material will be taken as 410 MPa.

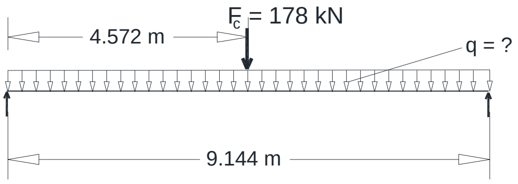

Fig. 5: Beam2 loading (After [30])

7. DESIGN SOLUTIONS

For load analysis (please refer to section 3.), the maximum shear force and maximum bending moment on the joist were obtained using standard formulas. Software was used for the load analysis of the girder. This took care of Step 1 of the design procedure. The principal failure modes of standard steel I-beams were analyzed above which took care of Step 2 of the design procedure, except the reliability target. This is addressed below. Step 3 of the design procedure is skipped as the beam material is specified in the problem. Step 4a is covered in section 4: Design Basis. Steps 4b to 6 of the design procedure are implemented by coding the formulas derived in the previous section in Excel spreadsheet.

In reliability-based design, a reliability target is used for design adequacy assessment. In ASD method, a design factor based on experience is specified and minimum yield strength is used to characterize the material capability. According to Mischke [3, p. 2.3], minimum strength is a percentile strength often placed at 1% failure level (99% reliability), sometimes called the ASTM minimum. Hence for this study, the target reliability was taken as 99% for bending stress design. Note that average or mean strength is used in probabilistic studies, not minimum strength.

While yield strength is a material property for structural beams, limit for beam deflection is not. Deflection limits are largely based on experience and values are stipulated for combined dead and live loads or for live load only. For floor joists and girders, the total load deflection is typically limited to $L/240$ while the live load only deflection is limited to $L/360$ [23, p. 290]. Traditionally, no design factor is used in the evaluation of beam deflection. In this study, the beam deflection criterion for combined dead and live loads was used for design sizing with a design factor applied. Then the live load only deflection was used to verify the design with a unity design factor. The reliability was then evaluated when the live load deflection satisfied the typical limitation.

7.1 Analysis Factors for Design Examples

7.1.1 Examples 1

The equations above were coded in Microsoft Excel for the design of the joist carrying only uniformly distributed load as shown in Fig. For joist, so β_d = 0.125, 8; $\beta_d^{/}$ = 0.50, and K_d = 76.8.

7.1.2 Example 2

The transferred load from the joists of Fig. 2 to the girder as concentrated loads as shown in Fig. 3. Note the distributed load in Fig. 3 is unknown (q = ?) at the beginning of the design. For the girder design β_c = 0.25, $\beta_c^{/}$ = 0.50, and K_c = 20.1 [23].

7.1.3 Example 3

For Fig. 4 girder design β_c = 0.25, $\beta_c^{/}$ = 0.50. For the beam self-weight, K_d = 76.8 and K_c = 20.1 for 18 kN concentrated load at three locations. At midspan, additional load 6 kN is require for which K_c = 48 [23]. Because the load on the beam is not classified into dead and live loads, live load is assumed so as to be conservative.

7.1.4 Example 4

For Fig. 5 design β_c = 0.25, $\beta_c^{/}$ = 0.50. For the beam self-weight, K_d = 76.8 and K_c = 48 for midspan concentrated load [23]. Because the load on the beam is not classified into dead and live loads, live load is assumed so as to be conservative.

7.2 Load Analysis Results

The results of the load analysis are summarized in Tables 2 and 3. Table 2 gives the results for the maximum shear forces acting on the beams. Table 3 gives the results for the maximum bending moments acting on the beams. Note that the self-weight of the beams in Examples 2, 3, and 4 are neglected at this stage because the beam sizes are not known yet. If they are summed in a design situation, they should be treated as distributed dead loads and can then be included in the appropriate cells in the table. The self-weight for example is said to be included in the loading for example 1 in the problem statement.

