DESIGN OF HYBRID QRD ARCHITECTURE USING MIMO-OFDM SYSTEMS

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Abstract

QR decomposition has been widely used in many signal processing applications to solve linear inverse problems. However, QR decomposition is considered a computationally expensive process, and its sequential implementations fail to meet the requirements of many time-sensitive applications. We propose a deeply pipelined reconfigurable architecture that can be dynamically configured to perform either approach in a manner that takes advantage of the strengths of each. At runtime, the input matrix is first partitioned into numerous sub matrices. This paper proposes a fully parallel VLSI architecture under fixed-precision for the inverse computation of a real square matrix using QR decomposition with Hybrid QRD Gram-Schmidt orthogonalization. The Hybrid QRD based on algorithm is stable and accurate to the integral multiples of machine precision under fixed-precision for a well-conditioned non-singular matrix. For typical matrices (4x4) found in MIMO communication systems, the proposed architecture was able to achieve a clock latency of 38 clocks.

Keywords:—*Architecture, FPGA, QR decomposition, Gram-Schmidt.*

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1. INTRODUCTION

Due to significant performance gains provided by MIMO, it is being widely adopted in most of the current and next generation wireless communication systems. To exploit the full potential of gains offered by MIMO, computationally efficient design of a wireless baseband communication receiver has become difficult and challenging. Signal processing circuits involved in a MIMO receiver have to be designed for high data throughput and low latency owing to their application in real-time wireless systems. The computational accuracy of signal detection has direct consequence on the throughput and reliability achieved in the receiver.

QR decomposition has been widely used in many signal processing applications such as MIMO systems [1], beam forming [2] and image recovery [3] to calculate the inverse of matrices or solve linear systems. However, its inherent computational complexity makes it unlikely to satisfy the requirements of many time-sensitive designs, especially when the system operates on a large-scale dataset.

The Gram-Schmidt process, Householder transformation and Givens rotation are known as the most popular algorithms for QR decomposition [4], among which, the Householder transformation and the Givens rotation are considered numerical stable algorithms, while the Gram-Schmidt process provides an opportunity to perform successive orthogonalizations. Parallel designs have been previously investigated to accelerate QR decomposition on traditional multi-core systems [5], [6], GPUs [7] and reconfigurable computing platforms [8].

In this paper, we propose a reconfigurable architecture for QR decomposition, which can be dynamically configured to perform Gram-Schmidt are deeply pipelined. To process large data sets, the input matrix is partitioned into multiple columns and rows of sub-matrices. The sub-matrix columns and rows are processed successively.

The rest of this paper is organized as follows: Section II offers a brief overview of the channel model for MIMO systems and the need for matrix. Section III explains QR decomposition based on hybrid QRD using Gram-Schmidt algorithm. Section IV presents the proposed architecture for hybrid QR decomposition. Section V discusses the latency, operations involved and the throughput achieved. Section VI concludes with the summary of key results of this paper.

2. MIMO SYSTEMS

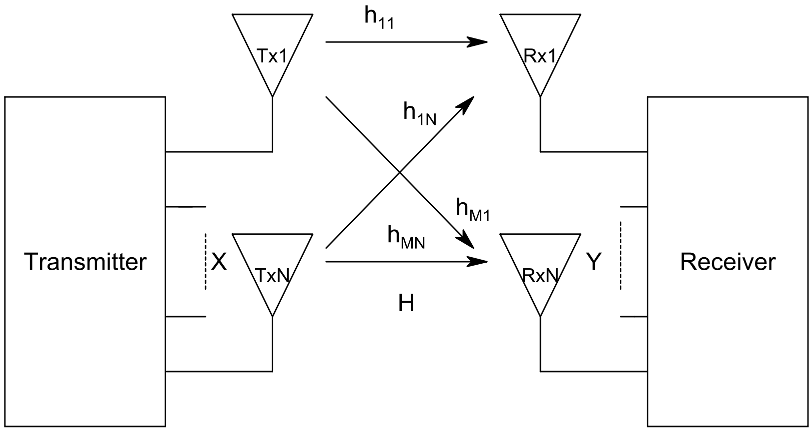

Fig. 1 . Block Diagram of MIMO Systems

The block diagram of a N transmit and M receive antennae MIMO system is shown in Fig.1.

