BOUNDARY LAYER FLOW AND HEAT TRANSFER OF A DUSTY FLUID OVER A VERTICAL PERMEABLE STRETCHING SURFACE

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Abstract

The steady boundary layer free convective flow of a dusty fluid past a vertical permeable stretching surface is studied .The governing equations are converted into first order ordinary differential equations using similarity transformations. These equations are solved numerically by using Runge kutta forth order method. The effects of physical parameters like fluid-particle interaction parameter, local Grashof number, suction parameters, Prandtl number, radiation parameter and Eckert number on the flow and heat transfer characteristics are computed and presented graphically. Also the rate of heat transfer at the surface is discussed. The present results are compared with the previous study and there is a good agreement.

AMS classification *76T10, 76T15*

Keywords: *Volume fraction, Interaction parameter, Dusty fluid, Thermal radiation, suction parameter, steady flow and heat transfer, Boundary layer flow, Numerical solution.*

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NOMENCLATURE

E_c Eckert number
q_r radiation heat flux
q_{r_p} radiation heat flux of particle phase
F_r Froud number
G_r Grashof number
P_r Prandtl number
T_∞ temperature at large distance from the wall.
T_p temperature of particle phase.
T_w wall temperature
$U_w(x)$ stretching sheet velocity
c_p specific heat of fluid
c_s specific heat of particles
k_s thermal conductivity of particle
u_p , v_p velocity component of the particle along x-axis and y-axis
A constant
Ra Thermal radiation
c stretching rate
f_0 suction parameter
g acceleration due to gravity
k thermal conductivity of fluid
l characterstic length
T temperature of fluid phase.
u,v velocity component of fluid along x-axis and y-axis
x,y cartesian coordinate
K* Mean absorption co-efficient

GREEK SYMBOLS

φ volume fraction
β fluid particle interaction parameter
β^* volumetric coefficient of thermal expansion
σ^* the Stefan Bolzman constant
ρ density of the fluid
ρ_p density of the particle phase
ρ_s material density
η similarity variable
θ fluid phase temperature
θ_p dust phase temperature
μ dynamic viscosity of fluid
ν kinematic viscosity of fluid
γ ratio of specific heat
τ relaxation time of particle phase
τ_T thermal relaxation time i.e. the time required by the dust particle to adjust its temperature relative to the fluid.
τ_p velocity relaxation time i.e. the time required by the dust particle to adjust its velocity relative to the fluid.
ε diffusion parameter
ω density ratio

1. INTRODUCTION

The study of laminar flow and heat transfer over a stretching sheet has a considerable interest because of its ever increasing industrial applications and important bearing on several technological processes. Such applications are in chemical industry, power and cooling industry for drying ,chemical vapour deposition on surface s and cooling of nuclear reactors etc. Such processes occur when the effect of buoyancy forces in free convection becomes significant. The problem of free convection under the influence of magnetic flied has attracted numerous researchers in view of its applications in geophysics and astrophysics

The study of the boundary layer flow over a stretched surface moving with a constant velocity was initiated by Sakiadis B.C.[20]in1961 .Then many researchers extended the above study with the effect of Heat Transfer .Tsou et.al [24] studied the effect of Heat Transfer and experimentally confirmed the numerical result of Sakiadis .Grubka et.al[6] investigated the temperature field in the flow over a stretching surface when subject to uniform heat flux .H.I.Anderson et.at [8] discussed a new similarity solution for the temperature fields .Sharidan et.al [21] presented similarity solutions for unsteady boundary layer flow and heat Transfer due to stretching sheet .A numerical solution for laminar thermal boundary over a flat plate with convective surface boundary condition was analyzed by Aziz[1].Chen [5] investigated mixed convection of a power law fluid past a stretching surface in presence of thermal radiation and magnetic field .Chakrabarti et.al[11] have studied note on boundary layer in a dusty gas.Crane [12] has obtained the flow past of stretching plate. B.J.Gireesha et.al[4] have studied the effect of hydrodynamic laminar boundary layer flow and heat Transfer of a dusty fluid over an unsteady stretching surface in presence of non uniform heat source/sink .They have examined the Heat Transfer characteristics for two type of boundary conditions namely variable wall temperature and variable Heat flux..G.K.Ramesh et.al [17] have investigated heat transfer in hydrodynamic flow of dusty boundary layer flow over an inclined stretching surface with non uniform heat source/sink .B.J. Gireesh et.al [3] also studied the mixed convective flow a dusty fluid over a stretching sheet in presence of thermal radiation, space dependent heat source/sink. V.R. Prasad, et.al[25] have studied Radiation on effect on MHD Unsteady free convection flow with a mass transfer past a vertical plate with variable surface temperature and concentration.

G.K.Ramesh et.al.[7] have studied on the radiation effects on a steady free convective boundary layer flow of a dusty fluid past a vertical permeable stretching surface**.** Robert et.al[19] have studied “Convective heattransfer ina conducting fluid over a permeable stretching surface with suction and internal heat generation/absorption.

In the present paper, the behavior of incompressible, laminar boundary- layer flows of a dusty fluid and heat transfer over a vertical permeable surface of stretching sheet with thermal radiation is studied. .By using similarity transformation we convert partial differential equations into first order ordinary differential equations and solved numerically Runge-Kutta fourth order method. In this case the thermal radiation effect plays a significant role in controlling heat transfer processes in polymer industry. The momentum equation for particulate phase in normal direction, heat due to conduction and viscous dissipation in the energy equation of the particle phase have been considered for better understanding of the boundary layer characteristics. The effects of volume fraction on skin friction, heat transfer and other boundary layer characteristics also have been studied.

