

MAGNETO-CONVECTIVE FLOW AND HEAT TRANSFER OF TWO IMMISCIBLE FLUIDS BETWEEN VERTICAL WAVY WALL AND A PARALLEL FLAT WALL

Mahadev M Biradar

Associate Professor, Dept. of Mathematics Basaveshwar Engineering College (Autonomous) Bagalkot, Karnataka INDIA
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Abstract

Convective flow and heat transfer between vertical wavy wall and a parallel flat wall consisting of two regions, one filled with electrically conducting and other with viscous fluid is analyzed. Governing equation of motion have been solved by linearization technique. Results are presented for various parameters such as Hartmann number, Grashof number, viscosity ratio, width ratio, conductivity ratio and source or sink. The effect of all the parameters except the Hartmann number and source or sink remains same for two viscous immiscible fluids. The effect of Hartmann number is to decrease the velocity at the wavy and flat wall. The suppression near the flat wall compared to wavy wall is insignificant. The velocity is large for source compared to sink for equal and different wall temperature.

Keywords: Convection, vertical, wavy wall, immiscible

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1. INTRODUCTION

Many transport process exists in natural and industrial applications in which the transfer of heat and mass occurs simultaneously as a result of buoyancy effect of thermal diffusion. Natural convection heat transfer plays an important role in the electronic components cooling since it has desirable characteristics in thermal equipments design; absence of mechanical or electromagnetic noise; low energy consumption, very important in portable computers; and reliability, since it has no elements to fail.

The optimization of the heat transfer has increasingly importance in electronic packaging due to the higher heat densities and to the electronic components and equipments miniaturization Sathe et.al. (1998). In spite of the abundant results about natural convection in electronic packaging, works dealing with heat transfer maximization is scarce in literature Landon et.al (1999) Da Silva et.al., (2004). This is, probably, due to the non-linear nature and to the difficult of natural convection simulation.

The corrugated wall channel is one of several devices employed for enhancing the heat transfer efficiency of industrial transport process. The problem of viscous flow in wavy channels was first treated analytically by Burns and Parks (1967) who expressed the stream function as a Fourier series under the assumption of Stokes flow. Wang and Vanka (1995) determined the rates of heat transfer for flow through a periodic array of wavy passages. They observed that in the

steady-flow regime, the average Nusselt numbers for the wavy-wall channel were only slightly larger than those for a parallel-plate channel. The problem of natural or mixed convection along a sinusoidal wavy surface extended previous work to complex geometries Yao (1983), Moulic et.al., (1989,1989) and has received considerable attention due to its relevance to real geometries. An example of such geometry is a "roughened" surface that occurs often in problem involving the enhancement of heat transfer.

The flow and heat transfer of electrically conducting fluids in channels under the effect of a transverse magnetic field occurs in MHD-generators, pumps, accelerators, nuclear-reactors, filtration, geo-thermal system and others. Recently there are experimental and theoretical studies on hydromagnetic aspects of two fluid flows available in literature. Lohrasbi and Sahai (1998) dealt with two-phase magnetohydrodynamic (MHD) flow and heat transfer in a parallel plate channel. Two-phase MHD flow and heat transfer in an inclined channel is investigated by Malashetty and Umavathi (1997). Recently Malashetty et.al, (2000, 2001) analyzed the problem of fully developed two fluid magnetohydrodynamic flows with and without applied electric field in an inclined channel. Chamakha (2000) considered the steady, laminar flow of two viscous, incompressible electrically conducting and heat generating or absorbing immiscible fluids in an infinitely – long, impermeable parallel-plate channel filled with a uniform porous medium.

In spite of the numerous applications of corrugated walls, much work is not seen in literature. Hence it is the objective of the present work, is to study the problem of flow and heat transfer between vertical wavy wall and a parallel flat wall consisting of two regions, one filled with electrically conducting fluid and second with electrically non-conducting fluid.

