CAHIT-8-EQUITABILITY OF CORONAS $C_n \circ K_1$

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Abstract

Cahit [4] proposed the concept of labeling the vertices and edges among the set of integers {0,1,2,…….k-1} as evenly as possible to obtain a generalization of graceful labeling as follows: For any graph G(V,E) and any positive integer k, assign vertex labels from {0,1,2,…….k-1} so that when the edge labels are induced by the absolute value of the difference of the vertex labels, the number of vertices labeled with i and the number of vertices labeled with j differ by at most one and the number of edges labeled with i and the number of edges labeled with j differ by at most one. Cahit called a graph with such an assignment of labels k-equitable.

We establish that Corona graphs $C_n \circ K_1$ are k-equitable as per Cahit's definition of k-equitability, k = 8.

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1. INTRODUCTION

Every labeling of the vertices of a graph with distinct natural numbers induces a natural labeling of its edges: the label of an edge uv is the absolute value of the difference of labels of u and v. A labeling of the vertices of graph of order p is minimally k-equitable if the vertices are labeled with 0,1,2,……p and in the induced labeling of its edges every label either occurs exactly k times or does not occur at all.

Bloom [2] used the term k-equitable to describe another kind of labeling. Hence we all use the term Cahit-k-equitable when the k-equitability is as per Cahit's definition. Bloom defined a labeling of the vertices of a graph to be k-equitable if in the induced labeling of its edges. Every labeling occurs exactly k-times if at all.

The Corona $G_1 \circ G_2$ of two graphs G_1 and G_2 was defined by Frucht and Harary[5] as the graph G obtained by taking one copy of G_1 which has p_1 vertices and p_1 copies of G_2 and then joining the ith vertex of G_1 to every vertex in the i^{th} copy of G_2. In [3] proved that the corona graphs $C_n \circ K_1$ k-equitable as per Cahit's definition of k-equitability k = 2, 3, 4, 5, 6. In addition in [1] we obtain the coronas $C_n \circ K_1$ are 7-equitable as per Cahit's definition. In this paper, we obtain that the coronas $C_n \circ K_1$, $n \geq 3$ are Cahit-8-equitable as per Cahit's definition.

2. CAHIT – 8 – EQUITABILITY OF CORONAS

<u>Theorem</u>: All Coronas Cahit-8-equitable

<u>**Proof**</u>: For Cahit-8-equitable the label set as well as the edge weight set is {0,1,2,3,4,5,6,7}. We have $|V(C_n \circ K_1)| = |E(C_n \circ K_1)| = 2n$. We consider different cases,

<u>**Case1**</u>: $2n \equiv 0 \pmod 8$
Let p = q = 2n = 8t,

<u>**Sub case 1.1**</u>: Suppose t is even. We give suitable labeling at the end of the proof for t = 2. So let $t \geq 4$. For Cahit – 8 – equitability of $C_n \circ K_1$ each label will have to be used 't' times such that each edge weight will occur 't' times.

We describe the labeling function $f: V(C_n \circ K_1)$ {0, 1,------,7}

$$f(u_1) = 6 \qquad f(v_1) = 5$$

$$f(u_{2i}) = 0 \qquad f(v_{2i}) = 4, 1 \leq i \leq \frac{t}{2}$$

$$f(u_{2i+1}) = 7 \qquad f(v_{2i+1}) = 2 \qquad 1 \leq i \leq \frac{t}{2} - 1$$

$$f(u_{t+1}) = 7 \qquad f(v_{t+1}) = 7$$

$$f(u_{2i}) = 0 \qquad f(v_{2i}) = 5 \qquad \frac{t}{2} + 1 \leq i \leq t$$

$f(u_{2i+1}) = 6$ $f(v_{2i+1}) = 5$ $\frac{t}{2} + 1 \leq i \leq t - 1$

$f(u_{2t+1}) = 6$ $f(v_{2t+1}) = 2$

$f(u_{2i+1}) = 4$ $f(v_{2i+1}) = 6$ $t + 1 \leq i \leq \frac{3t}{2} - 1$

$f(u_{2i}) = 1$ $f(v_{2i}) = 2$ $t + 1 \leq i \leq \frac{3t}{2} - 1$

$f(u_{3t}) = 1$ $f(v_{3t}) = 4$

$f(u_{2t+1}) = 3$ $f(v_{3t+1}) = 1$ $\frac{3t}{2} \leq i \leq 2t - 1$

$f(u_{3t+2}) = 3$ $f(v_{3t+2}) = 2$

$f(u_{2i}) = 3$ $f(v_{2i}) = 7$ $\frac{3t}{2} + 2 \leq i \leq 2t$

It can be directly verified that this labeling of $C_n \circ K_1$ is Cahit – 8 – equitable.