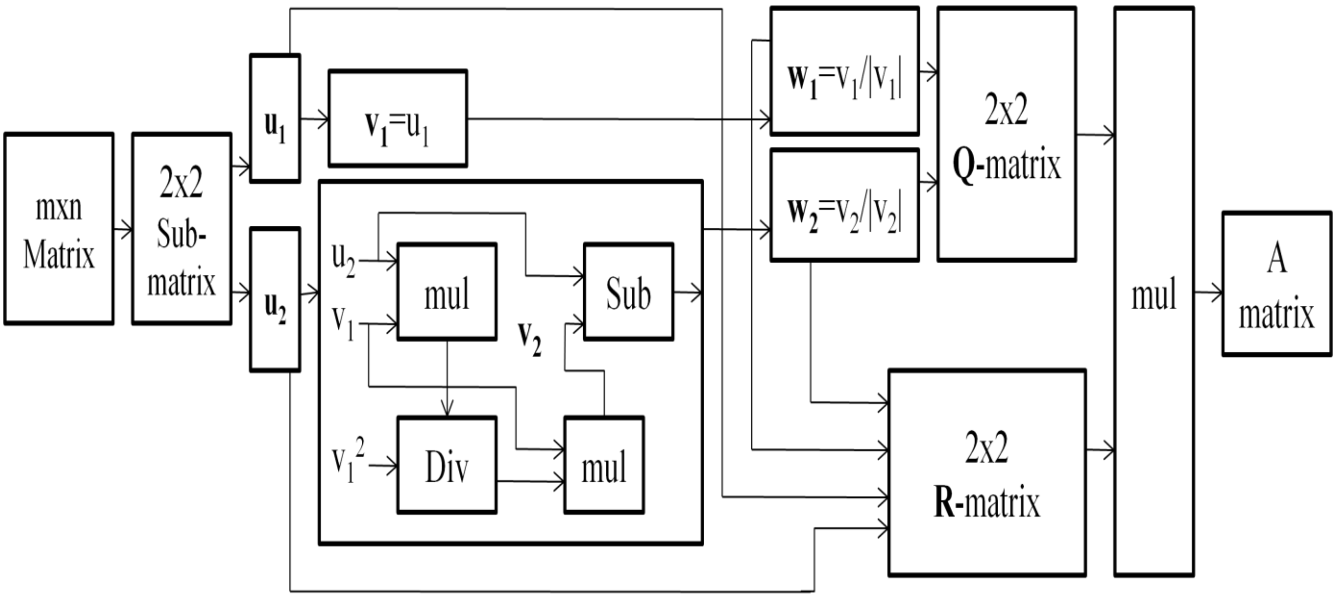

Fig. 2 .Block Diagram of Hybrid QR Decomposition

The channel model under a flat fading channel condition is given by Eq.1.

$$Y = HX + Z \quad (1)$$

where, Y is a (M × T) complex received matrix, His a (M × N) complex channel matrix, X is a (N × T) transmitted matrix whose elements are taken from a complex modulation constellation, and Z is a (M × T) complex additive white Gaussian noise matrix. Here, T is the number of symbol periods over which data is being transmitted. A good summary for these technique scan be found in [2]. As M, N increases beyond four, a MIMO system gains very little in performance for a disproportionate increase in receiver complexity. A typical MIMO system has M, N ≤ 4as performance gain achieved beyond four antennae is insignificant compared to the increased receiver complexity.

3. PROPOSED HYBRID QR DECOMPOSITION

QR decomposition is one of the popular matrix factorization methods. QR decomposition of an m × n matrix A has a form given by eq. (2)

$$A=QR \quad (2)$$

where Q is an m × m matrix, which is an orthogonal matrix such that $Q^T \cdot Q = I$ and R is an m × n upper triangular matrix. This amounts to finding Orthonormal basis for an(A) [9], [10]. QR decomposition can be used to solve full rank least squares problem. The original matrix A and decomposed matrices Q and R for m = n = 4 are represented by Eqs.3-5.

$$A = \begin{bmatrix} a_{11} a_{12}\ a_{13} a_{14} \\ a_{21} a_{22}\ a_{23}\ a_{24} \\ a_{31} a_{32} a_{33} a_{34} \\ a_{41} a_{42}\ a_{43}\ a_{44} \end{bmatrix} \quad (3)$$

$$Q = \begin{bmatrix} q_{11}\ q_{12}\ q_{13} q_{14} \\ q_{21} q_{22} q_{23} q_{24} \\ q_{31} q_{32}\ q_{33}\ q_{34} \\ q_{41} q_{42}\ q_{43}\ q_{44} \end{bmatrix} \quad (4)$$

$$R = \begin{bmatrix} r_{11} r_{12}\ r_{13} r_{14} \\ 0\ \ r_{22} r_{23} r_{24} \\ 0\ \ 0\ \ r_{33} r_{34} \\ 0\ \ 0\ \ 0\ \ r_{44} \end{bmatrix} \quad (5)$$

