NEW APPROACH FOR WOLFE'S MODIFIED SIMPLEX METHOD TO SOLVE QUADRATIC PROGRAMMING PROBLEMS

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Abstract

In this paper, an alternative method for Wolfe's modified simplex method is introduced. This method is easy to solve quadratic programming problem (QPP) concern with non-linear programming problem (NLPP). In linear programming models, the characteristic assumption is the linearity of the objective function and constraints. Although this assumption holds in numerous practical situations, yet we come across many situations where the objective function and some or all of the constraints are non-linear functions. The non-linearity of the functions makes the solution of the problem much more involved as compared to LPPs and there is no single algorithm like the simplex method, which can be employed to solve efficiently all NPPs.

Keywords: *Quadratic programming problem, New approach, Modified simplex method, and Optimal solution.*

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1. INTRODUCTION

Quadratic programming problems (QPP) deals with the non-linear programming problem (NLPP) of maximizing (or minimizing) the quadratic objective function subject to a set of linear inequality constraints.

In general QPP be:

$$\text{Maximize} Z = f(x) = \sum_{j=1}^{n} c_j x_j + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} c_{jk} x_j x_k$$

Subject to the constraints:

$$\sum_{j=1}^{n} a_{ij} x_j \leq \ b_i \,, x_j \geq 0 \ (\mathrm{i} = 1,2,\dots,\mathrm{m}, \mathrm{j} = 1,2,\dots,\mathrm{n})$$

where $c_{jk} = c_{kj}$ for all j,k and $b_i \geq 0$ for all $i = 1,2,\dots,m$.

Also, assume that the quadratic form

$$\sum_{j=1}^{n}\sum_{k=1}^{n} c_{jk} x_j x_k$$

be negative semi-definite.

Terlaky's algorithm is active set method, start from a primal feasible solution to construct dual feasible solution which is complimentary to the primal feasible solution. Terlaky [11] proposed an algorithm which does not require the enlargement of the basic table as Frank-Wolfe [4] method. Wolfe Philip [13] has given algorithm which based on fairly simple modification of simplex method and converges in finite number of iterations. Dantzig [3] suggestion is to choose that entering vector corresponding to which $z_j - c_j$ is most negative. Khobragade et al. [7] suggestion is to choose that entering vector corresponding to which $\frac{(z_j - c_j)}{\sum x_i}$ is most negative, where $\sum x_i$ is the sum of corresponding column to each $z_j - c_j$.

In this paper, an attempt has been made to solve quadratic programming problem (QPP) by new method which is an alternative for Wolfe's method. This method is different from Terlaky, Wolfe, Khobragade et al. method.

2. AN ALTERNATIVE ALGORITHM FOR WOLFE'S MODIFIED SIMPLEX METHOD

To find optimal solution of any QLPP by an alternative method for Wolfe's modified simplex method, algorithm is given as follows:

Step 1. First, convert the inequality constraints into equations by introducing slack-variables $\boldsymbol{y_i^2}$ $(\boldsymbol{i = 1,2,\dots m}) \geq \boldsymbol{0}$ in the $\boldsymbol{i^{th}}$ constraints and the non-negative restrictions by introducing slack variables $\boldsymbol{s_j^2}$ $(\boldsymbol{j = 1,2,\dots n}) \geq \boldsymbol{0}$ in the $\boldsymbol{j^{th}}$ restrictions.

Step 2. Construct the Lagrangian function

$$\boldsymbol{L(x,y,s,\lambda,\mu) = f(x) - \sum_{i=1}^{m} \lambda_i \left[\sum_{j=1}^{n} a_{ij} x_j - \ b_i + q_i^2\right] - \sum_{j=1}^{n} \mu_j [- x_j + s_j^2].}$$

Differentiating this Lagrangian function $\boldsymbol{L(x,y,s,\lambda,\mu)}$ with respect to the components of $\boldsymbol{x,y,s,\lambda,\mu}$ and equating the first order partial derivatives to zero, derive Kuhn-Tucker conditions from the resulting equations.