We give below a suitable labeling for t = 2 which corresponds to n = 8

3. CAHIT – 8 – EQUITABLE LABELING OF $C_8 \circ K_1$

Here p = q = 16, t = 2, n = 8

(v_1) 5 4 7 5 2 4 2 1 (v_8)

(u_1) 6 0 7 0 6 1 3 3 (u_8)

Sub Case 1.2 Suppose t is odd. We give suitable labeling at the end of the proof for t = 3. So let t ≥ 5. In this case each label will have to be used ‘t’ times such that each edge weight will occur ‘t’ times.

We describe the labeling function f f: V ($C_n \circ K_1$) {0,1,------,7}

$f(u_{2i-1}) = 7$ $f(v_{2i-1}) = 4$ $1 \leq i \leq \frac{t+1}{2}$

$f(u_{2i}) = 0$ $f(v_{2i}) = 2$ $1 \leq i \leq \frac{t+1}{2}$

$f(u_{2i+1}) = 6$ $f(v_{2i+1}) = 1$ $\frac{t+1}{2} \leq i \leq t$

$f(u_{2i+2}) = 0$ $f(v_{2i+2}) = 5$ $\frac{t+1}{2} \leq i \leq t - 1$

$f(u_{2t+2}) = 5$ $f(v_{2t+2}) = 5$

$f(u_{2i+1}) = 1$ $f(v_{2i+1}) = 5$ $t + 1 \leq i \leq \frac{3t-3}{2}$

$f(u_{2i}) = 2$ $f(v_{2i}) = 4$ $t + 2 \leq i \leq \frac{3t-1}{2}$

$f(u_{3t}) = 1$ $f(v_{3t}) = 2$

$f(u_{3t+1}) = 3$ $f(v_{3t+1}) = 4$

$f(u_{2i+1}) = 3$ $f(v_{2i+1}) = 7$ $\frac{3t+1}{2} \leq i \leq 2t - 1$

$f(u_{2i}) = 3$ $f(v_{2i}) = 6$ $\frac{3(t+1)}{2} \leq i \leq 2t$

It can be directly verified that each label and each edge weight occur ‘t’ times.

We give below a suitable labeling for t = 3 which corresponds to n = 12.

4. CAHIT – 8 – EQUITABLE LABELING OF $C_{12} \circ K_1$

Here p = q = 24, t = 3, n = 12

(v_1) 4 2 4 2 1 5 1 5 2 4 7 6 (v_{12})

(u_1) 7 0 7 0 6 0 6 5 1 3 3 3 (v_{12})

Case 2 : 2n ≡ 2 (mod 8)
Let p = q = 2n = 8t + 2

Sub Case 2.1: Suppose t is even. We give suitable labeling at the end of the proof for t = 2. So let t ≥ 4. For Cahit – 8 – equitability of $C_n o K_1$ six labels will have to be used ‘t’ times each, and two labels will have to be used ‘t+1’ times each such that six edge weights will occur ‘t’ times each and two edge weights will occur ‘t+1’ times each.

We describe the labeling function f: V ($C_n \circ K_1$) $\longrightarrow$ {0,1,------,7}

$f(u_1) = 0$ $f(v_1) = 1$

$f(u_{2i}) = 7$ $f(v_{2i}) = 4$ $1 \leq i \leq \frac{t}{2}$

$f(u_{2i+1}) = 0$ $f(v_{2i+1}) = 2$ $1 \leq i \leq \frac{t}{2}$

$f(u_{2i}) = 6$ $f(v_{2i}) = 1$ $\frac{t}{2} + 1 \leq i \leq t$

$f(u_{2i+1}) = 0$ $f(v_{2i+1}) = 5$ $\frac{t}{2} + 2 \leq i \leq t$

$f(u_{2t+2}) = 5$ $f(v_{2t+2}) = 5$

$f(u_{2i+1}) = 1$ $f(v_{2i+1}) = 5$ $t + 1 \leq i \leq \frac{3t}{2} - 1$

$f(u_{2i}) = 2$ $f(v_{2i}) = 4$ $t + 2 \leq i \leq \frac{3t}{2}$

$f(u_{3t+1}) = 1$ $f(v_{3t+1}) = 2$

$f(u_{2i}) = 3$ $f(v_{2i}) = 6$ $\frac{3t}{2} + 1 \leq i \leq 2t$

$f(u_{2i+1}) = 3 \quad f(v_{2i+1}) = 7 \quad \frac{3t}{2} + 1 \le i \le 2t$

It can be directly verified that six labels and six edge-weights occur 't' times each and two labels and two edge weights occur 't+1' times each.