2. MATHEMATICAL FORMULATION

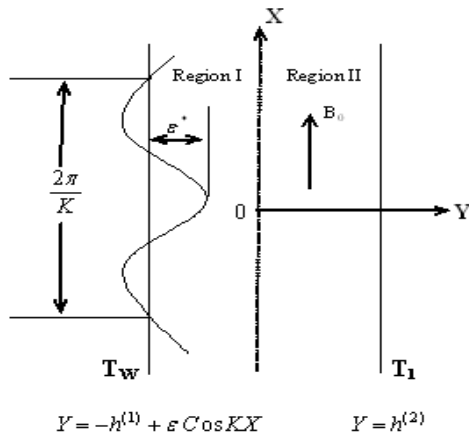


Fig-1: Physical Configuration

Consider the channel as shown in figure 4.1, in which the X axis is taken vertically upwards and parallel to the flat wall while the Y axis is taken perpendicular to it in such a way that the wavy wall is represented by $Y = -h^{(1)} + \varepsilon * \cos KX$ and the flat wall by $Y = h^{(2)}$. The region $0 \leq y \leq h^{(1)}$ is occupied by viscous fluid of density $\rho^{(1)}$, viscosity $\mu^{(1)}$, thermal conductivity $k^{(1)}$ and the region $h^{(2)} \leq y \leq 0$ is occupied another viscous fluid of density $\rho^{(2)}$, viscosity $\mu^{(2)}$, thermal conductivity $k^{(2)}$. The wavy and flat walls are maintained at constant and different temperatures T_w and T_1 respectively. We make the following assumptions:

- (i) that all the fluid properties are constant except the density in the buoyancy-force term;
- (ii) that the flow is laminar, steady and two-dimensional;
- (iii) that the viscous dissipation and the work done by pressure are sufficiently small in comparison with both the heat flow by conduction and the wall temperature;
- (iv) that the wavelength of the wavy wall, which is proportional to $1/K$, is large.

Under these assumptions, the equations of momentum, continuity and energy which govern steady two-dimensional flow and heat transfer of viscous incompressible fluids are

Region –I

$$\rho^{(1)} \left(U^{(1)} \frac{\partial U^{(1)}}{\partial X} + V^{(1)} \frac{\partial U^{(1)}}{\partial Y} \right) = -\frac{\partial P^{(1)}}{\partial X} + \mu^{(1)} \nabla^2 U^{(1)} - \rho^{(1)} g \tag{2.1}$$

$$\rho^{(1)} \left(U^{(1)} \frac{\partial V^{(1)}}{\partial X} + V^{(1)} \frac{\partial V^{(1)}}{\partial Y} \right) = -\frac{\partial P^{(1)}}{\partial Y} + \mu^{(1)} \nabla^2 V^{(1)} \tag{2.2}$$

$$\frac{\partial U^{(1)}}{\partial X} + \frac{\partial V^{(1)}}{\partial Y} = 0 \tag{2.3}$$

$$\rho^{(1)} C_p^{(1)} \left(U^{(1)} \frac{\partial T^{(1)}}{\partial X} + V^{(1)} \frac{\partial T^{(1)}}{\partial Y} \right) = k^{(1)} \nabla^2 T^{(1)} + Q^{(1)} \tag{2.4}$$

Region –II

$$\rho^{(2)} \left(U^{(2)} \frac{\partial U^{(2)}}{\partial X} + V^{(2)} \frac{\partial U^{(2)}}{\partial Y} \right) = -\frac{\partial P^{(2)}}{\partial X} + \mu^{(2)} \nabla^2 U^{(2)} - \rho^{(2)} g - \sigma B_0^2 U^{(2)} \tag{2.5}$$

$$\rho^{(2)} \left(U^{(2)} \frac{\partial V^{(2)}}{\partial X} + V^{(2)} \frac{\partial V^{(2)}}{\partial Y} \right) = -\frac{\partial P^{(2)}}{\partial Y} + \mu^{(2)} \nabla^2 V^{(2)} \tag{2.6}$$

$$\frac{\partial U^{(2)}}{\partial X} + \frac{\partial V^{(2)}}{\partial Y} = 0 \tag{2.7}$$

$$\rho^{(2)} C_p^{(2)} \left(U^{(2)} \frac{\partial T^{(2)}}{\partial X} + V^{(2)} \frac{\partial T^{(2)}}{\partial Y} \right) = k^{(2)} \nabla^2 T^{(2)} + Q^{(2)} \tag{2.8}$$

Where the superscript indicates the quantities for regions I and II, respectively. To solve the above system of equations, one needs proper boundary and interface conditions. We assume $C_p^{(1)} = C_p^{(2)}$

