REMAINING FATIGUE LIFE AND BEHAVIOUR OF DAMAGED RCBEAMS-REVIEW

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Abstract

Reinforced concrete bridges are often adopted in Railway construction of shorter spans to achieve simplicity in casting for required shape and size, easy to strengthen, riding comfort and speed of construction. These bridges consist of girders and deck slabs. When the speed and type of loading vary to transport ores, commodities and passengers, the bridges are subjected to higher stress range of fatigue than that designed. Assessment of remaining fatigue life of bridges either for retaining or strengthening or discarding purposes. Discarding the bridge can result in obstruction of traffic, reduction of revenue and loss of time, strategic needs and monumental structures. Experimental studies have been carried out for assessing the remaining fatigue cycles and flexural behavior of Damaged (cracked) Beams by fracture of tension reinforcement. The parameters of the study are; strength of concrete, percentage steel reinforcement and extent of damage in rebars. Load-deformation response and variation of strains across the depth at the level of center of beam, tension and compression reinforcements, extreme compressive strain in concrete, and crack propagation under fatigue load were analyzed. Review of Previous research on the fatigue life of bridges is presented. The test results are compared with the current ACI SP 75 design practice.

Keywords: *Damage, RC beams, Compression reinforcement, tension reinforcement, Fatigue*

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1. INTRODUCTION

To estimate the remaining service life of damaged RC beams of railway bridges incorporating various parameters including the size effect several attempts have been made. The provisions for fatigue design of RC beams for railway bridges are laid down in various standards such as IRS Bridge Manual (1998), Bridge Rule (2008), and Concrete Bridge Code (1997). An attempt has been made to estimate the remaining service life of beams of bridges with various parameters. An analytical model has been proposed to predict the service life of RC beams subjected to cyclic loading. The parameters of this are; ratio of effective cover-to-effective depth (depth ratio), crack depth-to-depth or diameter of reinforcement (rebar size ratio), based on fracture mechanics approach, and fatigue provisions in Euro codes(2003)and BS-5400(1999). It is also planned to propose design method for RC beams of optimum weight and long service life.

2. STATE OF ART

Provisions for design of structures for Fatigue loading are shown in BS-5400 (Part 10), Euro Code (Part 3), and IRS concrete bridge code of railways. Understanding Paris equation is very important for the design of concrete structures. This review enlightened the use of linear damage theory for metals and alloys in the elements of road and railway bridges. But these codes do not explain about fatigue life of RC beams subjected to cyclic loading based on damage in the beams.SPR34-Report describes straight/skew/curved bridge geometry, layout and proportions of member as well as specifications for materials. Methods to apply database for structural analysis has also been furnished. Field study and analysis as per AASHTO specifications for prediction of remaining life of beams are presented. Finite element model for RC beams with diagonal tension in 1962 were segregated to study the fatigue life, which induces diagonal tension cracks and S-N curve for shear reinforcement (stirrups) is developed to assess the remaining life of RC beams of these bridges. This project was carried out for Oregon Department of Transportation Research Unit, Oregon and Federal Highway Administration, Washington. This report failed to emphasize a study of fatigue effect on RC beams in flexural behavior and on how to ascertain remaining life of RC beams of bridges using fracture mechanics.

For prediction of crack width in precast-reinforced concrete slabs, under repeated loadings, fracture mechanics approach has been used to analyze the crack depth at initial loading and the fatigue crack growth rate, at 10Hz load frequency with respect to steel stress ratio, and reinforcement ratio (Carpinteri et al. 2006).This paper established a design equation to predict the crack width and spacing of steel. Experimental model consists of a simply supported beam or slab subjected to cyclic loading and deflection to measure crack width. Analytical model was developed for a slab with crack of V-notch profile. The force along with the opening moment is acting at the ends. Crack depth is estimated using Paris-Erdogan equation. The relationships between crack width vs. number of cycles, and crack width vs. life time, are established (Soehardjono et al.2006).This paper does not consider the size effect, size of crack in rebar, and does not consider loading of Railway Bridge. A survey of the state of-the-art for homogenous materials on cumulative fatigue damage and life prediction theories has been carried out (Fatemi et al.1997).This report provides the comprehensive review of cumulative fatigue damage theories for metals and alloys. This report represents about linear damage rules, on-linear damage curve, two stage linearization approach, Life curve modification methods, Approaches based on crack growth concepts, continuous damage mechanism models (Energy based theories).This paper does not describe about fatigue of heterogeneous materials like reinforced concrete and its service life estimation.