We give below a suitable labeling for t=2 which corresponds to n = 9.

5. CAHIT – 8 – EQUITABLE LABELING OF $C_9 \circ K_1$

Here p = q = 18, t = 2, n = 9

(v_1) 1 4 2 1 4 5 2 6 7 (v_9)
(u_1) 0 7 0 6 0 5 1 3 3 (u_9)

Case 2.2 Assume t is odd. We give suitable labeling at the end of the proof for t = 1. So let t ≥ 3. For Cahit-8-equitability of $C_n o K_1$ six labels will have to be used 't' times each and two labels will have to be used 't+1' times each such that six edge weights will occur t times each and two edge weights will occur t+1 times each.

We describe the labeling function f: V ($C_n \circ K_1$) $\longrightarrow$ {0,1,------,7}

$f(u_{2i-1}) = 7 \quad f(v_{2i-1}) = 4 \quad 1 \le i \le \frac{t+1}{2}$

$f(u_{2i-2}) = 0 \quad f(v_{2i-2}) = 2 \quad 2 \le i \le \frac{t+3}{2}$

$f(u_{2i+1}) = 6 \quad f(v_3) = 1 \quad \frac{t+1}{2} \le i \le t$

$f(u_{2i}) = 0 \quad f(v_{2i}) = 5 \quad \frac{t+3}{2} \le i \le t$

$f(u_{2t+2}) = 5 \quad f(v_{2t+2}) = 5$

$f(u_{2i}) = 1 \quad f(v_{2i}) = 5 \quad t+1 \le i \le \frac{3(t-1)}{2}$

$f(u_{2i+4}) = 2 \quad f(v_{2i+4}) = 4 \quad t \le i \le \frac{3(t-1)}{2} - 1$

$f(u_{3t}) = 1 \quad f(v_{3t}) = 2$

$f(u_{3t+1}) = 3 \quad f(v_{3t+1}) = 2$

$f(u_{3t+2}) = 3 \quad f(v_{3t+2}) = 4$

$f(u_{2i}) = 3 \quad f(v_{2i}) = 6 \quad \frac{3(t+1)}{2} \le i \le 2t$

$f(u_i) = 2 \quad f(v_i) = 1 \quad \frac{3(t+1)}{2} \le i \le 2$

It can be directly verified that six labels and six edge weights occur 't' times each and two labels and two edge weights occur 't+1' times each.

We give below a suitable labeling for t = 1 which corresponds to n = 5.

6. CAHIT – 8 – EQUITABLE LABELING OF $C_5 \circ K_1$

Here p = q = 10, t = 1, n = 5

v_1	v_2	v_3	v_4	v_5
1	4	2	5	3
0	7	2	0	6
u_1	u_2	u_3	u_4	u_5

Case3 2n ≡ 4 (mod8)
Let p = q = 2n = 8t + 4

Sub Case 3.1 Assume t is even. We give suitable labeling at the end of the proof for t = 2. So let t ≥ 4. For Cahit-8-equitability of $C_n \circ K_1$ four labels will have to be used 't' times each, and four labels will have to be used 't+1' times each such that four edge weights will occur t times each and four edge weights will occur 't+1' times each.