The physical hydro dynamic conditions are

$$U^{(1)} = 0 \quad V^{(1)} = 0, \quad \text{at } Y = -h^{(1)} + \varepsilon * \cos KX$$

$$U^{(2)} = 0 \quad V^{(2)} = 0, \quad \text{at } Y = h^{(2)}$$

$$U^{(1)} = U^{(2)} \quad V^{(1)} = V^{(2)}, \quad \text{at } Y = 0$$

$$\mu^{(1)} \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^{(1)} = \mu^{(2)} \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^{(2)} \quad \text{at } Y = 0 \tag{2.9}$$

The boundary and interface conditions on temperature are

$$\begin{aligned} T^{(1)} = T_w & \quad \text{at} \quad Y = -h^{(1)} + \varepsilon^* \cos KX \\ T^{(2)} = T_1 & \quad \text{at} \quad Y = h^{(2)} \\ T^{(1)} = T^{(2)} & \quad \text{at} \quad Y = 0 \\ k^{(1)} \left(\frac{\partial T}{\partial Y} + \frac{\partial T}{\partial X} \right)^{(1)} & = k^{(2)} \left(\frac{\partial T}{\partial Y} + \frac{\partial T}{\partial X} \right)^{(2)} \quad \text{at } Y = 0 \end{aligned} \quad (2.10)$$

The conditions on velocity represent the no-slip condition and continuity of velocity and shear stress across the interface. The conditions on temperature indicate that the plates are held at constant but different temperatures and continuity of heat and heat flux at the interface.

The basic equations (4.2.1) to (4.2.8) are made dimensionless using the following transformations

$$\begin{aligned} (x, y)^{(1)} & = \frac{l}{h^{(1)}} (X, Y)^{(1)}; \quad (u, v)^{(1)} = \frac{h^{(1)}}{v^{(1)}} (U, V)^{(1)} \\ (x, y)^{(2)} & = \frac{l}{h^{(2)}} (X, Y)^{(2)}; \quad (u, v)^{(2)} = \frac{h^{(2)}}{v^{(2)}} (U, V)^{(2)} \\ \theta^{(1)} & = \frac{(T^{(1)} - T_s)}{(T_w - T_s)}; \quad \theta^{(2)} = \frac{(T^{(2)} - T_s)}{(T_w - T_s)} \\ \bar{P}^{(1)} & = \frac{P^{(1)}}{\rho \left(\frac{v^{(1)}}{h^{(1)}} \right)^2}; \quad \bar{P}^{(2)} = \frac{P^{(2)}}{\rho \left(\frac{v^{(2)}}{h^{(2)}} \right)^2} \end{aligned} \quad (2.11)$$

Where T_s is the fluid temperature in static conditions.

Region -I

$$u^{(1)} \frac{\partial u^{(1)}}{\partial x} + v^{(1)} \frac{\partial u^{(1)}}{\partial y} = -\frac{\partial \bar{P}^{(1)}}{\partial x} + \frac{\partial^2 u^{(1)}}{\partial x^2} + \frac{\partial^2 u^{(1)}}{\partial y^2} + G\theta^{(1)} \quad (2.12)$$

$$u^{(1)} \frac{\partial v^{(1)}}{\partial x} + v^{(1)} \frac{\partial v^{(1)}}{\partial y} = -\frac{\partial \bar{P}^{(1)}}{\partial y} + \frac{\partial^2 v^{(1)}}{\partial x^2} + \frac{\partial^2 v^{(1)}}{\partial y^2} \quad (2.13)$$

$$\frac{\partial u^{(1)}}{\partial x} + \frac{\partial v^{(1)}}{\partial y} = 0 \quad (2.14)$$

$$u^{(1)} \frac{\partial \theta^{(1)}}{\partial x} + v^{(1)} \frac{\partial \theta^{(1)}}{\partial y} = \frac{1}{Pr} \left(\frac{\partial^2 \theta^{(1)}}{\partial x^2} + \frac{\partial^2 \theta^{(1)}}{\partial y^2} \right) + \frac{\alpha}{Pr} \quad (2.15)$$