2. FLOW ANALYSIS OF THE PROBLEM

Consider an steady two dimensional laminar boundary layer of an incompressible viscous dusty fluid over a vertical stretching sheet .The flow is generated by the action of two equal and opposite forces along the x-axis and y-axis being normal to the flow .The sheet being stretched with the velocity $U_w(x)$ along the x-axis, keeping the origin fixed in the fluid of ambient temperature T. Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size and number density of the dust particle s taken as a constant throughout the flow. Unlike previous studies, we have included particle-particle interaction terms in momentum and every equation

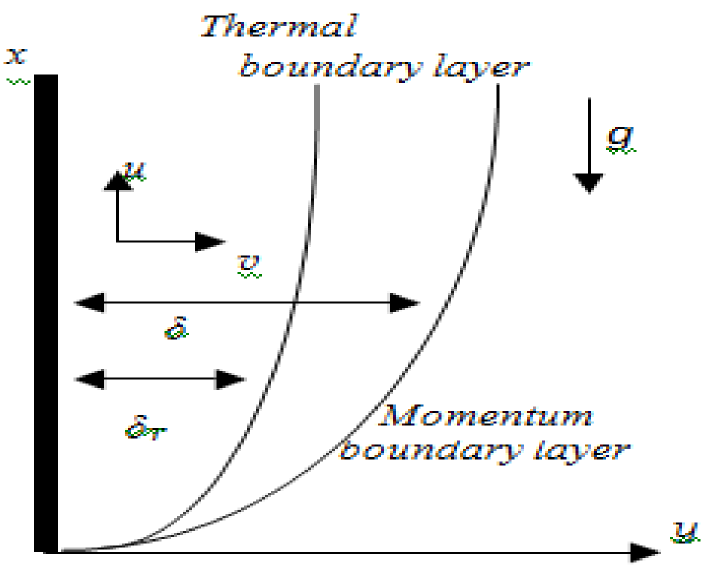


Fig. 1

The governing equations of steady two dimensional boundary layer incompressible flows of dusty fluids are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial}{\partial x}\left(\rho_p u_p\right)+\frac{\partial}{\partial y}\left(\rho_p v_p\right)=0 \qquad (2)$$

$$u\frac{\partial u}{\partial x}\ +v\frac{\partial v}{\partial y}=\frac{1}{\rho}\ \mu\frac{\partial^2 u}{\partial y^2}-\frac{1}{(1-\varphi)\rho}\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)+g\beta^*\left(T-T_\infty\right) \qquad (3)$$

$$\varphi\rho_s\left(u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\left(u-u_p\right)+\varphi\left(\rho_s-\rho\right)g \qquad (4)$$

$$\varphi\rho_s\left(u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial v_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s\left(v-v_p\right) \qquad (5)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\varphi\rho_s c_s}{(1-\varphi)c_p}\frac{1}{\rho\,\tau_T}\left(T_p-T\right)+\frac{\varphi\rho_s}{(1-\varphi)}\frac{1}{\rho c_p}\frac{1}{\tau_p}\left(u_p-u\right)^2+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \qquad (6)$$

$$u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}=-\frac{1}{\tau_p}\left(T_p-T\right)+\frac{1}{\varphi\rho_s c_s}\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)-\frac{1}{\tau_p}\frac{1}{c_s}\left(u-u_p\right)^2+\frac{\mu_s}{\rho_s c_s}\left[u_p\frac{\partial^2 u_p}{\partial y^2}+\left(\frac{\partial u_p}{\partial y}\right)^2\right]-\quad\varphi\ \frac{\partial q_{r_p}}{\partial y} \qquad (7)$$

Where $(u,v)\ and\ \left(u_p,v_p\right)$ are the velocity components of the fluid and dust particle phases along x and y directions respectfully. μ , ρ and ρ_p, are the co-efficient of viscosity of the fluid, density of the fluid and particle phase, of the particle phase respectfully.

With boundary conditions

$$\left.\begin{array}{c} u=U_\omega(x)=cx\,, v=-v_\omega(x)\ \ at\ y=0 \\ \rho_p=\omega\rho, u=0\,, u_p=0\,, v_p\to v s\ y\to\infty \end{array}\right\} \qquad (8)$$

Where ω is the density ratio in the main stream.

In order to solve (6) and (7), we consider non –dimensional temperature boundary conditions as follows

$$\left.\begin{array}{c} T=T_w=T_\infty+A\left(\frac{x}{l}\right)^2\ at\ y=0 \\ and\ T\to T_\infty\ ,T_p\to T_\infty\ \ as\ y\to\infty \end{array}\right\} \qquad (9)$$

Where A is a positive constant, $l=\sqrt{\frac{v}{c}}$ is a characteristic length.

Using the Rosseland approximation for radiation heat flux is simplified as

$$q_r=-\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y} \qquad (10)$$

Where σ^* and K^* are the Stefan Bolzman constant and the mean absorption co-efficient respectfully.