Step 3. Introduce non-negative artificial variables $\boldsymbol{a_j}, (\boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \ldots, \boldsymbol{n})$ in the Kuhn-Tucker conditions

$$\boldsymbol{c_j + \sum_{k=1}^{n} c_{jk}x_k - \sum_{i=1}^{m} \lambda_i a_{ij} + \mu_j = 0}$$

for $\boldsymbol{j = 1, 2, \ldots, n}$ and construct an objective function

$$\boldsymbol{Z = a_1 + a_2 + \cdots + a_n}.$$

Step 4. Obtain the initial basic feasible solution to the LPP:

$$\text{Min } \boldsymbol{Z = a_1 + a_2 + \cdots + a_n}$$

Subject to the constraints:

$$\boldsymbol{\sum_{k=1}^{n} c_{jk}x_k - \sum_{i=1}^{m} \lambda_i a_{ij} + \mu_j = -\ c_j; (j = 1, 2, \ldots, n)}$$

$$\boldsymbol{\sum_{j=1}^{n} a_{ij}x_j + q_i^2 = \ b_i; (i = 1, 2, \ldots, m)}$$

$$\boldsymbol{\lambda_i, \mu_j, x_j, y_i, a_j \geq 0, \qquad (i = 1, \ldots, m; j = 1, \ldots, n)}$$

and satisfying the slackness condition:

$$\boldsymbol{\lambda_i\, y_i = 0} \text{ and } \boldsymbol{\mu_j x_j = 0}.$$

Step 5. Solve this LPP by an alternative two-phase method. Choose greatest coefficient of decision variables.
(i) If greatest coefficient is unique, then variable corresponding to this column becomes incoming variable.
(ii) If greatest coefficient is not unique, then use tie breaking technique.

Step 6. Compute the ratio with $\boldsymbol{X_B}$. Choose minimum ratio, then variable corresponding to this row is outgoing variable. The element corresponding to incoming variable and outgoing variable becomes pivotal (leading) element.

Step 7. Use usual simplex method for this table and go to next step.

Step 8. Ignore corresponding row and column. Proceed to step 5 for remaining elements and repeat the same procedure either an optimal solution is obtain or there is an indication of an unbounded solution**.**

Step 9. If all rows and columns are ignored, current solution is an optimal solution. Thus optimum solution is obtained and which is optimum solution of given QPP also.

3. SOLVED PROBLEMS

3.1. Problem 1:

Solve the following quadratic programming problem:

Maximize $Z = 4x_1 + 2x_2 - x_1^2 - x_2^2 - 5$
Subject to: $x_1 + x_2 \leq 4$
$x_1,\ x_2 \geq 0.$

Solution: First, we convert the inequality constraint into equation by introducing slack variable s_1^2. Also the inequality constraints $x_1,\ x_2 \geq 0$, we convert them into equations by introducing slack variables s_2^2 and s_3^2. So the problem becomes

Maximize $Z = 4x_1 + 2x_2 - x_1^2 - x_2^2 - 5$
Subject to: $x_1 + x_2 + s_1^2 = 4$
$-x_1 + s_2^2 = 0$
$-x_2 + s_3^2 = 0.$

Now, Construct the Lagrangian function

$$L(x_1, x_2, s_1, s_2, s_3, \lambda_1, \lambda_2, \lambda_3) = (4x_1 + 2x_2 - x_1^2 - x_2^2 - 5) - \lambda_1(x_1 + x_2 + s_1^2 - 4) - \lambda_2(-x_1 + s_2^2) - \lambda_3(-x_2 + s_3^2)$$

By Khun-Tucker conditions, we get

$$2x_1 + \lambda_1 - \lambda_2 = 4, \qquad 2x_2 + \lambda_1 - \lambda_3 = 2$$

$$\lambda_1 s_1 = \lambda_2 s_2 = \lambda_3 s_3 = 0$$

$$x_1 + \ x_2 \ + s_1^2 = 4, \quad -x_1 + s_2^2 = 0, \qquad -x_2 + s_3^2 = 0$$

where $x_1,\ x_2, s_1^2, \lambda_i \geq 0, i = 1, \ldots, 3$ satisfying the complementary slackness conditions

$$\lambda_1 s_1^2 + x_1\lambda_2 + x_2\lambda_3 = 0.$$

Now, introducing the artificial variables $a_1, a_2 \geq 0$ the given QPP is equivalent to:

Minimize $Z = a_1 + a_2$
Subject to: $2x_1 + \lambda_1 - \lambda_2 + a_1 = 4$
$2x_2 + \lambda_1 - \lambda_3 + a_2 = 2$
$x_1 + \ x_2 \ + s_1^2 = 4$

where $x_1,\ x_2, s_1^2, a_1, a_2, \lambda_i \geq 0, i = 1, \ldots, 3.$

Simplex table:

C_B	BVS	X_B	x_1	x_2	λ_1	λ_2	λ_3	a_1	a_2	s_1^2	Ratio
1	a_1	4	**2**	0	1	-1	0	1	0	0	2→
1	a_2	2	0	2	1	0	-1	0	1	0	-
0	s_1^2	4	1	1	0	0	0	0	0	1	4
0	x_1	2	1	0	1/2	-1/2	0	1/2	0	0	-
1	a_2	2	0	**2**	-1	0	-1	0	1	0	1→
0	s_1^2	2	0	1	-1/2	1/2	0	-1/2	0	1	2

0	x_1	2	1	0	1/2	-1/2	0	1/2	0	0	
1	x_2	1	0	1	1/2	0	-1/2	0	1/2	0	
0	s_1^2	1	0	0	-3/2	1/2	1/2	-1/2	-1/2	1	

Current solution is an optimal solution. $x_1 = 2,\ x_2 = 1$. Max. $Z = 0$.

3.2. Problem 2:

Use Wolfe's method to solve the following quadratic programming problem:

Minimize $Z = 6 - 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$

Subject to: $x_1 + x_2 \le 2$

$x_1,\ x_2 \ge 0.$

Solution: First, we convert the inequality constraint into equation by introducing slack variable s_1^2. Also the inequality constraints $x_1,\ x_2 \ge 0$, we convert them into equations by introducing slack variables s_2^2 and s_3^2. So the problem becomes

Maximize $Z = -6 + 6x_1 - 2x_1^2 + 2x_1x_2 - 2x_2^2$

Subject to: $x_1 + x_2 + s_1^2 = 2$

$-x_1 + s_2^2 = 0$

$-x_2 + s_3^2 = 0.$

Now, Construct the Lagrangian function

$$L(x_1, x_2, s_1, s_2, s_3, \lambda_1, \lambda_2, \lambda_3) = (-6 + 6x_1 - 2x_1^2 + 2x_1x_2 - 2x_2^2) - \lambda_1(x_1 + x_2 + s_1^2 - 2) - \lambda_2(-x_1 + s_2^2) - \lambda_3(-x_2 + s_3^2)$$

By Khun-Tucker conditions, we get

$$4x_1 - 2x_2 + \lambda_1 - \lambda_2 = 6, \qquad -2x_1 + 4x_2 + \lambda_1 - \lambda_3 = 0$$

$$\lambda_1 s_1 = \lambda_2 s_2 = \lambda_3 s_3 = 0$$

$$x_1 + x_2 + s_1^2 = 2, \quad -x_1 + s_2^2 = 0, \quad -x_2 + s_3^2 = 0$$

where $x_1,\ x_2, s_1^2, \lambda_i \ge 0, i = 1, \dots, 3$ satisfying the complementary slackness conditions

$$\lambda_1 s_1^2 + x_1\lambda_2 + x_2\lambda_3 = 0.$$

Now, introducing the artificial variables $a_1, a_2 \ge 0$ the given QPP is equivalent to:

Minimize $Z = a_1 + a_2$

Subject to: $4x_1 - 2x_2 + \lambda_1 - \lambda_2 + a_1 = 6$

$-2x_1 + 4x_2 + \lambda_1 - \lambda_2 + a_2 = 0$

$x_1 + x_2 + s_1^2 = 2$

where $x_1,\ x_2, s_1^2, a_1, a_2, \lambda_i \ge 0, i = 1, \dots, 3.$

Simplex table:

C_B	BVS	X_B	x_1	x_2	λ_1	λ_2	λ_3	a_1	a_2	s_1^2	Ratio
1	a_1	6	**4**	-2	1	-1	0	1	0	0	3/2→
1	a_2	0	-2	4	1	0	-1	0	1	0	-
0	s_1^2	2	1	1	0	0	0	0	0	1	2
0	x_1	3/2	1	-1/2	1/4	-1/4	0	1/4	0	0	-
1	a_2	3	0	3	3/2	-1/2	-1	1/2	1	0	1→
0	s_1^2	1/2	0	**3/2**	-1/4	1/4	0	-1/4	0	1	1/3
0	x_1	5/3	1	0	1/6	-1/6	0	1/6	0	1/3	10
1	a_2	2	0	0	**2**	-1	-1	1	1	-2	1→
0	x_2	1/3	0	1	-1/6	1/6	0	-1/6	0	2/3	-
0	x_1	3/2	1	0	0	-1/2	1/12	1/12	1/12	1/2	
0	λ_1	1	0	0	1	-1/2	-1/2	1/2	1/2	-1	
0	x_2	1/2	0	1	0	1/12	-1/12	-1/12	1/12	1/2	

Current solution is an optimal solution. $x_1 = \frac{3}{2},\ x_2 = \frac{1}{2}$. Max. $Z = \frac{1}{2}$.

3.3. Problem 3:

Apply Wolfe's method to solve the QPP:

Maximize $Z = 2x_1 + 3x_2 - 2x_1^2$

Subject to: $x_1 + 4x_2 \le 4$

$x_1 + x_2 \le 2$

$x_1,\ x_2 \ge 0.$

Solution: First, we convert the inequality constraints into equations by introducing slack variables s_1^2 and s_2^2 respectively. Also the inequality constraints $x_1,\ x_2 \ge 0$, we convert them into equations by introducing slack variables s_3^2 and s_4^2. So the problem becomes

Maximize $Z = 2x_1 + 3x_2 - 2x_1^2$

Subject to: $x_1 + 4x_2 + s_1^2 = 4$

$x_1 + x_2 + s_2^2 = 2$

$-x_1 + s_3^2 = 0$

$-x_2 + s_4^2 = 0.$

Now, Construct the Lagrangian function

$$L(x_1, x_2, s_1, s_2, s_3, s_4, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (2x_1 + 3x_2 - 2x_1^2) - \lambda_1(x_1 + 4x_2 + s_1^2 - 4) - \lambda_2(x_1 + x_2 + s_2^2 - 2) - \lambda_3(-x_1 + s_3^2) - \lambda_4(-x_2 + s_4^2)$$

By Khun-Tucker conditions, we get

$$4x_1 + \lambda_1 + \lambda_2 - \lambda_3 = 2, \quad 4\lambda_1 + \lambda_2 - \lambda_4 = 3$$

$$\lambda_1 s_1 = \lambda_2 s_2 = \lambda_3 s_3 = \lambda_4 s_4 = 0$$

$$x_1 + 4x_2 + s_1^2 = 4, \quad x_1 + x_2 + s_2^2 = 2,$$

$$-x_1 + s_3^2 = 0, \quad -x_2 + s_4^2 = 0$$

where $x_1, x_2, s_1^2, s_2^2, \lambda_i \geq 0, i = 1, \ldots, 4$ satisfying the complementary slackness conditions

$$\lambda_1 s_1^2 + \lambda_2 s_2^2 + x_1\lambda_3 + x_2\lambda_4 = 0.$$

Now, introducing the artificial variables $a_1, a_2 \geq 0$ the given QPP is equivalent to:

Minimize $Z = a_1 + a_2$

Subject to: $4x_1 + \lambda_1 + \lambda_2 - \lambda_3 + a_1 = 2$

$4\lambda_1 + \lambda_2 - \lambda_4 + a_2 = 3$

$x_1 + 4x_2 + s_1^2 = 4$

$x_1 + x_2 + s_2^2 = 2$

where $x_1, x_2, s_1^2, s_2^2, a_1, a_2, \lambda_i \geq 0, i = 1, \ldots, 4$.