Gram-Schmidt orthogonalization [9], [10] is a direct method to compute Q and R. The Gram-Schmidt process for a matrix A proceeds as:

$$v_i = u_i - \left(\frac{u_i\, v_1}{v_1 v_1}\right) v_1 - \left(\frac{u_i\, v_2}{v_2 v_2}\right) v_2 \ldots \ldots . \left(\frac{u_i v_{i-1}}{v_{i-1} v_{i-1}}\right) v_{i-1} \quad (6)$$

$$w_i = \frac{v_i}{|v_i|} \quad (7)$$

$$Q = [w_1, w_2, \ldots . w_n] \quad (8)$$

$$R = [r_{ji}], \quad r_{ji} = u_i\, w_j \quad (9)$$

where, $v_1 = u_1$ and $v_2 = u_2 - \left(\frac{u_2\, v_1}{v_1 v_1}\right) v_1$

u_i,i=1,2….n, are column vectors of matrix A. w_i,i=1,2…n, are column vectors of matrix Q. The original matrix A and decomposed matrices Q and R for m = n = 4 using hybrid QR decomposition are represented by Eqs.3-5.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (10)$$

$$A_{11} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, A_{12} = \begin{bmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}, A_{22} = \begin{bmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{bmatrix}$$

$$u_1 = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}, \qquad u_2 = \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}$$

$$v_1 = u_1$$

$$v_1 = \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \quad (11)$$

$$v_2 = u_2 - \left(\frac{u_2\, v_1}{v_1 v_1}\right) v_1$$

$$v_2 = \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} \quad (12)$$

$$w_{11} = \frac{v_{11}}{\sqrt{v_{11}^2 + v_{21}^2}}\ , w_{12} = \frac{v_{21}}{\sqrt{v_{11}^2 + v_{21}^2}}\ ,$$

$$w_{21} = \frac{v_{12}}{\sqrt{v_{12}^2 + v_{22}^2}}\ , w_{22} = \frac{v_{22}}{\sqrt{v_{12}^2 + v_{22}^2}} \quad (13)$$

$$Q = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \quad (14)$$

$$r_{11} = u_1 w_1\ , r_{12} = u_2 w_1\ , r_{22} = u_2 w_2$$

$$R = \begin{bmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{bmatrix} \quad (15)$$

$$A = QR$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{bmatrix} \quad (16)$$

The input matrix A is subdivided into$A_{11}, A_{12}, A_{21}, A_{22}$ sub-matrices, where u_1 is the first column vectors of A_{11} matrix, u_2 is the second column vectors of A_{11} matrix,$v_1 = u_1$ and $w_i = \frac{v_i}{|v_i|}$, where $w_{11}, v_2 = u_2 - \left(\frac{u_2\, v_1}{v_1 v_1}\right) v_1, w_{21}$are the first column vectors of the matrix Q_{11} and w_{12}, w_{22} are the second column vectors of the matrix Q_{11}. FindingR_{11} by $r_{ji} = u_i\, w_j$.

Hence input matrix A_{11} is proved by multiplying matrixQ_{11} and R_{11}. Similarly can find A_{12}, A_{21}, A_{22} by these method.

4. ARCHITECTURE FOR HYBRID QR DECOMPOSITION

In Hybrid QR Decomposition block diagram, input matrix is divided into m x n sub-matrices [8]. Here we taken 4x4 matrix as a input, it is divided into four 2x2 sub-matrices. Among these we solved for one 2x2 sub-matrix.

To find Q matrix we need to find u, v and w. where u_1 is the first column of the input matrix A, $u_2$is the second column of the input matrix A. where $v_1 = u_1$ and to find v_2 , initially multiply v_1 and u_1, then divide by $v_1^2$and multiply with v_1, finally subtract with u_2. To find w_1 divide $v_1$by $|v_1|$. Where $|v_1|$ is square root of v_{11}^2 sum with v_{21}^2. Similarly to find w_2 divide v_2 by $|v_2|$. Where $|v_2|$ is square root of v_{12}^2 sum with v_{22}^2. Now w_1 is the first column of the matrix Q and w_2 is the second column of the matrix Q.

To find R matrix we need to find r_{ji}, where $r_{ji}=u_i w_j$. Here upper triangular matrix is used, hence $r_{21}=0$. Now we can find R matrix by product of u and w.

To find A matrix multiply the matrix Q and R. By using sub-matrices we can solve matrices easily and it requires less time.