Nonlinear analysis and remaining fatigue life of reinforced concrete bridges are studied for a European Commission project (Plos et al. 2007).This paper describes review on various models, methods of structural analysis, FE modeling, material properties, experimental validation, fatigue of reinforced concrete, methodology for fatigue safety assessment and remaining life assessment of reinforced concrete railway bridges**.** This paper considers only round shaped mild steel with cracks in rebar. While estimating fatigue life, the relationship between sizes, crack in rebar and concrete, effect of ribs and use of square rebars has not been considered. The fatigue assumes significance in the design of railway bridges where dynamic impacts are considerable. It is therefore necessary to design new railway bridges on Indian Railways taking into due consideration of the fatigue effects. Since the existing bridge code does not contain such provisions, use of Euro norms may be considered. The Euro norms contain explicit provisions for accounting fatigue effects in concrete structures (Goel, 2008).The Stress Intensity Factors in RC beams under cyclic loading depends on the size of the member. The stress intensity factor for failure of beams under cyclic loading is less than that of beams subjected to monotonic loadings. Variation of stress intensity factor under dynamic loading is based on d_o(size effect factor), as in the size effect law, size of beam (d),fracture toughness at peak load and LEFM relation in Paris-Erdogan equation for 80% of monotonic loading. In addition to the above, the stress intensity factor is to be modified based on crack propagation in concrete. Number of cycles to fracture is a function of stress intensity factor. In turn service life is function of size of specimen, and crack depth in concrete. Here the effect of cover and rebar diameter, effect of ribs are not considered (Bezant, 1992).

3. RESEARCH SIGNIFICANCE

In the case of RC beams of girder bridges as shown in Figs.1 and 2), remaining life equations have been formulated only for those with welded reinforcement. However, for RC beams of girder bridges with reinforcement tied with binding wire are required to be formulated. The effect of fatigue in combination with higher ratio of dead load-to-live load, and dynamic loading impact are significant, which can be brought down to the required level of remaining service life by developing an appropriate equation. For prediction of remaining service life with minimum weight of RC beams of girders is viable to design, easily constructed and transported in case of precast elements with less erection stresses (IRS Bridge Manual, 1998). The objective of the present study is to predict the remaining service life of the damaged railway RC girder bridges.

4. METHODOLOGY FOR DEVELOPMENT OF ANALYTICAL MODEL

4.1. Remaining Life Concept

Within the permissible stress limits in steel and concrete, the remaining life of beam depends on the remaining safe strength of uncracked portion of beam. Remaining safe strength is the product of balance safe stress and uncracked area of cross section of beam. Balance safe stress in concrete is the difference between permissible safe stress in concrete and actual stress in the remaining uncracked concrete section. Using Raina's method, actual stress in the remaining concrete of cracked beam is evaluated, under the condition that the crack depth in steel reaches a value equal to 1.2 times the radius of bar. A magnitude of steel stress d(Δσ) equal to the product of modular ratio and above cited balance safe stress in the remaining concrete on compression side hampers further crack in steel and concrete. d (Δσ) is the contribution of concrete strength to service life, in addition to service life offered by reinforcement. Hence equation for evaluating N_{ij} (cycles to propagate crack from a_i to a_j) needs to be modified with reduction factor (a_{jk}) for Δσ using the following equation.

$$(\Delta\sigma)^{(n_s-a_{jk})} = \left(\left((\Delta\sigma) + d\,(\Delta\sigma)\right)\right)^{n_s} \quad (1)$$

$$a_{jk} = n_s - \left[\frac{n_s \times \log((\Delta\sigma) + d(\Delta\sigma))}{\log(\Delta\sigma)}\right] \quad (2)$$

Where n is material constant for steel. In addition, a modification factor $\left(\frac{a_j}{a_i}\right)$ is introduced for $\left(\frac{Y_S}{\lambda_S}\right)$ which is Geometrical Modification factor. In the equation for N_{ij} , in order to formulate the equation in terms of other parameters GMT (Gross Million Tonnage), Base line-L, for present models. It is deduced that $\left(\frac{a_{jj}}{a_i}\right)$ is a function of compressive strength of concrete.

$$\left(\frac{Y_S}{\lambda_S}\right)^{\left(n_s-\left(\frac{a_{jj}}{a_i}\right)\right)} = \left[\left(\frac{Y_S}{\lambda_S} \times \left(1 + \frac{d\,(\Delta\sigma)}{(\Delta\sigma)}\right)\right)^{n_s}\right] \quad (3)$$

$$\left(\frac{a_{jj}}{a_i}\right) = n_s - \left[\frac{n_s \times \log\left(\frac{Y_S}{\lambda_S} \times \left(1 + \frac{d\,(\Delta\sigma)}{(\Delta\sigma)}\right)\right)}{\log\left(\frac{Y_S}{\lambda_S}\right)}\right] \quad (4)$$

4.2. Definition of Research Parameters

In the estimation of service life of RC beams

1. Reliable fatigue damage accumulation method is still lacking (*Plos et al. 2007*)
2. Contribution of concrete strength is not evaluated so far (*Herwig,2008*)
3. Effect of ribs and bar shape are yet to be considered in the fatigue life evaluation

4. Size effect on RC beams subjected to cyclic loading is to be studied further considering for beams of different sizes with the same geometrical similarity

4.3 Existing Models

1. N_{Bazant} (fracture of concrete) = Function of (effective depth of beam (d), aggregate size factor (d_0), crack depth in concrete (a) *(Bazant, 1992)*
2. $N_{Sohardjeno}$ (fracture of concrete) = Function of (crack depth in concrete, a, steel ratio, stress ratio, material constants) *(Sohardjeno et al. 2006)*
3. N_{Plos} (fracture of steel) = Function of (crack depth in steel, a, radius of steel, r) *(Plos et al. 2007)*

4.4 Formulation of Proposed Model

The present model is a function of the following parameters
N_{sp} (fracture of steel) = Function of (a,r, d', d, d_0, s, z, GMT, L)

where a= crack depth in steel, r = radius of steel, d' =effective cover, d = effective depth, d_0 = aggregate size effect factor, s = crack depth in concrete, z = size of bar with ribs, GMT = gross million ton of loads, L = base line of influence length effect.