Simplex table:

C_B	BVS	X_B	x_1	x_2	λ_1	λ_2	λ_3	λ_4	a_1	a_2	s_1^2	s_2^2	Ratio
1	a_1	2	**4**	0	1	1	-1	0	1	0	0	0	1/2→
1	a_2	3	0	0	4	1	0	-1	0	1	0	0	-
0	s_1^2	4	1	4	0	0	0	0	0	0	1	0	4
0	s_2^2	2	1	1	0	0	0	0	0	0	0	1	2
0	x_1	1/2	1	0	1/4	1/4	-1/4	0	1/4	0	0	0	-
1	a_2	3	0	0	4	1	0	-1	0	1	0	0	-
0	s_1^2	7/2	0	**4**	-1/4	-1/4	1/4	0	-1/4	0	1	0	7/8→
0	s_2^2	3/2	0	1	-1/4	-1/4	1/4	0	-1/4	0	0	1	3/2
0	x_1	1/2	1	0	1/4	1/4	-1/4	0	1/4	0	0	0	2
1	a_2	3	0	0	**4**	1	0	-1	0	1	0	0	3/4→
0	x_2	7/8	0	1	-1/16	-1/16	1/16	0	-1/16	0	1/4	0	-
0	s_2^2	5/8	0	0	-3/16	-3/16	3/16	0	-3/16	0	-1/4	1	-
0	x_1	5/16	1	0	0	3/16	-1/4	1/16	1/4	-1/16	0	0	
0	λ_1	3/4	0	0	1	1/4	0	-1/4	0	1/4	0	0	
0	x_2	59/64	0	1	0	-3/64	1/16	-1/64	-1/16	1/64	1/4	0	
0	s_2^2	49/64	0	0	0	-9/64	3/16	-3/64	-3/16	3/64	-1/4	1	

Current solution is an optimal solution. $x_1 = \frac{5}{16}$, $x_2 = \frac{59}{16}$. Max. Z = 3.19.

3.4. Problem 4:

Solve by Wolfe's method:

Maximize $Z = 2x_1 + x_2 - x_1^2$

Subject to: $2x_1 + 3x_2 \leq 6$

$2x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0.$

Solution: First, we convert the inequality constraints into equations by introducing slack variables s_1^2 and s_2^2 respectively. Also the inequality constraints $x_1, x_2 \geq 0$, we convert them into equations by introducing slack variables s_3^2 and s_4^2. So the problem becomes

Maximize $Z = 2x_1 + x_2 - x_1^2$

Subject to: $2x_1 + 3x_2 + s_1^2 = 6$

$2x_1 + x_2 + s_2^2 = 4$

$-x_1 + s_3^2 = 0$

$-x_2 + s_4^2 = 0.$

Now, Construct the Lagrangian function

$$L(x_1, x_2, s_1, s_2, s_3, s_4, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (2x_1 + x_2 - x_1^2) - \lambda_1(2x_1 + 3x_2 + s_1^2 - 6) - \lambda_2(2x_1 + x_2 + s_2^2 - 4) - \lambda_3(-x_1 + s_3^2) - \lambda_4(-x_2 + s_4^2)$$

By Khun-Tucker conditions, we get

$$2x_1 + 2\lambda_1 + 2\lambda_2 - \lambda_3 = 2, \quad 3\lambda_1 + \lambda_2 - \lambda_4 = 1$$

$$\lambda_1 s_1 = \lambda_2 s_2 = \lambda_3 s_3 = \lambda_4 s_4 = 0$$

$$2x_1 + 3x_2 + s_1^2 = 6, \quad 2x_1 + x_2 + s_2^2 = 4,$$

$$-x_1 + s_3^2 = 0, \quad -x_2 + s_4^2 = 0$$

where $x_1, x_2, s_1^2, s_2^2, \lambda_i \geq 0, i = 1, \dots, 4$ satisfying the complementary slackness conditions

$$\lambda_1 s_1^2 + \lambda_2 s_2^2 + x_1\lambda_3 + x_2\lambda_4 = 0.$$

Now, introducing the artificial variables $a_1, a_2 \geq 0$ the given QPP is equivalent to:

Minimize $Z = a_1 + a_2$

Subject to: $2x_1 + 2\lambda_1 + 2\lambda_2 - \lambda_3 + a_1 = 2$

$$3\lambda_1 + \lambda_2 - \lambda_4 + a_2 = 1$$
$$2x_1 + 3x_2 + s_1^2 = 6$$
$$2x_1 + x_2 + s_2^2 = 4$$

where $x_1, x_2, s_1^2, s_2^2, a_1, a_2, \lambda_i \geq 0, i = 1, \dots, 4$.

Simplex table:

C_B	BVS	X_B	x_1	x_2	λ_1	λ_2	λ_3	λ_4	a_1	a_2	s_1^2	s_2^2	Ratio
1	a_1	2	2	0	2	2	-1	0	1	0	0	0	1
1	a_2	1	0	0	**3**	1	0	-1	0	1	0	0	1/3→
0	s_1^2	6	2	3	0	0	0	0	0	0	1	0	-
0	s_2^2	4	2	1	0	0	0	0	0	0	0	1	-
1	a_1	4/3	2	0	0	4/3	-1	2/3	1	-2/3	0	0	-
0	λ_1	1/3	0	0	1	1/3	0	-1/3	0	1/3	0	0	-
0	s_1^2	6	2	**3**	0	0	0	0	0	0	1	0	2→
0	s_2^2	4	2	1	0	0	0	0	0	0	0	1	4
1	a_1	4/3	**2**	0	0	4/3	-1	2/3	1	-2/3	0	0	2/3→
0	λ_1	1/3	0	0	1	1/3	0	-1/3	0	1/3	0	0	-
0	x_2	2	2/3	1	0	0	0	0	0	0	1/3	0	3
0	s_2^2	2	4/3	0	0	0	0	0	0	0	-1/3	1	3/2
0	x_1	2/3	1	0	0	2/3	-1/2	1/3	1/2	-1/3	0	0	
0	λ_1	1/3	0	0	1	1/3	0	-1/3	0	1/3	0	0	
0	x_2	14/9	0	1	0	-4/9	1/3	-2/9	-1/3	2/9	1/3	0	
0	s_2^2	10/9	0	0	0	-8/9	2/3	-4/9	-2/3	4/9	-1/3	1	

Current solution is an optimal solution. $x_1 = \frac{2}{3},\ x_2 = \frac{14}{9}$. Max. $Z = \frac{22}{9}$.

3.5. Problem 5:

Solve the following quadratic programming problem:

Minimize $Z = -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$

Subject to: $2x_1 + x_2 \leq 6$

$$x_1 - 4x_2 \leq 0$$
$$x_1,\ x_2 \geq 0.$$

Solution: First, we convert the inequality constraints into equations by introducing slack variables s_1^2 and s_2^2 respectively. Also the inequality constraints $x_1, x_2 \geq 0$, we convert them into equations by introducing slack variables s_3^2 and s_4^2. So the problem becomes

Maximize $Z = 4x_1 - x_1^2 + 2x_1x_2 - 2x_2^2$

Subject to: $2x_1 + x_2 + s_1^2 = 6$

$$x_1 - 4x_2 + s_2^2 = 0$$
$$-x_1 + s_3^2 = 0$$
$$-x_2 + s_4^2 = 0.$$

Now, Construct the Lagrangian function

$$L(x_1, x_2, s_1, s_2, s_3, s_4, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (4x_1 - x_1^2 + 2x_1x_2 - 2x_2^2) - \lambda_1(2x_1 + x_2 + s_1^2 - 6) - \lambda_2(x_1 - 4x_2 + s_2^2) - \lambda_3(-x_1 + s_3^2) - \lambda_4(-x_2 + s_4^2)$$