We describe the labeling function f: V ($C_n \circ K_1$) {0,1,------,7}

$f(u_{2i-1}) = 0 \quad f(v_{2i-1}) = 2 \quad 1 \le i \le \frac{t}{2} + 1$

$f(u_2) = 7 \quad f(v_2) = 4 \quad 1 \le i \le \frac{t}{2}$

$f(u_{2i+1}) = 6 \quad f(v_{2i+1}) = 1 \quad \frac{t}{2} + 1 \le i \le t$

$f(u_{2i}) = 7 \quad f(v_{2i}) = 4 \quad \frac{t}{2} + 1 \le i \le t$

$f(u_{2t+2}) = 5 \quad f(v_{2t+2}) = 5$

$f(u_{2i+3}) = 1 \quad f(v_{2i+3}) = 5 \quad t \le i \le \frac{3t-2}{2}$

$f(u_{2i}) = 2 \quad f(v_{2i}) = 4 \quad t+2 \le i \le \frac{3t}{2}$

$f(u_{3t+1}) = 1 \quad f(v_{2t+2}) = 2$

$f(u_{3t+2}) = 3 \quad f(v_{3t+2}) = 4$

$f(u_{2i}) = 3 \quad f(v_{2i}) = 7 \quad \frac{3t}{2} + 1 \le i \le 2t$

$f(u_{3t+1}) = 3 \quad f(v_{3t+1}) = 6 \quad \frac{3t}{2} + 2 \le i \le 2t + 1$

It can be directly verified that four labels and four edge weights occur 't' times each and four labels and four edge weights occur 't+1' times each.

We give below a suitable labeling for t = 2 which corresponds to n = 10.

7. CAHIT – 8 – EQUITABLE LABELING OF $C_{10}\circ K_1$

Here p = q = 20, t = 2, n = 10

v_1	v_2	v_3	v_4	v_5	v_6	v_7
	v_8	v_9	v_{10}			
2	4	2	1	5	5	2
	4	7	6			
0	7	0	6	0	5	1
	3	3	3			
u_1	u_2	u_3	u_4	u_5	u_6	u_7
	u_8	u_9	u_{10}			

Sub Case 3.2 Assume t is odd. We give suitable labeling at the end of the proof for t=1. So let t ≥ 3. For Cahit-8-equitabilty of $C_n\circ K_1$ four labels will have to be used ‘t’ times each and four labels will have to be used ‘t+1’ times each such that four edge weights will occur ‘t’ times each and four edge weights will occur ‘t+1’ times each.

We describe the labeling function f: V ($C_n\circ K_1$) {0,1,------,7}

$f(u_{2i-1}) = 7 \quad f(v_{2i-1}) = 4 \quad 1 \le i \le \frac{t+1}{2}$

$f(u_{2i}) = 0 \quad f(v_{2i}) = 2 \quad 1 \le i \le \frac{t+1}{2}$

$f(u_{2i+1}) = 0 \quad f(v_{2i+1}) = 4 \quad \frac{t+1}{2} \le i \le t$

$f(u_{2i}) = 6 \quad f(v_{2i}) = 5 \quad \frac{t+3}{2} \le i \le t$

$f(u_{2t+2}) = 5 \quad f(v_{2t+2}) = 5$

$f(u_{2i+1}) = 1 \quad f(v_{2i+1}) = 5 \quad t+1 \le i \le \frac{3t-1}{2}$

$f(u_{2i}) = 0 \quad f(v_{2i}) = 6 \quad t+2 \le i \le \frac{3t+1}{2}$

$f(u_{3t+2}) = 3 \quad f(v_{3t+2}) = 1$

$f(u_{3t+3}) = 3 \quad f(v_{3t+3}) = 4$

$f(u_{2i+1}) = 3 \quad f(v_{2i+1}) = 6 \quad \frac{3(t+1)}{2} \le i \le 2t$

$f(u_{2i}) = 3 \quad f(v_{2i}) = 7 \quad \frac{3t+3}{2} + 1 \le i \le 2t+1$

It can be easily verified that four labels and four edge weights occur ‘t’ times each and four labels and four edge weights occur ‘t+1’ times each.

We give below a suitable labeling for t=1 which correspond to n = 6.

8. CAHIT – 8 – EQUITABLE LABELING OF $C_6\circ K_1$

Here p = q = 12, t = 1, n = 6

v_1	v_2	v_3	v_4	v_5	v_6
2	2	5	3	4	1
7	0	5	6	0	3
u_1	u_2	u_3	u_4	u_5	u_6

Case 4 $2n \equiv 6 \pmod 8$
Let p = q = 2n = 8t + 6

Sub Case 4.1 Suppose t is even. We give suitable labeling at the end of the proof for t = 2. So let t ≥ 4. For Cahit-8-equitabilty of $C_n\circ K_1$ two labels will have to be used ‘t’ times each and six labels will have to be used ‘t+1’ times each such that two edge weights will occur ‘t’ times each and six edge weight will occur ‘t+1’ times each.