In this model, a, r, d', d, d_0, s, L and z are all geometric parameters, which influence the correction factor. The stress intensity factor depends on crack depth, stress range, and correction factor.

The stress range is function of geometry, maximum and minimum loads. The correction factor is function of geometry and crack depth. The crack depth is function of load cycles and geometry. Fatigue life of RC beams depends on fatigue life of rebars. As the rebar is a homogenous material, LEFM principles are adopted. The rate of propagation of crack is arrived by Paris-Erdogan equation

$$\frac{da}{dN} = D_s \; X(\Delta K)^{n_s} \qquad (5)$$

Stress intensity factor =

$$= Y_S X \Delta\sigma_S X \sqrt{(\pi \; X \; a)} \qquad (6)$$

Substituting the value of ΔK in Paris-Erdogan equation and integrating for N, the following equation is derived.

$$N_{ij} = \int_{a_i}^{a_j} dN = \int_{a_i}^{a_j} \frac{1}{D_s X \; \Delta K^{n_s}} da = \int_{a_i}^{a_j} \frac{1}{D_s X \; Y^{n_s} X \Delta\sigma^{n_s} X \pi^{\frac{n_s}{2}} X a^{\frac{n_s}{2}}} da \qquad (Plos\ et\ al\ 2007)$$

$$N_{ij} = \int_{a_i}^{a_j} \frac{1}{D_s X \; Y^{n_s} X \Delta\sigma^{n_s} X \pi^{\frac{n_s}{2}} X a^{\frac{n_s}{2}}} X \left(1 - \frac{a_i}{ajj}\right)^{\alpha} \qquad (7)$$

$$\textit{Experimental Equation for Number of Cycle Range to Fracture } N_{ij} = \left[\frac{1}{\left[\left(\frac{n_s}{2}\right)-1\right] \times D_s \times a_i^{\left[\left(\frac{n_s}{2}\right)-1\right]}}\right] \times \left[1 - \left\{\frac{a_i}{a_{jj}}\right\}^{\left[\left(\frac{n_s}{2}\right)-1\right]}\right] \times \left[\frac{1}{\pi^{\frac{n_s}{2}}}\right] \times \left[\frac{1}{\Delta\sigma^{\left(n_s - a_{jk}\right)}}\right] \times \left[\frac{1}{\left\{\frac{\left(f_{ck} \times b \times d \times \left(1 + \frac{d'}{d} + \frac{\overline{d_0}}{d}\right) \times a_{jc}^{n_s}\right) x \frac{1}{m} + \left(f_y \times A_{st} \times \left(\frac{s}{z}\right) \times a_{js}^{n_c}\right) + \left(\frac{E_C \times I_C \times \delta}{L^2 \times (e - e')}\right)^{\frac{n_s}{n_c}}}{GMT}\right\}^{\left(n_s - \frac{a_{jj}}{a_i}\right)}}\right] \qquad (8)$$

5. VALIDATION OF ANALYTICAL MODEL

Analysis and design of RC beam has been carried out according to the Indian Railways concrete bridge code 1997 for the Input Data Base arrived. Analysis and design of RC beams has been carried out using fatigue deign loads for the Input Data Base arrived. Stress-strain response, shear force, bending moment, and crack width for the above cases are compared. It has been observed that RC beams designed based on fatigue design results in lighter sections for the same reinforcement and less reinforcement is required for same cross-section vice-versa. Hence it is possible to arrive at light weight sections if suitable fatigue design procedure is adopted. For the given Input Data Base, minimum stress is calculated when the self-weight is acting and the maximum stress is calculated in steel and concrete when the self-weight along with live load and impact are acting. Live load is the equivalent point load which induces same bending moment as that of the equivalent uniformly distributed load at the midspan.

5.1. Fracture Mechanics Approach

Using the fracture mechanics approach, the geometric modification factor $\left(\frac{Y_S}{\lambda_S}\right)$ has been assessed based on the following equations. A trial value of Y_S is calculated from equation of *(Plos et al. 2007)* for σ_{rmax} and N_{rmax} from AREMA equation. Trial value of λ_s is calculated from equation of stress intensity factor equation. $\left(\frac{Y_S}{\lambda_S}\right)$ is calculated for equivalent steel and concrete sections as described in model calculations using modular ratio at appropriate equations. Final value of $\left(\frac{Y_S}{\lambda_S}\right)$ is calculated for equivalent steel section.