By Khun-Tucker conditions, we get

$$2x_1 - 2x_2 + 2\lambda_1 + \lambda_2 - \lambda_3 = 4,$$
$$-2x_1 + 4x_2 + \lambda_1 - 4\lambda_2 - \lambda_4 = 0$$

$$\lambda_1 s_1 = \lambda_2 s_2 = \lambda_3 s_3 = \lambda_4 s_4 = 0$$

$$2x_1 + x_2 + s_1^2 = 6, \qquad x_1 - 4x_2 + s_2^2 = 0,$$

$$-x_1 + s_3^2 = 0, \qquad -x_2 + s_4^2 = 0$$

where $x_1, x_2, s_1^2, s_2^2, \lambda_i \geq 0, i = 1, \dots, 4$ satisfying the complementary slackness conditions

$$\lambda_1 s_1^2 + \lambda_2 s_2^2 + x_1\lambda_3 + x_2\lambda_4 = 0.$$

Now, introducing the artificial variables $a_1, a_2 \geq 0$ the given QPP is equivalent to:

Minimize $Z = a_1 + a_2$

Subject to:

$$2x_1 - 2x_2 + 2\lambda_1 + \lambda_2 - \lambda_3 + a_1 = 4$$
$$-2x_1 + 4x_2 + \lambda_1 - 4\lambda_2 - \lambda_4 + a_2 = 0$$
$$2x_1 + x_2 + s_1^2 = 6$$
$$x_1 - 4x_2 + s_2^2 = 0$$

where $x_1, x_2, s_1^2, s_2^2, a_1, a_2, \lambda_i \geq 0, i = 1, \dots, 4$.

Simplex table:

C_B	BVS	X_B	x_1	x_2	λ_1	λ_2	λ_3	λ_4	a_1	a_2	s_1^2	s_2^2
1	a_1	4	2	-2	2	1	-1	0	1	0	0	0
1	a_2	0	-2	**4**	1	-4	0	-1	0	1	0	0
0	s_1^2	6	2	1	0	0	0	0	0	0	1	0
0	s_2^2	0	1	-4	0	0	0	0	0	0	0	1
1	a_1	4	1	0	5/4	-1	-1	-1/2	1	1/2	0	0
0	x_2	0	-1/2	1	1/4	-1	0	-1/4	0	1/4	0	0
0	s_1^2	6	**5/2**	0	-1/4	1	0	1/4	0	-1/4	1	0
0	s_2^2	0	-1	0	1	-4	0	-1	0	1	0	1
1	a_1	8/5	0	0	**13/5**	-7/5	-1	-3/5	1	3/5	-2/5	0
0	x_2	6/5	0	1	1/5	-4/5	0	-1/5	0	1/5	1/5	0
0	x_1	12/5	1	0	-1/10	2/5	0	1/10	0	-1/10	2/5	0
0	s_2^2	12/5	0	0	9/10	-18/5	0	-9/10	0	9/10	2/5	1
0	λ_1	8/13	0	0	1	-7/13	-5/13	-3/13	5/13	1/26	0	0
0	x_2	14/13	0	1	0	-9/13	1/13	-2/13	-1/13	5/26	0	0
0	x_1	32/13	1	0	0	9/26	-1/26	1/13	1/26	-5/52	1	0
0	s_2^2	24/13	0	0	0	-81/26	9/26	-9/13	-9/26	45/52	0	1

Current solution is an optimal solution. $x_1 = \frac{32}{13}$, $x_2 = \frac{14}{13}$. Min. $Z = -\frac{88}{13}$.

4. CONCLUSION

An alternative method for Wolfe's method to obtain the solution of quadratic programming problems has been derived. An algorithm that performs well on one type of the problem may perform poorly on problem with a different structure. A number of algorithms have been developed, each applicable to specific type of NPPP only. The numbers of application of non-linear programming are very large and it is not possible to give a comprehensive survey of all of them. However, an efficient method for the solution of general NLPP is still. This technique is useful to apply on numerical problems, reduces the labour work and save valuable time.

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