We describe the labeling function f: V ($C_n\circ K_1$) $\longrightarrow$ {0,1,------,7}

$f(u_{2i-1}) = 0 \quad f(v_{2i-1}) = 2 \quad 1 \le i \le \frac{t}{2}$

$f(u_{2i}) = 7 \quad f(v_{2i}) = 4 \quad 1 \le i \le \frac{t}{2}$

$f(u_{t+1}) = 0 \quad f(v_{t+1}) = 4$

$f(u_{2i}) = 6 \quad f(v_{2i}) = 1 \quad \frac{t}{2} + 1 \le i \le t$

$f(u_{2i+1}) = 0 \quad f(v_{2i+1}) = 5 \quad \frac{t}{2} + 1 \le i \le t$

$f(u_{2t+2}) = 5 \quad f(v_{2t+2}) = 5$

$f(u_{2i+3}) = 1 \quad f(v_{2i+3}) = 5 \quad t \le i \le \frac{3t}{2} - 2$

$f(u_{2i+4}) = 2 \quad f(v_{2i+4}) = 4 \quad t \le i \le \frac{3t}{2} - 1$

$f(u_{3t+1}) = 1 \quad f(v_{3t+1}) = 2$

$f(u_{3t+3}) = 3 \quad f(v_{3t+3}) = 1$

$f(u_{2i+2}) = 3 \quad f(v_{2i+2}) = 7 \quad \frac{3t}{2} + 1 \le i \le 2t$

$f(u_{2i+1}) = 3 \quad f(v_{2i+1}) = 6 \quad \frac{3t}{2} + 2 \le i \le 2t+1$

It can be obviously checked that two labels and two edge weights occur ‘t’ times each and six labels and six edge weights occur ‘t+1’ times each.

We obtain below a suitable labeling for t = 2 which correspond to n = 11.

9. CAHIT – 8 – EQUITABLE LABELING OF $C_{11}\circ K_1$

Here p = q = 22, t = 2, n = 11

(v_1) 2 4 4 1 5 5 2 4 1 7 6 (v_{11})
(u_1) 0 7 0 6 0 5 1 2 3 3 3 (u_{11})

Sub Case 4.2 Assume t is odd. We obtain labeling at the end of the proof for t = 1. So that let t ≥ 3. For Cahit-8-equitability of $C_n\circ K_1$ two labels will have to be used 't' times each and six labels will have to be used 't+1' times each such that two edge weights will occur 't' times each and six edge weights will occur 't+1' times each.

We define the labeling function f: V ($C_n\circ K_1$) $\longrightarrow$ {0,1,------,7}

$f(u_{2i-1}) = 7 \quad f(v_{2i-1}) = 4 \quad 1 \le i \le \frac{t+1}{2}$

$f(u_{2i}) = 0 \quad f(v_{2i}) = 2 \quad 1 \le i \le \frac{t+1}{2}$

$f(u_{2i+1}) = 6 \quad f(v_{2i+1}) = 1 \quad \frac{t+1}{2} \le i \le t$

$f(u_{2i}) = 0 \quad f(v_{2i}) = 5 \quad \frac{t+3}{2} \le i \le t+1$

$f(u_{2t+1}) = 5 \quad f(v_{2t+2}) = 5$

$f(u_{2i+1}) = 2 \quad f(v_{2i+1}) = 4 \quad t+1 \le i \le \frac{3t+1}{2}$

$f(u_{2i}) = 1 \quad f(v_{2i}) = 5 \quad t+2 \le i \le \frac{3t+1}{2}$

$f(u_{3t+3}) = 3 \quad f(v_{3t+3}) = 2$

$f(u_{2i+1}) = 3 \quad f(v_{2i+1}) = 6 \quad \frac{3t+3}{2} \le i \le 2t+1$

$f(u_{2i}) = 3 \quad f(v_{2i}) = 7 \quad \frac{3t+5}{2} \le i \le 2t+1$

It can be directly verified that two labels and two edge weights occur 't' times each and six labels and six edge weights occur 't+1' times each.

We give below a suitable labeling for t = 1 which corresponds to n = 7.

10. CAHIT – 8 – EQUITABLE LABELING OF $C_7\circ K_1$

Here p = q = 14, t = 1, n = 7

v_1	v_2	v_3	v_4	v_5	v_6	v_7
3	1	5	2	2	1	3
7	0	5	0	6	4	4
u_1	u_2	u_3	u_4	u_5	u_6	u_7

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