5. IMPLEMENTATION AND RESULT

Our design is implemented in verilog HDL on Xilinx 9.1i XSE. Our architecture uses 4-bits input values that are used for multipliers, subtractors, dividers, square roots for Gram-Schmidt process. In QR Decomposition method, it is more complexity to find the orthogonal matrix and upper triangular matrix. By using Hybrid QR Decomposition we can reduce the clock latency than other methods. In Fig. 3, it is shown that the clock latency reduced to 38 clocks than other methods. In Ref.[11] the clock latency is given as 88 clocks, as well as in Ref.[12] the clock latency is given as 67 clocks. Hence our design has less latency than other methods.

An RTL (Register Transfer Logic)view of finding Q matrix is shown in Fig.4. By using input matrix A we can find Q matrix using Gram-Schmidt algorithm.

In an TABLE I, the comparison results are shown. The parameters are shown that the clock latency, order, technology and algorithm which are used in this paper and Ref.[11] and Ref.[12].

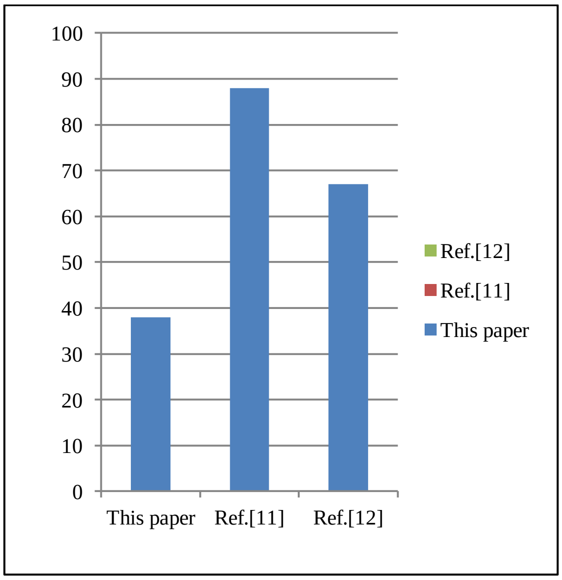

Fig. 3 Comparison of clock latencies

Table 1 Comparison of implementation results

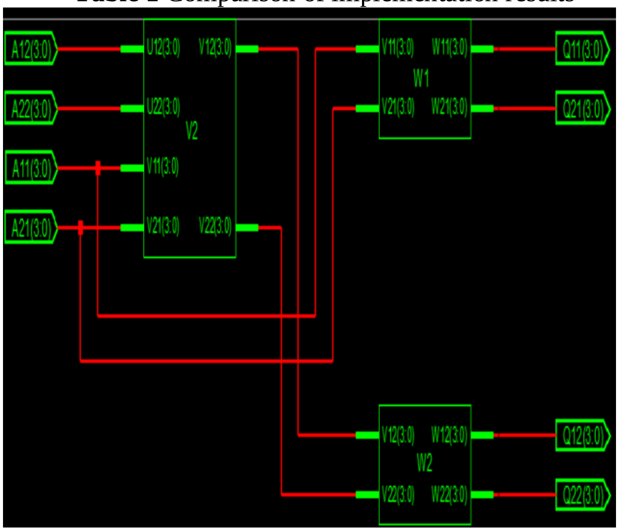

Fig. 4 RTL view of finding Q matrix.

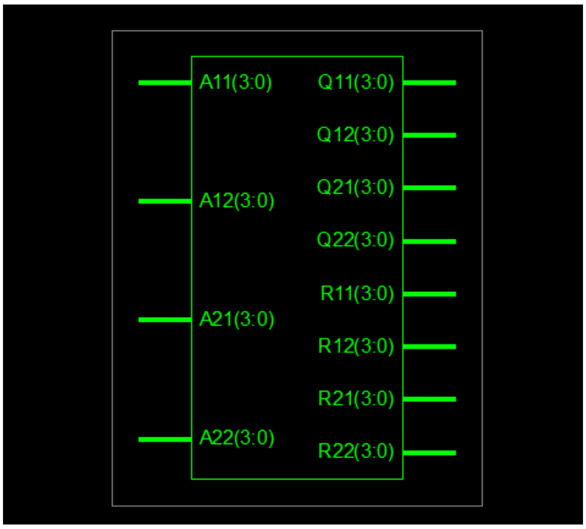

Fig. 5 RTL view of Q and R matrix

6. CONCLUSION

This paper has presented an implementation of the Hybrid QR decomposition based on Gram-Schmidt algorithm for MIMO-OFDM systems. The Hybrid QR Decomposition divides the input matrix into mxn sub-matrices. The architecture of Hybrid QR decomposition reduces the hardware cost. The Hybrid QR Decomposition reduces the clock latency than QR Decomposition. The proposed architecture is implemented in ModelSim-Altera 6.3g_p1 (Quartus II 8.1) and verified by Xilinx 9.1i XSE.

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