.Assuming that the temperature differences within the flow such that term T^4 may be expressed as a linear fuction of the temperature.We expand T^4 in a Taylor series about T_∞ and neglecting the higher order terms beyond the first degree in $(T-T_\infty)$we get

$$T^4\cong 4T_\infty^4\text{T-}3T_\infty^4 \qquad (11)$$

For most of the gases $\tau_p\approx\tau_T$, $k_s=k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$ if $\frac{c_s}{c_p}=\frac{2}{3P_r}$

Introducing the following non dimensional variables in equation (1) to (7)

$$u=cxf'(\eta)\,, v=-\sqrt{cv}f(\eta)\,,\eta=\sqrt{\frac{c}{v}}\ y\,, u_p=cxF'(\eta)\,, v_p=\sqrt{cv}\ G(\eta)\,,\varphi\rho_r=H(\eta)$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\ ,\ \theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty}\ ,\beta=\frac{1}{c\tau_p}\ ,\epsilon=\frac{v_s}{v}\ ,P_r=\frac{\mu c_p}{k}\ ,E_c=\frac{c^2l^2}{Ac_p},R_a=\frac{16T_\infty^3\sigma^*}{3K^*k},$$

Where $T-T_\infty=A\left(\frac{x}{l}\right)^2\theta$,

$$T_p-T_\infty=A\left(\frac{x}{l}\right)^2\theta_p\,,\frac{\partial q_{r_p}}{\partial y}=-\frac{16T_\infty^3\sigma^*}{3K^*}\frac{\partial^2 T_p}{\partial y^2}\,,$$

c is the stretching rate and being a positive constant.
c_p is the specific heat of fluid phase.
k is the thermal conductivity.
β is the fluid particle interaction parameter.
$\beta^*$is the volumetric coefficient of thermal expansion.
We get the following non dimensional form.

$$HF + HG' + GH' = 0 \tag{12}$$

$$f'''(\eta) + f(\eta)f''(\eta) - [f'(\eta)]^2 + \frac{1}{(1-\varphi)}\beta\, H(\eta)[F(\eta) - f'(\eta)] + Gr\theta = 0 \tag{13}$$

$$G(\eta)F'(\eta) + [F(\eta)]^2 = \epsilon\, F''(\eta) + \beta[f'(\eta) - F(\eta)] + \frac{1}{Fr}\left(1 - \frac{1}{\gamma}\right) \tag{14}$$

$$G' = \in G'' - \beta[\, f + G\,] \tag{15}$$

$$\theta'' = \left(Pr(2f'\theta - f\theta') - \frac{2}{3}\frac{\beta}{1-\varphi}H[\theta_p - \theta] - \frac{1}{1-\varphi}PrE_c\beta H[F - f']^2 - PrE_c f''^2 \right) \frac{1}{(R_a+1)} \tag{16}$$

$$\theta_p''(\eta) = \left(2F\theta_p + G\theta_p' + \beta[\theta_p - \theta] + \beta EcPr[f' - F]^2 - \frac{3}{2}\epsilon EcPr[FF'' + (F')^2]\right) / \left(\frac{\epsilon}{Pr} + \frac{3}{2}\frac{R_a}{\gamma}\right) \tag{17}$$

Where $Gr = g\frac{\beta^*(T_\omega - T_\infty)}{c^2 x}$ *is the local Grashof number* .

With boundary conditions

$$\left.\begin{array}{r} H'(\eta) = 0\,, G'(\eta) = 0\,, f(\eta) = f_0\,, f'(\eta) = 1\,, F'(\eta) = 0\,, \theta(\eta) = 1\,, \theta_p' = 0\ as\ \eta \to 0 \\ f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta)\,, H(\eta) = \omega\,, \theta(\eta) \to 0\,, \theta_p(\eta) \to 0\ as\ \eta \to \infty \end{array}\right\} \tag{18}$$

Where $f_{0=}\frac{V_w}{(\nu C)^{1/2}}$ is the suction parameter

3. SOLUTION OF THE PROBLEM

Here in this problem the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ are not known but $f'(\infty) = 0, F(\infty) = 0, G(\infty) = -f(\infty), H(\infty) = \omega, \theta(\infty) = 0, \theta_p(\infty) = 0$ are given. We use Shooting method to determine the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ .We have supplied $f''(0) = \propto_0$ and $f''(0) = \propto_1$. The improved value of $f''(0) = \propto_2$ is determined by utilizing linear interpolation formula. Then the value of $f'(\propto_2, \infty)$ is determined by using Runge-Kutta method. If $f'(\propto_2, \infty)$ is equal to $f'(\infty)$ up to a certain decimal accuracy, then $\propto_2$ i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\propto_0 = \propto_1$ *and* $\propto_1 = \propto_2$ until a correct $\propto_2$ is obtained. The same procedure described above is adopted to determine the correct values of $F(0), G(0), H(0), \theta'(0), \theta_p(0)$.

The essence of Shooting technique to solve a boundary value problem is to convert the boundary value problem into initial value problem. In this problem the missing value of $\theta'(0)$ and $f''(0)$ for different set of values of parameter are chosen on hit and trial basis such that the boundary condition at other end i.e. the boundary condition at infinity (η_∞)are satisfied.A study was conducting to examine the effect of step size as the appropriate values of step size $\Delta\eta$ was not known to compare the initial values of $\theta'(0)$ and $f''(0)$ .If they agreed to about 6 significant digits, the last value of η_∞ used was considered the appropriate value; otherwise the procedure was repeated until further change in η_∞ did not lead to any more change in the value of $\theta'(0)$ and $f''(0)$ .The step size $\Delta\eta$ =0.1 has been found to ensure to be the satisfactory convergence criterion of 1×10^{-6}.The solution of the present problem is obtained by numerical computation after finding the infinite value for η. It has been observed from the numerical result that the approximation to $\theta'(0)$ and $f''(0)$ are improved by increasing the infinite value of η which is finally determined as η =10.0 with a step length of 0.1 beginning from η = 0. Depending upon the initial guess and number of steps N. the values of $f''(0)$ and $\theta'(0)$ are obtained from numerical computations which are given in table – 2 for different parameters.