$a_s = a_c = a_j = 1.2\ X\ r,$ *where* " r " is the radius of bar.

$$K_s = 3000 \frac{N}{mm^{1.5}} \ , K_c = 1194.88 \frac{N}{mm^{1.5}} \qquad (9)$$

5.1.1. Case-1 Equivalent steel beam of same depth as that of RC beam.

$$K_s = \lambda_S X \sigma_S X \sqrt{(\pi\ X\ a_s)}$$

$$\left[\frac{Y_S}{\lambda_S}\right] = \frac{\left[\ (\ h_c X\ \ a_{jc}{}^{n_s})/m + (\ X\ \beta_s\ X\ a_{js}{}^{n_c}) + (P_C{}^{\frac{n_s}{n_c}})\right]}{GMT} \qquad (10)$$

5.1.2. Case-2 Equivalent RC Beam $\left(A_{eq} = A_c + m\ XA_{st}\right)$

$$K_c = \lambda_c X \sigma_c X \sqrt{(\pi\ X\ a_c)}$$

$$\left[\frac{Y_S}{\lambda_S}\right] X \left[\frac{E_C}{E_S}\right] = \left[\ (\ h_c\ X\ \ a_{jc}{}^{n_s}) + (m\ \ X\ \beta_s\ X\ a_{js}{}^{n_c}) \ + (P_C{}^{\frac{n_s}{n_c}})\right] / GMT \qquad (11)$$

Solve for $a_{jc}{}^{n_s}$ and $a_{js}{}^{n_c}$**.,**to determine, $\left(\frac{Y_S}{\lambda_S}\right)$

i. Evaluating the values of various new coefficients introduced in the service life equation and determined from the experiment. This involves simulating the cyclic loading on idealized RC beams for different stress ranges, effective cover-to-effective depth (depth ratio), crack depth-to-side or diameter of reinforcement (rebar size ratio), and percentage of steel reinforcement.

ii. Establishing final empirical equation to assess the remaining service life of RC beams of girder bridges for various GMT of railway loadings.

6. EXPERIMENTAL PROCEDURE

6.1. The beams designed in this study were tested using load controlled actuators under Sine Wave type of cyclic loading (compression). The beams were tested under four-point loading system. The displacements were monitored using Linearly Variable Differential Transformer (LVDT) under the loads and at mid span. The displacement, load, Cycle numbers and strain will be recorded using a Data Logging System. All the beams were tested for maximum of 1 million cycles or till failure, with load range of 1 to 14 tons and frequency of 3 Hz. The size of beams was arrived at based on Goodman's diagram*(Plos et al. 2007, ACI-343(Bridge and Structures, 10), Naaman 1982, and Balaguru et al 1982).*The dimensions of the beams are; 2400 x 250 x 400 mm.The maximum and minimum stress ranges can be estimated from Goodman's diagram*(Plos et al. 2007)*

6.2. All the beams were designed as under-reinforced beams, so that the failure is due to yielding or fracture of steel reinforcement. The failure of under-reinforced RC beams is controlled by the state of the steel reinforcement. Hence, it is essential to understand the failure of reinforcement provided with the designed damage under cyclic loading. Therefore, the service life is controlled by the service life of reinforcement with defined damage (Figs.5-7). Hence, the remaining service life can be predicted by the studies on service life of damaged reinforcement bars subjected to cyclic loading. Fracture in rebar due to corrosion is prevented by providing with proper cover. Hence the corrosion effect is not considered in this analysis.

6.3. For same size of beams, three beams for reference without damage (B16,B17,B18) and fifteen damaged beams are provided with 0.3r mm, 0.9r mm and 1.0r mm cracks. Hence total number of three beams is to be tested under static loading. Other fifteen beams are to be tested under monotonic and cyclic loading.

6.4. The number of cycles to fracture of rebar with respect to theoretical strain in steel (N_{frac}) and relevant crack widths and depth were observed in N-counter and

Data Logging System of UTM of Structural Engineering Laboratory respectively, for same sizes of beams with different concrete mixes, steel grades and crack ranges. Concretes used were M25, M45, M35and M55 with different type of steel conforming to Fe415(MS), Fe415 and Fe500 grades. The experimental set-up and specimen details are shown in (Figs. 3 and 4) especially with respect to depth ratio and rebar size ratio. For each percentage of damage in rebar, $N_{fracture}$ verses ultimate stress range ($\Delta\sigma$) causing fracture of each depth ratios is plotted. For each percentage of damage in the rebar, $N_{fracture}$verses crack depth (s) in concrete on ultimate strain (ε_s) of each depth ratios is plotted.