Region -II

$$\begin{aligned} u^{(2)} \frac{\partial u^{(2)}}{\partial x} + v^{(2)} \frac{\partial u^{(2)}}{\partial y} & = -\frac{\partial \bar{P}^{(2)}}{\partial x} + \frac{\partial^2 u^{(2)}}{\partial x^2} + \frac{\partial^2 u^{(2)}}{\partial y^2} \\ & + \beta m^2 r^2 h^3 G\theta^{(2)} - M^2 m h^2 u^{(2)} \end{aligned} \quad (2.16)$$

$$u^{(2)} \frac{\partial v^{(2)}}{\partial x} + v^{(2)} \frac{\partial v^{(2)}}{\partial y} = -\frac{\partial \bar{P}^{(2)}}{\partial y} + \frac{\partial^2 v^{(2)}}{\partial x^2} + \frac{\partial^2 v^{(2)}}{\partial y^2} \quad (2.17)$$

$$\frac{\partial u^{(2)}}{\partial x} + \frac{\partial v^{(2)}}{\partial y} = 0 \quad (2.18)$$

$$\begin{aligned} u^{(2)} \frac{\partial \theta^{(2)}}{\partial x} + v^{(2)} \frac{\partial \theta^{(2)}}{\partial y} & = \frac{km}{Pr} \left(\frac{\partial^2 \theta^{(2)}}{\partial x^2} + \frac{\partial^2 \theta^{(2)}}{\partial y^2} \right) \\ & + \frac{mh^2 \alpha Q}{Pr} \end{aligned} \quad (2.19)$$

The dimensionless form of equation (2.9) and (2.10) using (2.11) become

$$u^{(1)} = 0; \quad v^{(1)} = 0 \quad \text{at} \quad y = -1 + \varepsilon \cos \lambda x$$

$$u^{(2)} = 0; \quad v^{(2)} = 0 \quad \text{at} \quad y = 1$$

$$u^{(1)} = \frac{1}{rmh} u^{(2)}; \quad v^{(1)} = \frac{1}{rmh} v^{(2)} \quad \text{at} \quad y = 0$$

$$\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^{(1)} = \frac{l}{rm^2 h^2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^{(2)} \quad \text{at} \quad y = 0 \quad (2.20)$$

$$\theta^{(1)} = 1 \quad \text{at} \quad y = -1 + \varepsilon \cos \lambda x$$

$$\theta^{(2)} = \bar{\theta} \quad \text{at} \quad y = 1$$

$$\theta^{(1)} = \theta^{(2)} \quad \text{at} \quad y = 0$$

$$\left(\frac{\partial\theta}{\partial y} + \frac{\partial\theta}{\partial x}\right)^{(1)} = \frac{k}{h}\left(\frac{\partial\theta}{\partial y} + \frac{\partial\theta}{\partial x}\right)^{(2)} \text{ at } y = 0 \quad (2.21)$$

3. SOLUTIONS

The governing equations (2.12) to (2.15), (2.16) to (2.19) along with boundary and interface conditions (2.20) to (2.21) are two dimensional, nonlinear and coupled and hence closed form solutions can not be found. However approximate solutions can be found using the method of regular perturbation. We take flow field and the temperature field to be

Region-I

$$\begin{aligned} u^{(1)}(x, y) &= u_0^{(1)}(y) + \varepsilon u_1^{(1)}(x, y) \\ v^{(1)}(x, y) &= \varepsilon v_1^{(1)}(x, y) \\ \bar{P}^{(1)}(x, y) &= P_0^{(1)}(x) + \varepsilon P_1^{(1)}(x, y) \\ \theta^{(1)}(x, y) &= \theta_0^{(1)}(y) + \varepsilon \theta_1^{(1)}(x, y) \end{aligned} \quad (3.1)$$

Region -II

$$\begin{aligned} u^{(2)}(x, y) &= u_0^{(2)}(y) + \varepsilon u_1^{(2)}(x, y) \\ v^{(2)}(x, y) &= \varepsilon v_1^{(2)}(x, y) \\ \bar{P}^{(2)}(x, y) &= P_0^{(2)}(x) + \varepsilon P_1^{(2)}(x, y) \\ \theta^{(2)}(x, y) &= \theta_0^{(2)}(y) + \varepsilon \theta_1^{(2)}(x, y) \end{aligned} \quad (3.2)$$

Where the perturbations $u_1^{(i)}, v_1^{(i)}, P_1^{(i)}$ and $\theta_1^{(i)}$ for $i=1, 2$ are small compared with the mean or the zeroth order quantities. Using equation (4.3.1) and (4.3.2) in the equation (2.12) to (2.15), (2.16) to (2.19) become