Table-1: Comparison results for the wall temperature gradient $-\theta'(0)$ in case of β=0,Gr=0,Ra=0, f_0=0, and Ec=0

Pr	Present Study $-\theta'(0)$	G.K. Ramesh , B.J.G&C.S.B [2012]	MS Able[2008]	Chen[1998]
0. 71	1. 0823306	1.0886	1.0886	1.08885
1.0	1.333400	1.3333	1.3333	1.3333

4. RESULTS AND DISCUSSION

The equations (12) to (17) subjected to boundary conditions (18) have been solved numerically using Runge-Kutta method coupled with a shooting technique for some values of the parameters $\beta, \varphi, \epsilon, \gamma, F_r, E_C, P_r, G_r, R_a$ and f_0. In order to check the accuracy of our present numerical solution procedure used a comparison of wall temperature gradient $-\theta'(0)$ is made with those reported by with G.K. Ramesh , B.J.Gireesha & C.S.Bagewadi[7], MS Able[13] and Chen [5] for various values of Pradtl number in absence of other parameters which are given in table-1.Our present result are in a good agreement with the previous results.

Fig-2: Variation of u w.r.t β

Φ=0.01 ,Ec=1.0,Pr=0.71,Gr=0.01,Ra=3.0 ϵ=5.0 Fr=10.0, γ=1.0, f0=1.0

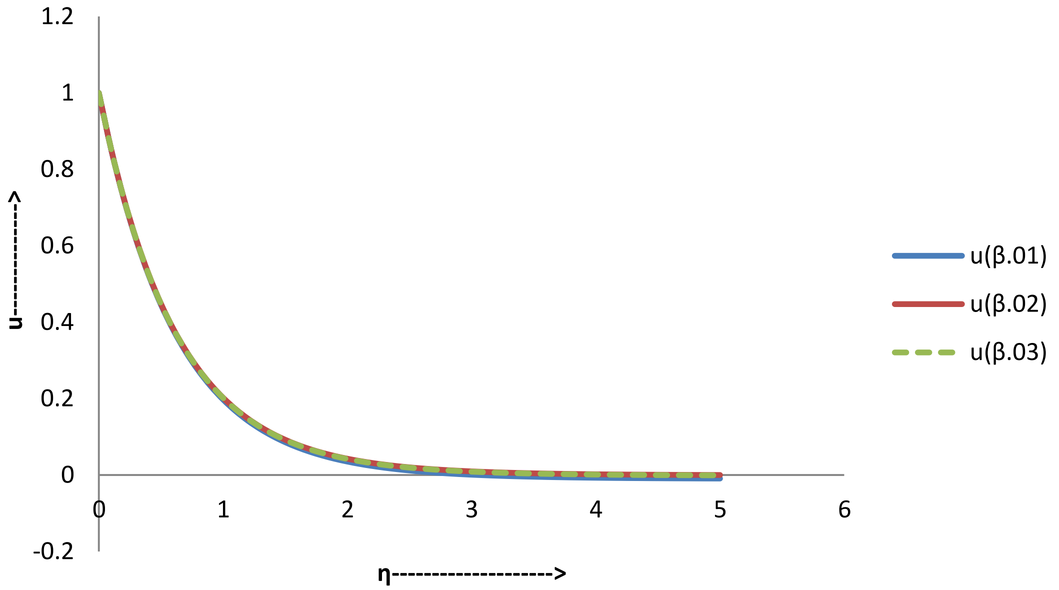


Fig-3: Variation of Up w.r.t β

Φ=0.01 ,Ec=1.0,Pr=0.71,Gr=0.01,Ra=3.0 ϵ=5.0 Fr=10.0, γ=1.0, f0=1.0

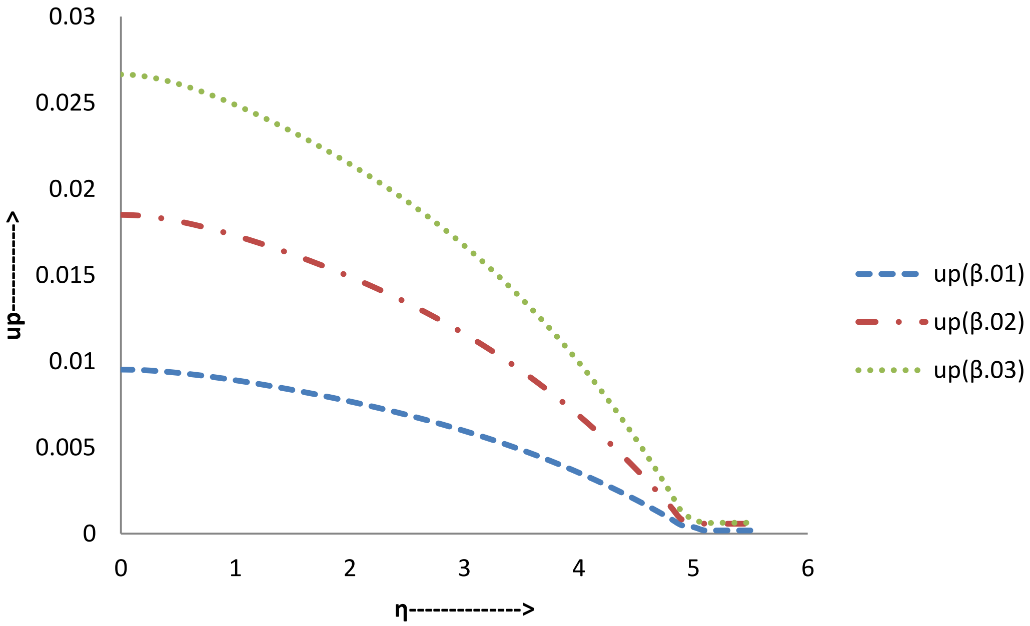


Fig-4: Variation of θ w.r.t Pr

Φ=0.01 ,Ec=1.0,Pr=0.71,Gr=0.01,Ra=3.0 ϵ=5.0 Fr=10.0, γ=1.0, f0=1.0

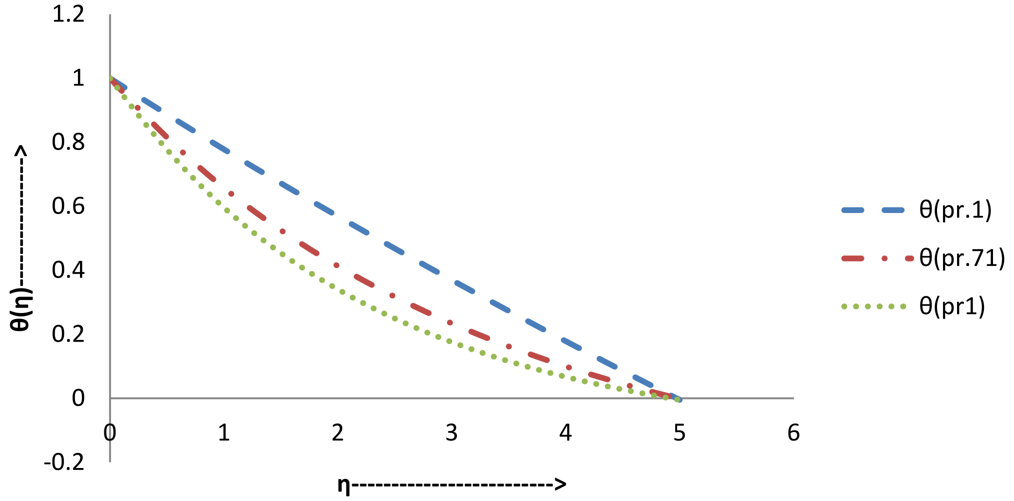


Fig-5Variation of θ w.r.t Gr

Φ=0.01 ,Ec=1.0,β=0.01,Pr=0.71,Ra=3.0 ϵ=2.0 Fr=10.0, γ=1.0, f0=1.0

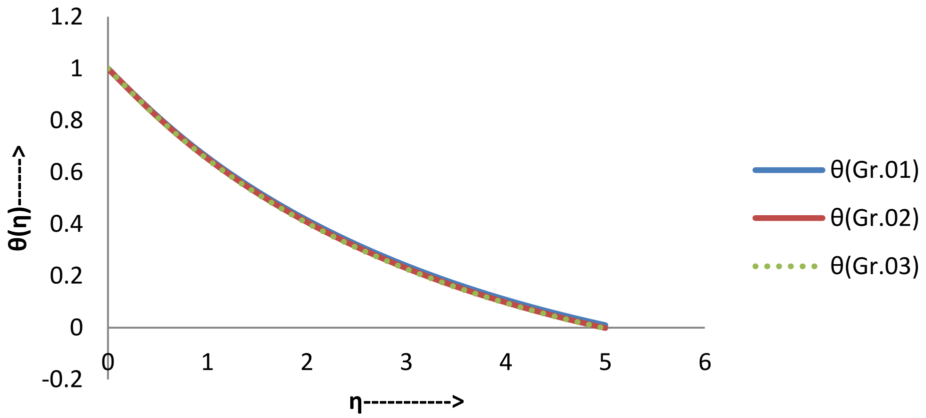


Fig-6 Variation of θp w.r.t Gr