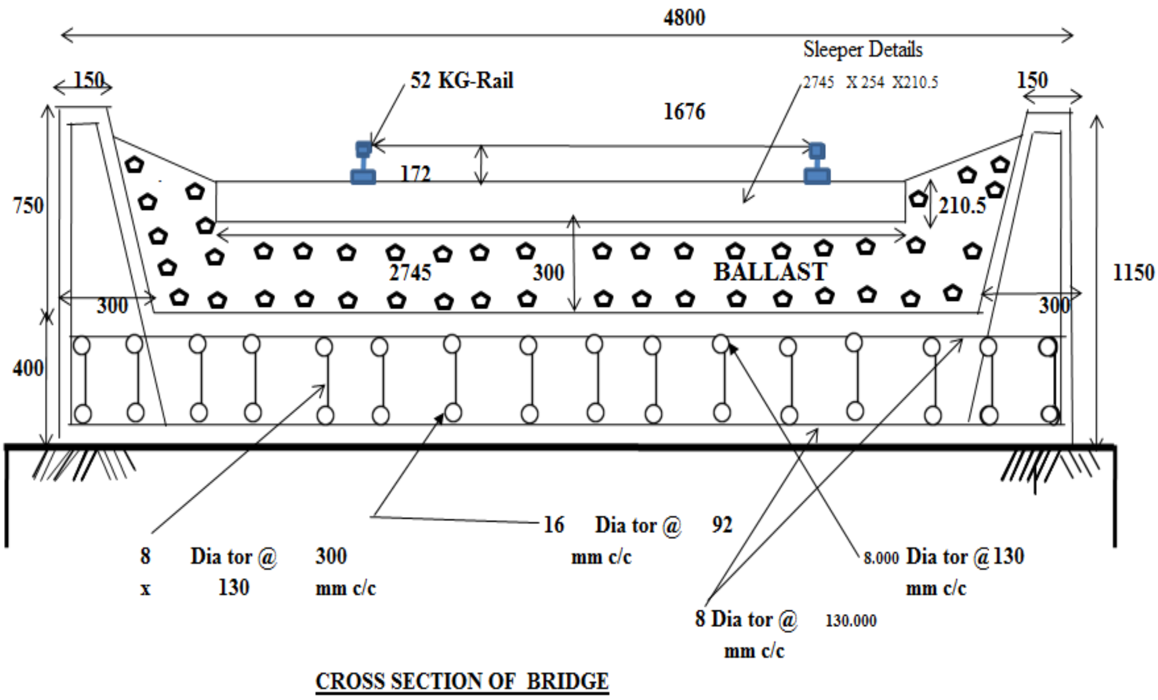

Fig. 1. Cross Section of Slab Bridge

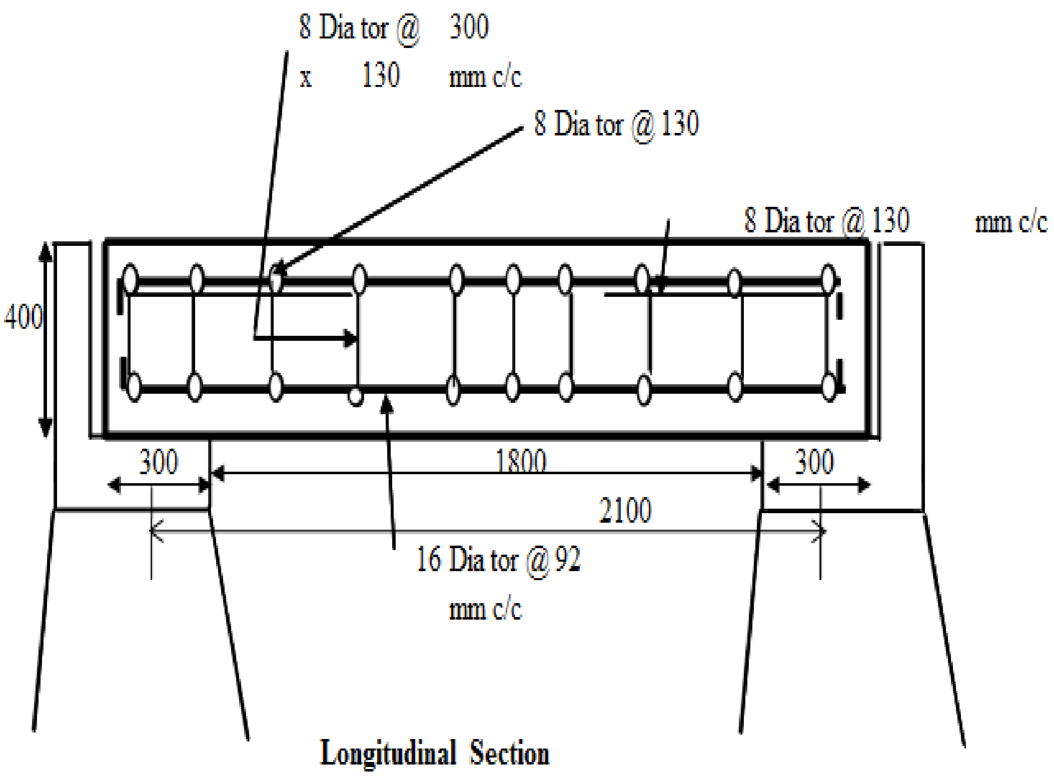

Fig. 2. Longitudinal Section of Bridge

BEAMS WITH Fe 500, Fe415, Fe415 (MS), Cover 50 mm, Side cover 25 mm, Rectangular-Notch Depth = a_i

