FUZZY LINEAR PROGRAMMING APPROACH FOR DETERMINING THE MANUFACTURING AMOUNT OF DIFFERENT PARTS IN AUTOMOTIVE SUPPLY INDUSTRY

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Abstract

In recent years, rapid and correct decision making is crucial for both people and enterprises. However, uncertainty makes decision-making difficult. Fuzzy logic is used for coping with this situation. Thus, fuzzy linear programming models are developed in order to handle uncertainty in objective function and the constraints. In this study, a problem of a factory in automotive supply industry is investigated, required data is obtained and the problem is figured out as a fuzzy linear programming model. The model is solved using Zimmerman approach which is one of the approaches for fuzzy linear programming. As a result, the solution gives the amount of manufacturing for each part type in order to gain maximum profit.

Keywords: *Fuzzy linear programming, fuzzy logic, linear programming, automotive supply industry*

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1. INTRODUCTION

Decision Making (DM) helps executives for determining the best alternative in any decision problem which comprise of procedures and criteria. DM generally depend on Decision Support Systems (DSS) tools [1].

Fuzzy set theory can be optimum approach to cope with uncertainties in decision problems modelled with linear programming. [2] There are different techniques in fuzzy linear programming. A common characteristic of these techniques is the preliminary defuzzification of the fuzzy variables [1].

Zadeh and Bellman propounded the notion of maximizing the decision for decision making problems. Tanaka et al adjusted it for mathematical programming problems. A fuzzy approach concerning multi-objective linear programming problems was introduced by Zimmermann. Also, Lai and Hwang, Tong Shaocheng, Buckley, among others, deal with that whole parameters are fuzzy. Studies in recent years suggest new techniques with the purpose of ranking fuzzy numbers and coming to an optimal solution. [3].

2. FUZZY SETS

2.1 Fuzzy Sets Theory

Fuzzy set theory is based on the logic underpinned the types of reasoning, which are ambiguous instead of precise [6]. In real life, classical logic is insufficient for many situations and events. When a person ask what the weather is, the answer can be very cold, cold, warm, hot, very hot and extremely hot. Style of thinking and data processing of people can process the uncertain data. However, traditional computer technology cannot overcome this type of data [4].

In fuzzy logic, a proposition can be either true or false. Moreover, it can be also partially true and partially false. That is, an element can has two membership in different sets simultaneously. In this case, trueness degree of the proposition range from zero to one. On the other hand, it also has '1-trueness degree' as degree of falsity [4].

System modelling is used for symbolising a real physical system. While this process perform, mathematical formula or equation is used. Most of physical systems include uncertainty and they are too sophisticated to model it completely. This type of systems can model with fuzzy logic approximately. Thus, fuzzy system modelling play an important role to describe complicated real world systems [5].

3. FUZZY LINEAR PROGRAMMING

3.1 Linear Programming

A Linear Programming (LP) problem is a special case of Mathematical Programming problem. From an analytical perspective, a mathematical program attempts to identify an extreme (minimum or maximum) point of a function, which furthermore satisfies a set of constraints. Linear programming is the objective function and the problem constraints are linear [6].

A classical model of LP, also called a crisp LP model, may have the following formulation:

Max Cx
s.t. $A_i x \leq b_i$, $i=1, \ldots, m$,
in which x is an $n \times 1$ alternative set, C is a $1 \times n$ coefficients of an objective function, A_i is an $m \times n$ matrix of coefficients of constraints and b_i is an $m \times 1$ right-hand sides.

The traditional problems of LP are solved with LINDO optimization software and obtain the optimal solution in a precise way. If coefficients of constraints, objective function or the right-hand sides are imprecise, in other words, being fuzzy numbers, traditional algorithms of LP are unsuitable to solve the fuzzy problem and to obtain the optimization.

In the real world, the coefficients are typically imprecise numbers because of insufficient information, for instance, technological coefficients. Many researchers formed Fuzzy Linear Programming of various types, invented approaches to convert them into crisp LP, and finally solved the problems with available software [7].