Φ=0.01 ,Ec=1.0,β=0.01,Pr=0.71,Ra=3.0 ϵ=2.0 Fr=10.0, γ=1.0, f0=1.0

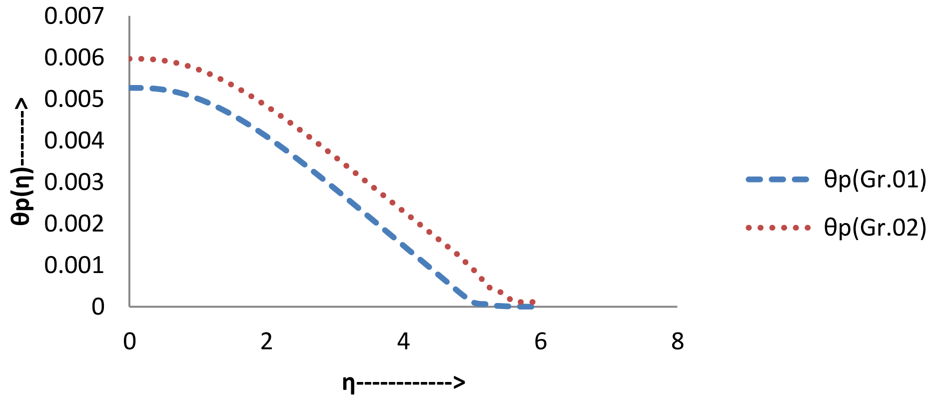


Fig-7 Variation of θ w.r.t Ra

Φ=0.01 ,Ec=1.0,β=0.01,Pr=0.71,Gr=0.01 ϵ=2.0 Fr=10.0, γ=1.0, f0=1.0

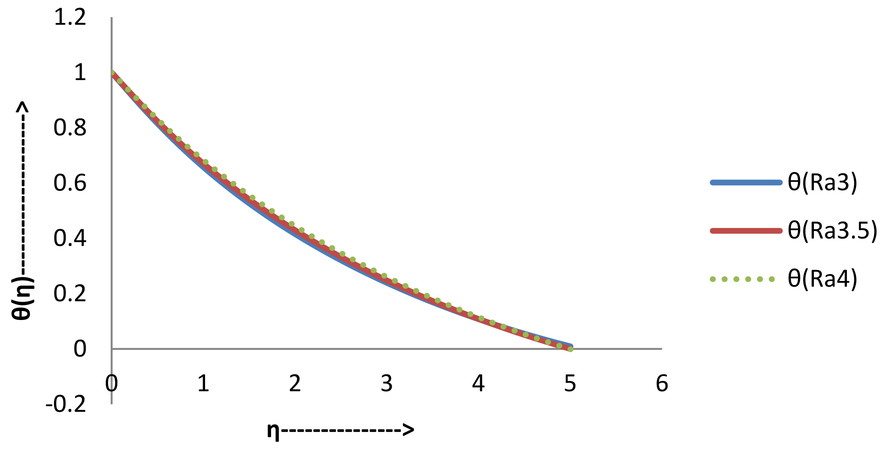


Fig-8 Variation of θp w.r.t Ra

Φ=0.01 ,Ec=1.0,β=0.01,Pr=0.71,Gr=0.01 ϵ=2.0 Fr=10.0, γ=1.0, f0=1.0

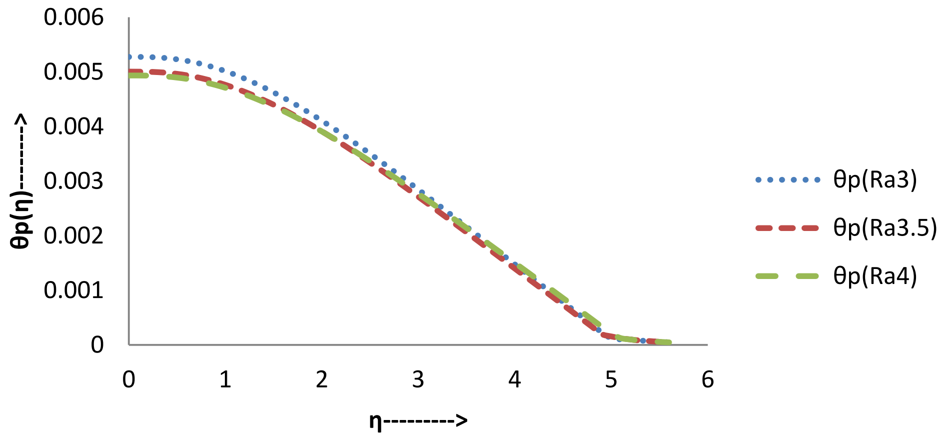


Fig-9 Variation of u w.r.t f0

Φ=0.01 ,Ec=1.0,Pr=0.71,Gr=0.01,Ra=3.0 ϵ=2.0 Fr=10.0, γ=1.0, β=0.01

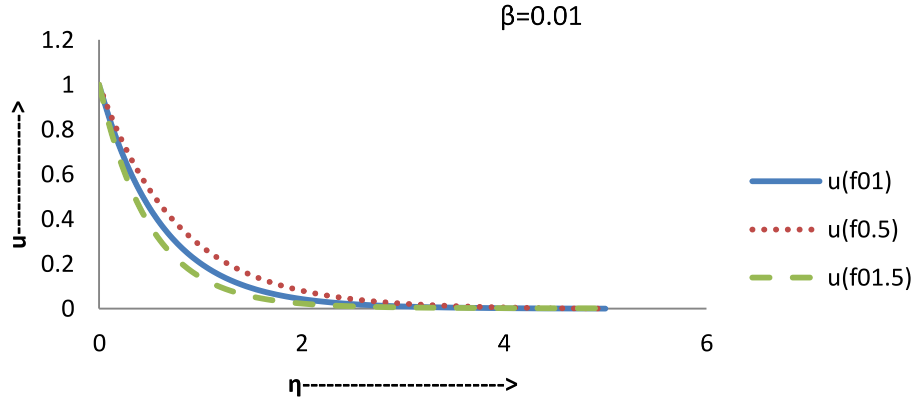


Fig-10 Variation of Up w.r.t f0
Φ=0.01 ,Ec=1.0,Pr=0.71,Gr=0.01,Ra=3.0 ϵ=2.0 Fr=10.0, γ=1.0,
β=0.01

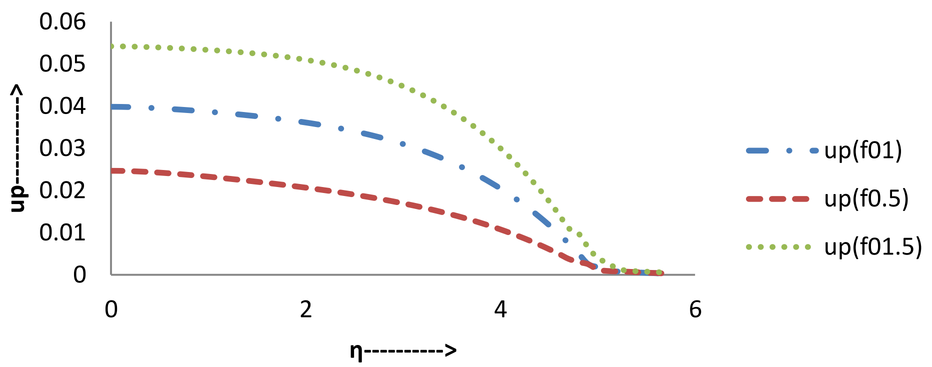


Fig-11 Variation of Up w.r.t f0
Φ=0.01 ,Ec=1.0,Pr=0.71,Gr=0.01,Ra=3.0 ϵ=2.0 Fr=10.0, γ=1.0,
β=0.01