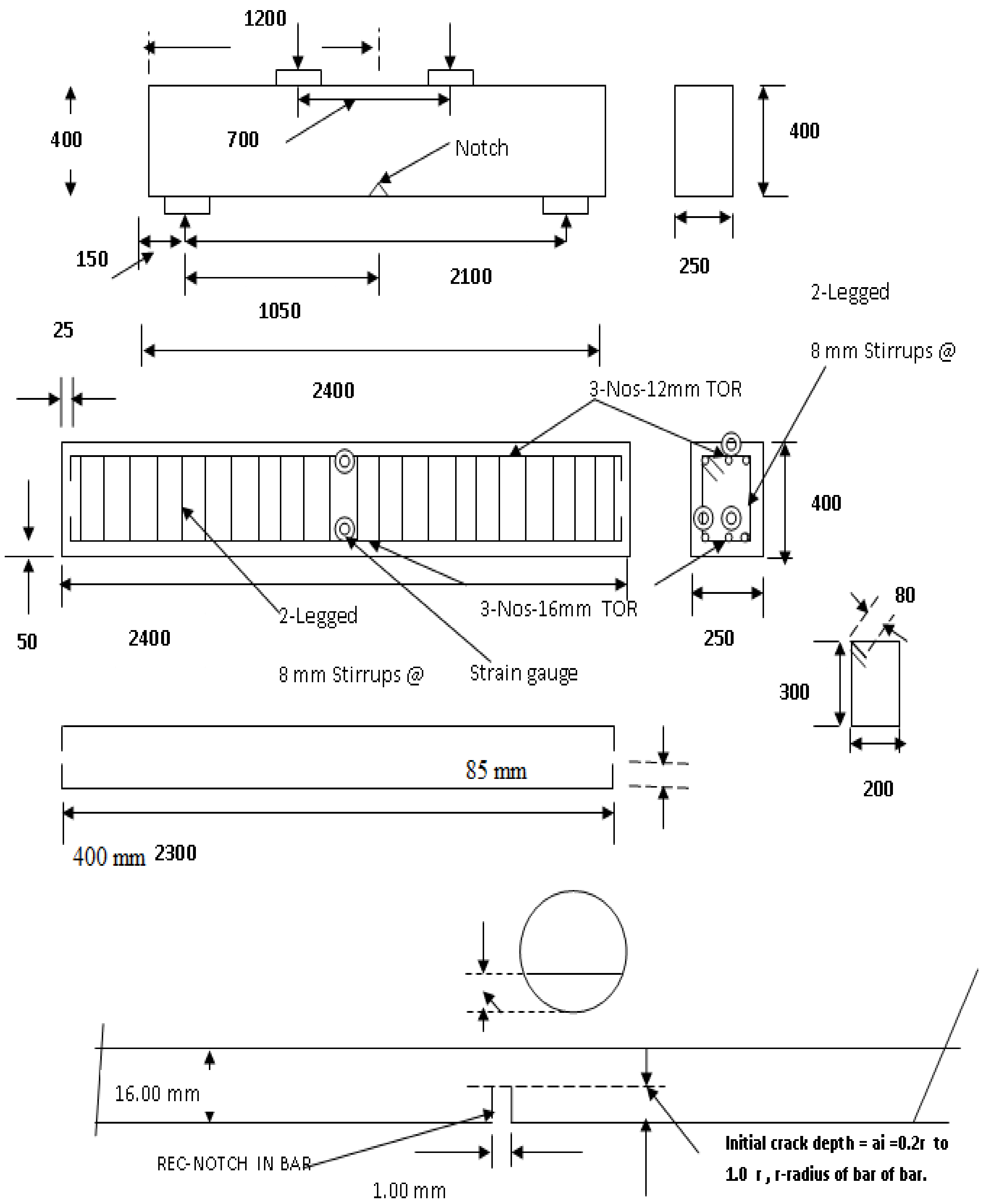

Fig. 3.RCC details of specimen beams for Fe500,Fe415,Fe415(MS) Grades of steel

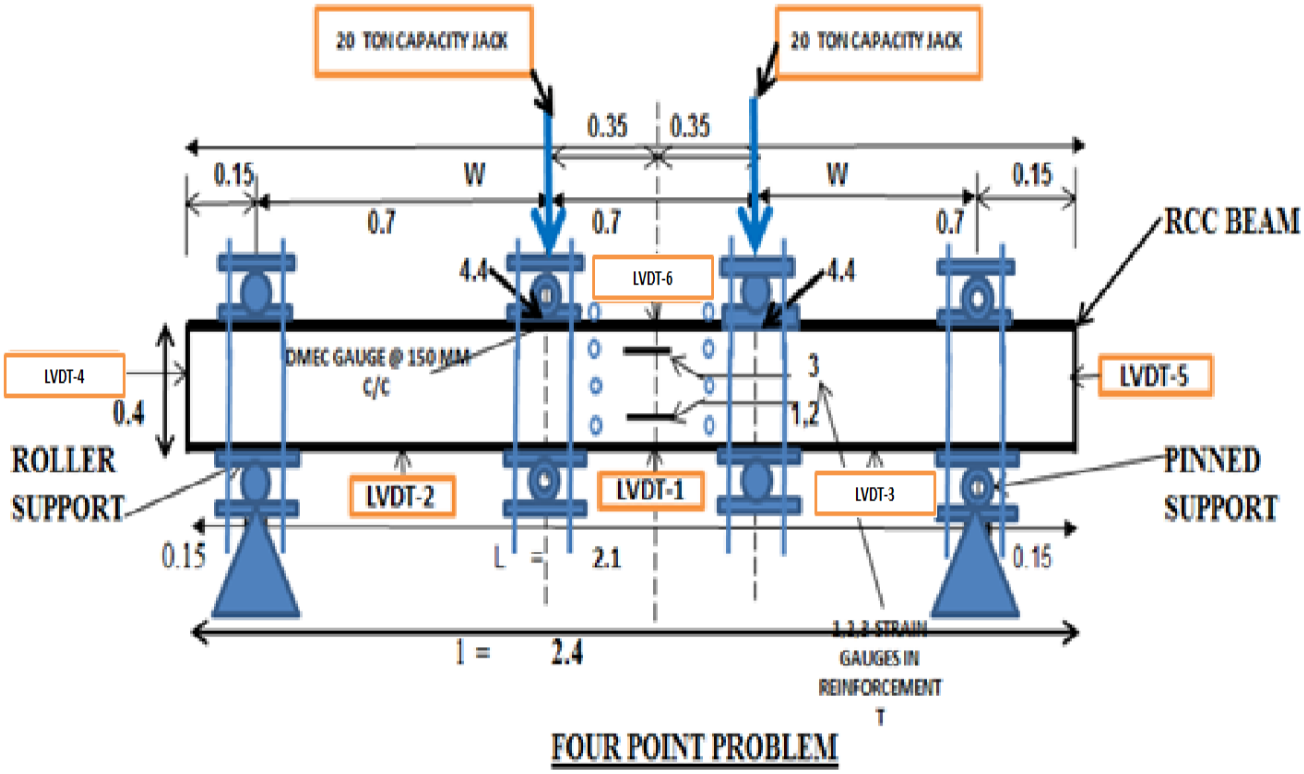

Fig.4.Four Point Bending Test Set up

ARRANGEMENT OF DMEC GAUGES AND LVDT AT TOP,BOTTOM AND SIDES

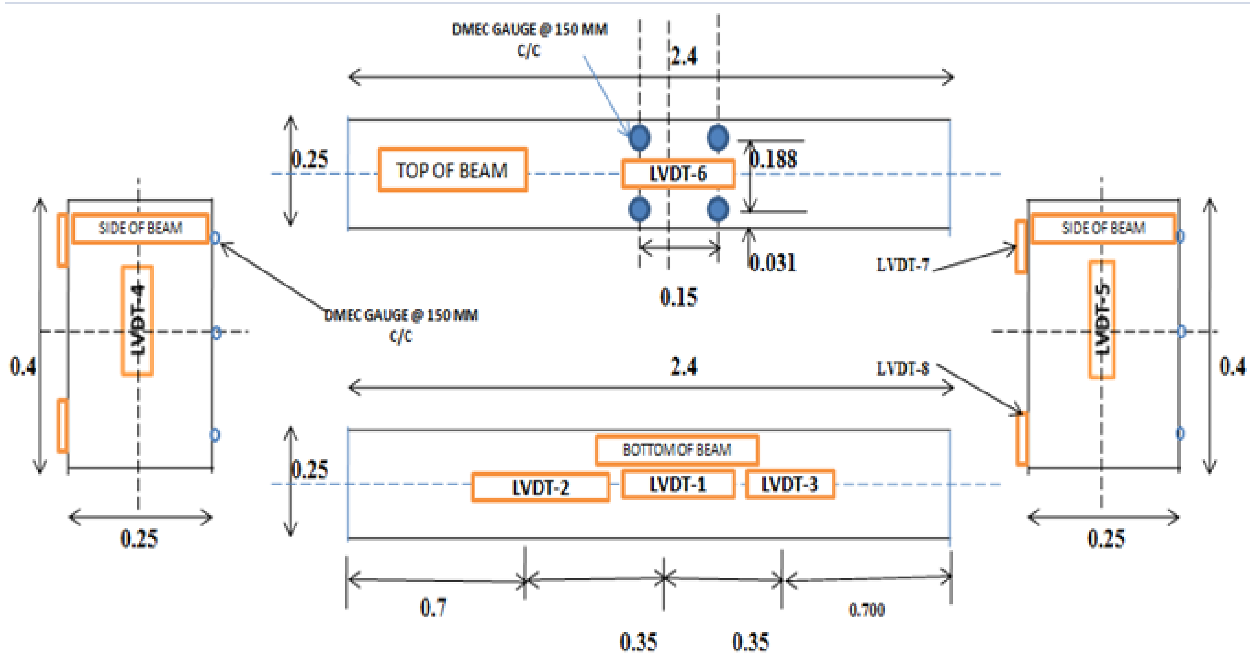

Fig.5. Arrangement of Instrumentation for Remaining Service Life Estimation


Fig.6. Actual Instrumentation of Test set up

7. MODIFIED EQUATION FOR RC BEAMS

7.1. The equations considered (from BS 5400-PART-10) for deciding the fatigue analysis of beams is $\sigma_u = \sigma_{rmax}$ where σ_u is ultimate stress range and $\boldsymbol{\sigma_{rmax}}$ is stress range for running condition. From S-N diagram for structural steel σ_u is evaluated and compared with σ_{rmax} actual stress range. If, the $\boldsymbol{\sigma_u > \sigma_{rmax}}$ reinforcement in the beam has fatigue life. If, $\boldsymbol{\sigma_u < \sigma_{rmax}}$ the reinforcement in the beam has no fatigue life, the cross section of RC beam is to be revised or strengthened.