Table 2: Shear Force Load Analysis

Design Example	Dead Load (kN)		Live Load (kN)	
	Dist.*	Conc.**	Dist.	Conc.
1	25	0	37.312	0
2	0	74.70	0	112.05
3	0	0	176	30
4	0	0	0	89
*Dist. = Distributed; **Conc. = Concentrated				

Table 3: Bending Moment Load Analysis

Design Example	Dead Load (kNm)		Live Load (kNm)	
	Dist.	Conc.	Dist.	Conc.
1	53.215	0	79.825	0
2	0	303.78	0	455.67
3	0	0	352	84
4	0	0	0	407

7.3 Design Sizing Results

Table 4 provides a summary of the design sizing results for the four examples. Values of Z_x are based on bending strength capability (Eq. (16)) while values of I_x are based on combined dead and live load deflection limitation of $L/240$ (Eqs. (26) and (31a)). These values were evaluated at 99% reliability. Note that the use of both Z_x and I_x in section property table search for standard steel beams reduces the search space.

Table 4: Summary of Design Sizing Results

Design Example	Section Properties Required (99% Reliability)	
	Z_x (10^4) mm^3	I_x (10^7) mm^4
1	68.47	23.29
2	393.55	166.89
3	223.88	71.85
4	165.65	62.07

7.4 Standard Beam Search Results

Table 5 summarizes the beam search results for the four design examples. Two candidate beams were chosen for each example demonstration purposes. The lighter ones of these two beams are the preferred since the design statements stipulated the selection of the lightest beams.

Table 5: Identification of Candidate Beams

Design Example	Candidate Beam*	Section Properties	
		Z_x **(10^4) mm^3**	I_x **(10^7) mm^4**
1	W460x52	94.39	21.23
	W410x60	92.59	18.65
2	W760x147	440.81	166.08
	W690x152	437.74	150.68
3	W610x101	252.36	76.17
	W530x109	274.44	66.60
4	W530x92	208.12	55.36
	W460x113	239.25	55.36
*Specification is converted to Metric from English units			

7.5 Design Verification Results

Table 6 provides a summary of the design verification results for bending stress for the design examples. All the candidate beams meet the target reliability of 99%, except the last one of design example 4. Notice that the least value of the reliability factor in this table is 1.81. This is higher than the minimum design factor value of 1.5 commonly used in ASD method. Hence the selected beams are acceptable from this perspective. Since the search is for the lightest beam, the first beam for each design example will be chosen.

Table 6: Bending Stress Design Verification

Design Example	Beam Candidate	n_z	R_z
1	W460x52	2.23	99.991
	W410x60	2.49	99.999
2	W760x147	1.82	99.700
	W690x152	1.81	99.661
3	W610x101	1.85	99.011
	W530x109	1.81	99.783
4	W530x92	2.25	99.698
	W460x113	1.86	98.041

Table 7 summarizes the design verification results based on live load only deflection. The maximum allowable deflection is the denominator in column 3 while the numerator is the expected deflection under the live loads. All the candidate beams meet the criterion, but the lighter ones will be chosen.

Table 7: Deflection Design Verification

Design Example	Beam Candidate	Live Load Deflection (mm)	R_z
1	W460x52	13.8/23.7*	95.926
	W410x60	13.6/23.7	96.339
2	W760x147	19.7/33.9	98.590
	W690x152	21.7/33.9	94.217
3	W610x101	18.8/22.2	99.819
	W530x109	21.5/22.2	97.946
4	W530x92	22.9/25.4	99.030
	W460x113	24.7/25.4	95.593
*Numerator is expected live load only deflection; denominator is maximum allowable live load only deflection.			

Table 8: Web Maximum Shear Stress Verification

Design Example	Beam	Max. Shear Stress (MPa)	n_z
1	W460x52	21.75	8.63
2	W760x147	22.65	8.28
3	W610x101	38.24	4.91
4	W530x92	18.77	12.61

Based on the design verification analysis, the chosen beams were checked for web maximum shear stress. Table 8 provides a summary of this check. In ASD method, a design factor of 1.5 is used as minimum for shear stress. The reliability factors in Table 8 are all above 4; therefore, the web maximum shear stresses are well below the shear yield strengths of the beam materials. Hence failure in shear is not anticipated.