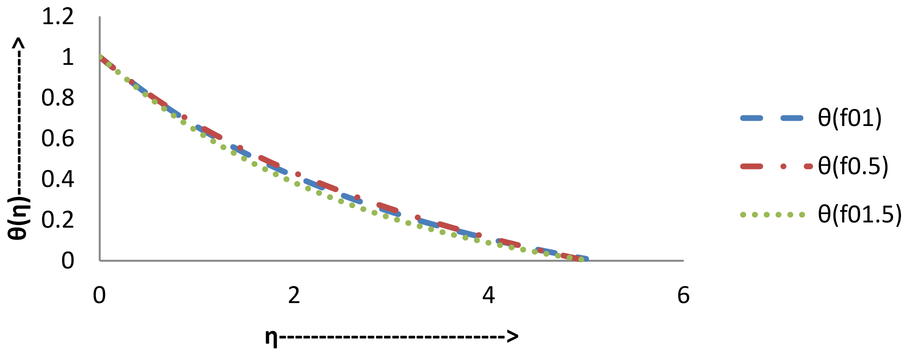


Fig-12 Variation of θp w.r.t f0
Φ=0.01 ,Ec=1.0,Pr=0.71,Gr=0.01,Ra=3.0 ϵ=2.0 Fr=10.0, γ=1.0,
β=0.01

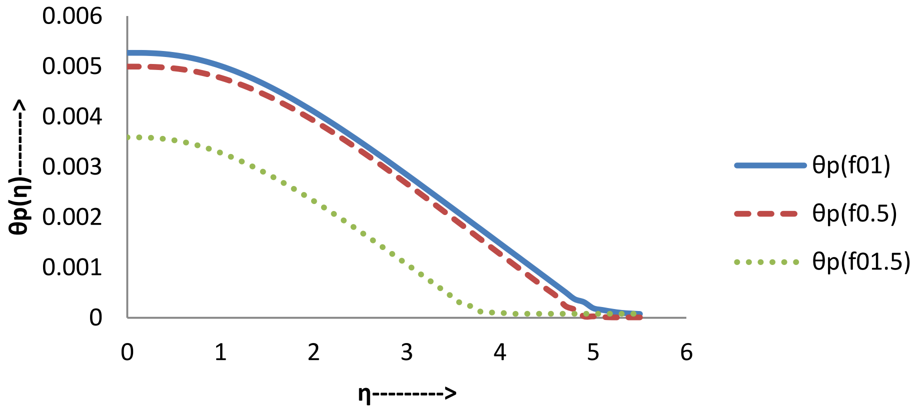


Fig-13 Variation of θ w.r.t Ec
Φ=0.01 ,f0=1.0,Pr=0.71,Gr=0.01,Ra=3.0 ϵ=2.0 Fr=10.0, γ=1.0, β=0.01

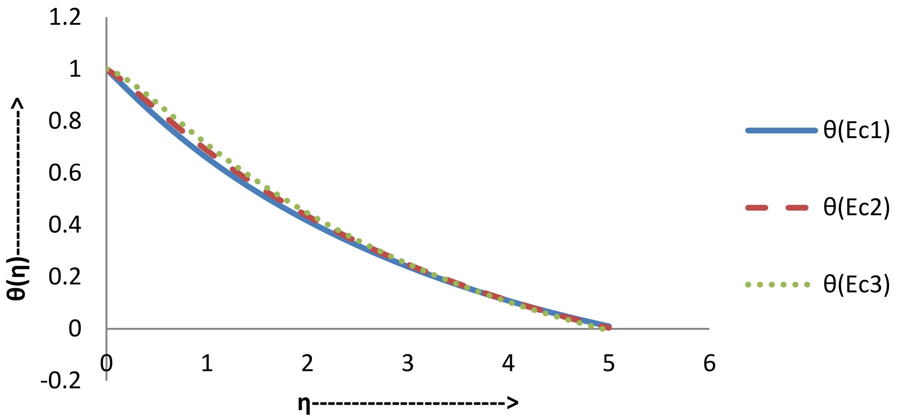
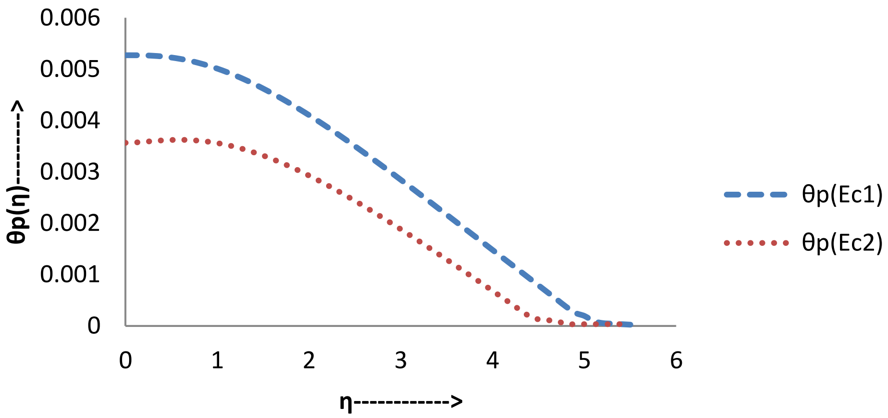


Fig-14 Variation of θp w.r.t Ec
Φ=0.01 ,f0=1.0,Pr=0.71,Gr=0.01,Ra=3.0 ϵ=2.0 Fr=10.0, γ=1.0, β=0.01



The variation of non dimensional velocity of fluid and particle phase is depicted in Figs. (2)&(3) for different values of β, where other values of parameters remain fixed. It is observed that there is no significant change in the fluid phase velocity u but the particle phase velocity u_p increases with the increase of β.