7.2. Ultimate stress range is

$$\sigma_u = \lambda_{1u} X \lambda_{2u} X \lambda_{3u} X \sigma_0$$

(From Appendix H of IRS Concrete Bridge Code) where λ_{1u}-Life factor, λ_{2u}-Base line factor, λ_{3u}-GMT load factor, and σ_0 = basic stress in reinforcement equal to 53 N/mm^2.λ_{rmax}- is life factor calculated from equation of

$$(\sigma_u - \sigma_{rmax}) = \lambda_{rmax} X \lambda_{2u} X \lambda_{3u} X \sigma_0$$

Ultimate GMT (U_{GMT}) is 27 for RC beams. Remaining GMT (R_{GMT}) of beam is the percentage of remaining GMT X 100. The percentage of remaining GMT is design GMT X remaining life /100.Remaining life is (1–damage ratio).Damage ratio is $\frac{\text{Number of cycles passed so far}(N_{rmax})}{\text{Number of cycles to fracture}(N_{1j})}$.Remaining life in years is $\text{MIN}\left\{120, 120 \text{ X } \left(\frac{(\lambda_{1u}-\lambda_{rmax})}{\lambda_{1u}}\right)^m\right\}$.Here the " m " is inverse slope of line in S-N curve and the value of equal to 3.(From Appendix Hof IRS concrete bridge code).

8. REFERENCES

[1] Indian Railway Standard (2008): *Bridge Rules, Research Designs & Standards Organization*, Ministry of Railways, Lucknow (U.P).

[2] Indian Railway Standard (1997): *Code of Practice for Plain, Reinforced, and Pre stressed Concrete for General Bridge Construction (concrete bridge code)*, Research Designs & Standards Organization, Min of Railways, Luc.(U.P).

[3] Indian Railway Standard (1998): *Bridge Manual*, Research Designs & Standards Organization, Ministry of Railways, Lucknow (U.P).

[4] British Standard Institution (1999): BS-5400: *Steel, Concrete and Composite Bridges-Part 10*-Code of Practice for Fatigue.

[5] European Committee for Standardization (2003) *Euro code 3: Design of Steel Structures - Part 1-9: Fatigue*.

[6] Broek, D. (1982): Elementary *Engineering Fracture Mechanics*, Martinus Nijhoff Publishers, Boston.

[7] Soehardjono Agoes, MD., Raka IGP., Suprobo Priyo. "*Crack Width Prediction for Precast Reinforced concrete Slabs under Repeated Loadings*", International Civil Engineering Conference towards Sustainable Civil Engg Practice, Surabaya, 2006.

[8] Fatemi, A., Yang, L., (1997), "*Cumulative Fatigue Damage and Life Prediction Theories: a survey of*

state of art for homogeneous materials", Univ of Toledo, USA.

[9] Higgins Christopher., Yim,Solomon C., Milller, Thomas H (2004), "*Remaining life of RC Beams with Diagonal Tension Cracks*", Final Report of SPR, 341, pp.1-110.

[10] Plos,M.,Lundgren,K.,Gylltoft,K.,Chalmers.,Cervenka,J.,Thelanderson,S.,Elfgren,L.,Herwig,A.,Bruhwller,E.,Rosell,E.,Gillesen,C.,(2007),"*Nonlinear analysis and remaining fatigue life of RC bridges*", Sustainable Bridges SB-4.5, pp.1-192.

[11] Naaman, A.E.,(1982), "*Fatigue in Partially Prestressed Concrete Beams*", Fatigue of Concrete Structures, Publication SP-75, ACI,Detroit,pp.25-46.

[12] Balaguru, P., Shah, S.P. (1982)," *A Method of Predicting Crack Widths and Deflections for Fatigue Loading", Fatigue of Concrete Structures*, SP-75, ACI, Detroit, 153-175.

[13] Bureau of Indian Standards (2008): "*Specification for high strength deformed steel bars and wires for concrete reinforcement*", IS, Manak Bhavan, New Delhi.

[14] Bureau of Indian Standards (2007):"*General construction in steel*- code of practice, Indian Standards, Manak Bhavan, New Delhi

[15] Goel, R.K.,(2008),"*Fatigue Assessment of Concrete Bridges Euro Norms*",IRICEN Journal of Civil Engineering. 2 (1), pp-46-48.

[16] Bazant, Z.P. (1992),"*Fracture Mechanics of Concrete Structures*," Elsevier Applied Science, Newyork and London,pp.1-140.