3.2 Fuzzy Linear Programming

Fuzzy linear programming (FLP) follows from the fact that classical linear programming is often insufficient in practical situations. In reality, certain coefficients that appear in classical LP problems may not be well-defined, either because their values depend on other parameters or because they cannot be precisely assessed and only qualitative estimates of these coefficients are available. FLP is an extension of classical linear programming and deals with imprecise coefficients by using fuzzy variables [8].

Now we consider the FLP Problem

Max $\tilde{Z} = \tilde{C}^{T}x$
s.t. $\tilde{A}x \lessapprox \tilde{b}$
$x \geq 0$.

The solution of this problem is to find the possibility distribution of the optional objective function Z. Many researchers had handled this problem by converting the fuzzy objective function and the fuzzy constraints into crisp ones [9].

Fuzzy linear programming model divide into parts in terms of fuzzy coefficients. For instance, while objective function is fuzzy, constraints cannot be fuzzy.

3.2.1. Objective Function is Fuzzy

In a real life, there are many situations that parameters of objective function (profit and cost) are imprecise. FLP model of this was propounded by Verdegay.

3.2.2. Right-Hand Sides are Fuzzy

There are two approach for this type of problem. While first approach concerning asimetric models belongs to Verdegay, second approach concerning simetric model belongs to Werners.

3.2.3 Right-Hand Sides and Coefficients of Constrains are Fuzzy

Negoita and Sularia developed an approach for this type of FLP model.

3.2.4 Objective Function and Constrains are Fuzzy

As it is understood the title, in this model, both objective function and constrains involve fuzziness. Zimmermann and Chanas have different approaches about it.

3.2.5 All Coefficients are Fuzzy

Sometimes, all coefficients can be fuzzy in the problem. Carlsson and Korhonen developed the approach for this.

3.3 Zimmermann Method

A LP with a fuzzy objective function and fuzzy inequalities shown by Zimmermann is indicated as follows: [7]

$c^{T}x \gtrapprox b_0$

$(Ax)_i \lessapprox b_i \quad i=1,2, \ldots, m$

$x \geq 0$

Inequality is a symmetrical model of which the objective function becomes one constraint. To write a general formulation, inequality is converted to a matrix form as [7]:

$-c^{T}x \lessapprox -b_0$

in which

$$B = \begin{bmatrix} -C \\ Ai \end{bmatrix} \text{ ve } b = \begin{bmatrix} -b0 \\ bi \end{bmatrix}$$

The inequalities of constraint signify "be as small as possible or equal" that can be allowed to violate the right-hand side b by extending some value. The degree of violation is represented by membership function as [7]:

$$\mu_0(x) = \begin{cases} 0 & ;\ if \quad cx \leq b0\text{-}d0 \\ 1\text{-}\frac{b0\text{-}cx}{d0} & ;\ if \quad b0\text{-}d0 \leq cx \leq b0 \\ 1 & ;\ if \quad cx \geq b0 \end{cases}$$

$$\mu_i(x) = \begin{cases} 0 & ;\ if \quad (Ax)i \geq bi+di \\ 1\text{-}\frac{(Ax)i\text{-}bi}{di} & ;\ if \quad bi \leq (Ax)i \leq bi+di \\ 1 & ;\ if \quad (Ax)i \leq bi \end{cases}$$

In which d is a matrix of admissible violation.

This problem can be transformed by introducing the auxiliary variable λ as follows:

$\mu_0(x) \geq \lambda$

$\mu_i(x) \geq \lambda$

$\lambda \in [0,1]$

This problem can be stated as linear programming as follows:

Max λ
s.t.