Fig-4 shows the variation of non dimensional temperature profile θ(η) for various values of Prandtl number Pr. It is observed that an increase in the value of Pr leads to a decrease of temperature profiles in the fluid phase. A higher Prandtl number fluid has a thinner thermal boundary layer and this increases the gradient of the temperature and also the surface heat transfer.

Figs-(5)&(6) display the temperature profiles for some values of Grashof number Gr for the fluid and particle phase flow. It is observed that there is no significant change in the fluid phase temperature θ but the particle phase temperature θp increases for the increasing values of Gr.

Figs.(7)&(8) represent the non dimensional temperature profile of both the fluid phase θ and the particle phase θp for different values of Ra. It is observed that a slight increase in temperature profile θ but a slight decrease in particle phase temperature θ_p .

The velocity and temperature profiles for both the phases are depicted in Figs.(9) to (12) for the various values of suction parameter f0. It is observed that the fluid velocity u decreases asymptotically but a significant increase in the particle phase velocity up with the increase of f0.The temperature θ and θp of fluid and particle phase respectfully decrease with the increasing value of f0.

The effect of Eckret number Ec , which signifies the viscous dissipation of the fluid,on the heat transfer ,is exhibited in Figs-(13)&(14). It is observed that an increase in viscous dissipation of the fluid tends to increase in fluid temperature with increase of Ec. The reason for this effect is that the viscosity of the fluid takes energy from motion of the fluid and transforms it into the internal energy of the fluid, which results in the heating of the fluid.The temperature θp of particle phase decreases with increasing value of Ec.

5. CONCLUSION

In this study, numerical analysis is presented to investigate the free convective heat transfer of a dusty fluid over a vertical permeable stretching sheet. Thermal radiation terms have been included in the energy equations. Velocity and temperature profiles are presented graphically and analyzed. Influence of physical parameters found to effect the problem under consideration are the fluid particle interaction parameter , local Grashof number , suction parameter ,radiation parameter ,Prandtl number and Eckert number. . Numerical components shows that the present values of wall temperature gradient is close agreement with those obtained by previous investigators in the absent of β,f0,Gr,Ra and Ec. On this basis of the above study we have the following observations:

Velocity of fluid phases has no significant effect as β increases but dust phase velocity increases as β increases.

1. Temperature of fluid decrease and dust phase increases as Gr increases.
2. For increasing value of f0 the velocity of fluid phase decreases and dust phase increases.
3. Temperature of fluid and dust phase decreases as f0 increases.
4. Wall temperature gradient increases with increasing Ra , f0 and Ec .While dencreases as the other parameters β, Gr and Pr increases.
5. Temperature of fluid phase decreases with increasing Pr, while increases with increasing the value of Ra and Ec .
6. Temperature of dust phase decreases as Ra and Ec increases

Table-2: Values of wall velocity gradient $-f''(0)$, temperature gradient $-\theta'(0)$ −F(0), −G(0),H(0), and $\theta_P(0)$ different values of β, E_c , G_r , P_r,Ra and f0 .

β	E_c	G_r	P_r	Ra	f0	$-f''(0)$	$-\theta'(0)$	-F(0)	-G(0)	H(0)	θp(0)
0.01	1.0	0.01	0.71	3.0	1.0	1.615392	0.381561	0.009527	1.04982	0.195286	-0.00033
0.02						1.608373	0.385960	0018501	1.029722	0.191971	0.008587
0.03						1.609931	0.385334	0.026646	1043498	0.188436	0009811
0.01	1.0	0.01	0.71	3.0	1.0	1.607616	0.382520	0.039853	1.054090	0.180973	0.005272
	2.0					1.612574	0.248861	0.039456	1.051767	0.175412	0.003565
	3.0					1.608629	0.124040	0.039918	1.054128	0.180439	0.003155
0.01	1.0	0.01	0.71	3.0	1.0	1.607616	0.382520	0.039853	1.054090	0.180973	0.005272
		0.02				1.596194	0.389850	0.040228	1.054307	0.184358	0.005469
		0.03				1.584169	0.394311	0.04062	1.055280	0.180883	0.005771
0.01	1.0	0.01	0.1	3.0	1.0	1.604693	0.226606	0.010168	1.016868	0.200407	0.001346
			0.71			1.615392	0.381561	0.009527	1.04982	0.195286	-0.00033
			1.0			1.605886	0.468703	0.009678	1.014235	0.198811	-7.5E-05
0.01	1.0	0.01	0.71	3.0	1.0	1.607616	0.382520	0.039853	1.054090	0.180973	0.005272
				3.5		1.607094	0.364308	0.039894	1.054010	0.176383	0.005004
				4.0		1.607113	0.347578	0.039966	1.054562	0.171996	0.004934
0.01	1.0	0.01	0.71	3.0	1.0	1.607616	0.382520	0.039853	1.054090	0.180973	0.005272
					1.5	1.986815	0.382112	0.054184	1.04508	0.170906	0.003587
					0.5	1.272791	0.392985	0.34693	1.022308	0.190083	0.004996

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