7.6 Comparison of Results

Table 9 shows a comparison of current design results with previous solutions. From Table 9, it is observed that the new method provides solutions that are identical for Design Examples 1 and 2. Design Example 3 solutions are practically the same, with the new solution beam lighter in weight. Solutions for Design Example 4 are similar in weight but the new has a deeper beam. The difference is attributed to the fact that the previous solution is based only on bending stress while deflection is considered along with bending stress in the new selection. In fact when the properties of W18x60 (previous solution) were used for live load only deflection design verification, it was found that the live load deflection exceeded the limit allowable. It is amazingly interesting that the new method provides so closely matching results with the ASD method.

Table 9: Comparison of Results

Design Example	Beam Selection	
	New Method	**ASD Method**
1	W460x52	W460x52
2	W760x147	W760x147
3	W610x101	W610x113
4	W530x92	W460x89

8. CONCLUSION

Table 10 gives the design reliability estimates for the selected beams based on the criteria of bending stress and transverse deflection. Note that all the chosen beams have a reliability of at least 99.73% for bending stress and at least 95.22% reliability for deflection. This table suggests that different reliability targets for different design criteria may be permissible for reliability-based design.

Table 10: Design Reliability

Design Example	Beam	Reliability	
		Bending Stress	**Deflection**
1	W460x52	99.99	95.93
2	W760x147	99.73	95.22
3	W610x101	99.81	99.71
4	W530x92	99.97	95.55

The results from this study suggest:

- Traditional engineering design methods can be transformed into probabilistic design methods by evaluating the design factor on probabilistic basis.
- A design or reliability factor depends only on the covs of design model parameters and the unit normal variate which represents a specific survival or failure probability.
- The traditional design or safety factor (reliability factor) can be quantitatively associated with survival or failure probability through this approach.
- A reliability-based design method for selecting standard structural beams assuming the lognormal probability distribution gives results that very closely match those of ASD method.
- Minimum reliability target of 99% is suggested for bending stress design for comparison with ASD method.
- An integrated steel beam deflection design approach using combined dead and live loads and live load only is developed that allows reliability estimation.
- Minimum reliability target of 95% is suggested for a deflection limit of $L/240$ for design sizing of floor beams using combined dead and live loads.
- Deflection design verification should be based on live load only deflection, limited to $L/360$. The reliability can then be estimated.
- Reliability-based design need not require expensive software or very specialized skills since Microsoft Excel: a spreadsheet program was used in the study.

It was observed during the study that the covs for loads, material properties, and analytical modeling appear to have more influence on reliability. The cov of geometric parameters appear to have negligible effects because of their relatively smaller values. Particularly, covs of area inertia, section modulus, and shape area are relatively more significant than the cov of beam length. The covs for practical design applications should be based on data from material vendors and historical data on loads. Engineering companies providing design services would have such data and a simple statistical analysis will give realistic cov values for use in their design practice.

Specific design requirements may necessitate reliability target higher than 99% for bending stress and 95% for deflection. Specifying reliability target in terms of the number of "nines" appears attractive because the failure rate decreases by one-order of magnitude for each additional nine. For instance, a reliability target of three nines (99.9%) has 10 times less failure rate than a reliability target of two nines (99%). Similarly, a reliability target of five nines (99.999%) has 10 times less failure rate than that of four nines (99.99%). A reliability target of 99% means 1 failure in 100 designs on the average, 99.9% means 1 failure in 1000, etc.

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NOMENCLATURE

COV, Cov, cov = coefficient of variation

$n_z =$ reliability factor

$s_n =$ standard deviation of design model

$n_o =$ design factor (approximation of n_z)