$\mu_0(x) \geq \lambda$

$\mu_i(x) \geq \lambda$

$\lambda \in [0,1]$

This problem was shown with membership functions of fuzzy objective function and fuzzy constrains as follows:

Max λ
s.t.

$$1 - \frac{b_0 - c_x}{d_0} \geq \lambda$$

$$1 - \frac{(Ax)_i - b_i}{d_i} \geq \lambda \quad ; \forall i$$

$\lambda \in [0,1]$

$x \geq 0$

After some simplification, fuzzy linear programming model obtain as follows:

Max λ *Max* λ
s.t. *s.t.*

$c^T x \geq b_0 - (1-\lambda)d_0$ $c^T x - \lambda d_0 \geq b_0 - d_0$

$(Ax)_i \leq b_i + (1-\lambda)d_i \quad ; \forall i$ $\equiv$ $(Ax)_i + \lambda d_i \leq b_i + d_i \quad ; \forall i$

$\lambda \in [0,1]$ $\lambda \in [0,1]$
$x \geq 0$ $x \geq 0$

4. APPLICATION

4.1 Problem Definition

Data used for the application was obtained a factory in automotive supply industry. It manufacture different part types. Since the expected profit and the demand of the product types are uncertain the problem is built as fuzzy linear programming model in order to determine manufacturing amounts per day for each part type for maximizing the profit. Data about the manufacturing and its constraints is given in Table 1:

Table 1: Data about the application

Variables				
	X_1	X_2	X_3	X_4
Variable name	Part 1	Part 2	Part 3	Part 4
Unit profits (TRY per part)	207.5	303	193	131
Expected demands (number of part)	163	172	187	135
Tolerances for demands (number of part)	14	18	25	10
Labour usage (second per part)	63.2	66.7	40	44.5
Expected profit (TRY)	170000			
Tolerance for profit (TRY)	20000			
Daily production capacity (number of part)	840			
Daily labour capacity (second)	42000			

4.2. FLP Model

Problem was modelled as daily basis. The fuzzy linear programming model of the problem is given below:

$c^T x = 207.5x_1 + 303x_2 + 193x_3 + 131x_4$

$b_0 = 170000$ $d_0 = 20000$
$b_1 = 163$ $d_1 = 14$
$b_2 = 172$ $d_2 = 18$
$b_3 = 187$ $d_3 = 25$
$b_4 = 135$ $d_4 = 10$

Max λ
st
$207.5x_1 + 303_{x2} + 193x_3 + 131x_4 - 20000t >= 150000$
$x_1 + x_2 + x_3 + x_4 <= 840$
$63.2x_1 + 66.7x_2 + 40x_3 + 44.5x_4 <= 42000$
$x_1 + 14t <= 177$
$x_2 + 18t <= 190$
$x_3 + 25t <= 212$
$x_4 + 10t <= 145$

$\lambda \in [0,1]$

$x_1, x_2, x_3, x_4 \geq 0$

4.3. Problem Solution

FLP model of the problem has been solved using Lindo optimization software. Results of the solution are given below:

Variable	Value	Reduced Cost
T	0.1111111	0.000000
X1	175.0000	0.000000
X2	188.0000	0.5555556E-01
X3	209.0000	0.000000
X4	143.0000	0.000000

Row	Slack or Surplus	Dual Price
1	0. 1111111	1.000000
2	124.2778	0.000000

3	125.0000	0.000000
4	3676.900	0.000000
5	0. 4444444	0.000000
6	0. 000000	0.5555556E-01
7	0. 2222222	0.000000
8	0. 8888889	0.000000
9	0. 1111111	0.000000
10	0. 8888889	0.000000

As can be seen from the solution, the factory should manufacture 175 part1, 188 part2, 209 part3, 143 part4. Total profit of the factory can be calculated as follows:

(175x207.5) + (188x303) + (209x193) + (143x131) = 152346.5 TRY

5. CONCLUSION

In this study, a problem of a factory in automotive supply industry was modelled by using fuzzy linear programming. Because the model has fuzziness in both objective function and constraints, it was solved using Zimmerman approach which is one of the approaches for fuzzy linear programming. As a result, the solution gives the amount of manufacturing for each part type in order to gain maximum profit.

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