s_{no} = approximation of s_n

$R_z =$ reliability at z-value

$z =$ unit normal variate

$\chi =$ generalized design model

$\mu_\chi =$ expected value of generalized design model

$\varpi_\chi =$ standard deviation of generalized design model

$\vartheta_\chi =$ cov of generalized design model

$\sum =$ summation symbol

$f(\ ) =$ generalized function representation

$q =$ combined distributed load per unit length

$L =$ span of beam

$q_L =$ load per unit length from live load

$q_D =$ load per unit length from dead load

$\beta_d =$ distributed load bending moment factor

$\beta_c =$ concentrated load bending moment factor

$\sigma =$ maximum expected stress
$Z_x =$ major section modulus
$S_Y =$ yield strength of beam material
$M_x =$ maximum bending moment
$M_{dL} =$ distributed live load bending moment
$M_{cL} =$ concentrated live load bending moment
$M_{dD} =$ distributed live load bending moment
$M_{cD} =$ concentrated dead load bending moment
$E =$ elastic modulus of material
$I_x =$ major area inertia of shape
$K_d =$ distributed load deflection factor
$K_c =$ concentrated load deflection factor
$\eta =$ span deflection factor
$\delta =$ maximum expected deflection
$\delta_A =$ maximum allowable deflection of beam
$\delta =$ total deflection
$\delta_L =$ live load deflection
$\delta_D =$ dead load deflection
$\delta_{dL} =$ distributed live load deflection
$\delta_{cL} =$ concentrated live load deflection
$\delta_{dD} =$ distributed dead load deflection
$\delta_{cD} =$ concentrated dead load deflection
$F_c =$ total concentrated load
$F_d =$ total distributed load
$F_{cDL} =$ concentrated dead load
$F_{cLL} =$ concentrated live load
$F_{dDL} =$ distributed dead load
$F_{dLL} =$ distributed live load
$F_\tau =$ maximum shear force
$F_{\tau D} =$ total dead load shear force
$F_{\tau L} =$ total live load shear force
$F_{\tau dD} =$ distributed dead load shear force
$F_{\tau cD} =$ concentrated dead load shear force
$F_{\tau dL} =$ distributed live load shear force
$F_{\tau cL} =$ concentrated live load shear force
$\beta_c^{/} =$ concentrated load shear force factor
$\beta_d^{/} =$ distributed load shear force factor
$h =$ beam depth
$h_w =$ web depth
$t_w =$ web thickness
$t_f =$ flange thickness
$A_w =$ area of web
$\tau_w =$ average web shear stress
$\vartheta_Y =$ cov for yield strength
$\vartheta_E =$ cov for elastic modulus
$\vartheta_l =$ cov for beam span
$\vartheta_A =$ cov of section area
$\vartheta_Z =$ cov for section modulus
$\vartheta_I =$ cov for area inertia
$\vartheta_m =$ cov for design model
$\vartheta_M =$ cov of design model expected value
$\vartheta_{Mo} =$ approximation of ϑ_M
$\vartheta_F =$ cov of serviceability failure value
$\vartheta_L =$ cov for live load
$\vartheta_D =$ cov for dead load
$\vartheta_{dD} =$ cov for distributed dead load
$\vartheta_{dL} =$ cov for distributed live load
$\vartheta_\sigma =$ cov for bending stress
$\vartheta_{Mx} =$ cov for bending moment
$\vartheta_{xD} =$ cov for dead load bending moment
$\vartheta_{xL} =$ cov for live load bending moment
$\vartheta_{xdD} =$ cov for distributed dead load bending moment
$\vartheta_{xdL} =$ cov for distributed live load bending moment
$\vartheta_{xcD} =$ cov for concentrated dead load bending moment
$\vartheta_{xcL} =$ cov for concentrated live load bending moment
$\vartheta_{\delta A} =$ cov of allowable deflection
$\vartheta_\delta =$ cov for total defection
$\vartheta_{\delta D} =$ cov for dead load deflection
$\vartheta_{\delta L} =$ cov for live load deflection
$\vartheta_{\delta dD} =$ cov for distributed dead load deflection
$\vartheta_{\delta dL} =$ cov for distributed live load deflection
$\vartheta_{\delta cD} =$ cov for concentrated dead load deflection
$\vartheta_{\delta cL} =$ cov for concentrated live load deflection
$\delta_{cD} =$ concentrated dead load deflection
$\vartheta_{\tau F} =$ cov for total shear force load
$\vartheta_{FD} =$ cov for dead shear force load
$\vartheta_{FL} =$ cov for live shear force load
$\vartheta_{\tau \max} =$ cov for web maximum shear stress