3.1 Zeroth Order

Region-I

$$\frac{d^2 u_0^{(1)}}{dy^2} = -G\theta_0^{(1)} \quad (3.3)$$

$$\frac{d^2 \theta_0^{(1)}}{dy^2} = -\alpha \quad (3.4)$$

Region-II

$$\frac{d^2 u_0^{(2)}}{dy^2} - mh^2 M^2 u_0^{(2)} = -\beta m^2 r^2 h^3 G\theta_0^{(2)} \quad (3.5)$$

$$\frac{d^2 \theta_0^{(2)}}{dy^2} = \frac{-\alpha Q h^2}{k} \quad (3.6)$$

3.2 First Order:

Region-I

$$u_0^{(1)} \frac{\partial u_1^{(1)}}{\partial x} + v_1^{(1)} \frac{du_0^{(1)}}{dy} = -\frac{\partial P_1^{(1)}}{\partial x} + \frac{\partial^2 u_1^{(1)}}{\partial x^2} + \frac{\partial^2 u_1^{(1)}}{\partial y^2} + G\theta_1^{(1)} \quad (3.7)$$

$$u_0^{(1)} \frac{\partial v_1^{(1)}}{\partial x} = -\frac{\partial P_1^{(1)}}{\partial y} + \frac{\partial^2 v_1^{(1)}}{\partial x^2} + \frac{\partial^2 v_1^{(1)}}{\partial y^2} \quad (3.8)$$

$$\frac{\partial u_1^{(1)}}{\partial x} + \frac{\partial^2 v_1^{(1)}}{\partial y^2} = 0 \quad (3.9)$$

$$\text{Pr} \left(u_0^{(1)} \frac{\partial \theta_1^{(1)}}{\partial x} + v_1^{(1)} \frac{d\theta_0^{(1)}}{dy} \right) = \frac{\partial^2 \theta_1^{(1)}}{\partial x^2} + \frac{\partial^2 \theta_1^{(1)}}{\partial y^2} \quad (3.10)$$

Region-II

$$\begin{aligned} u_0^{(2)} \frac{\partial u_1^{(2)}}{\partial x} + v_1^{(2)} \frac{du_0^{(2)}}{dy} &= -\frac{\partial P_1^{(2)}}{\partial x} + \frac{\partial^2 u_1^{(2)}}{\partial x^2} + \frac{\partial^2 u_1^{(2)}}{\partial y^2} \\ &+ \beta m^2 r^2 h^3 G\theta_1^{(2)} - mh^2 M^2 u_1^{(2)} \end{aligned} \quad (3.11)$$

$$u_0^{(2)} \frac{\partial v_1^{(2)}}{\partial x} = -\frac{\partial P_1^{(2)}}{\partial y} + \frac{\partial^2 v_1^{(2)}}{\partial x^2} + \frac{\partial^2 v_1^{(2)}}{\partial y^2} \quad (3.12)$$

$$\frac{\partial u_1^{(2)}}{\partial x} + \frac{\partial^2 v_1^{(2)}}{\partial y^2} = 0 \quad (3.13)$$

$$u_0^{(2)} \frac{\partial \theta_1^{(2)}}{\partial x} + v_1^{(2)} \frac{d\theta_0^{(2)}}{dy} = \frac{km}{\text{Pr}} \left(\frac{\partial^2 \theta_1^{(2)}}{\partial x^2} + \frac{\partial^2 \theta_1^{(2)}}{\partial y^2} \right) \quad (3.14)$$

Where $C^{(1)} = \frac{\partial(P_0 - P_s)^{(1)}}{\partial x}$ and $C^{(2)} = \frac{\partial(P_0 - P_s)^{(2)}}{\partial x}$

taken equal to zero (see Ostrach, 1952). With the help of (3.1) and (3.2) the boundary and interface conditions (2.20) to (2.21) become

$$u_0^{(1)}(-1) = 0 \quad \text{at } y = -1$$

$$u_0^{(2)}(1) = 0 \quad \text{at } y = 1$$

$$u_0^{(1)} = \frac{1}{rmh} u_0^{(2)} \quad \text{at } y = 0$$

$$\frac{du_0^{(1)}}{dy} = \frac{1}{rm^2 h^2} \frac{du_0^{(2)}}{dy} \quad \text{at } y = 0 \quad (3.15)$$

$$\theta_0^{(1)}(-1) = 1 \quad \text{at } y = -1$$

$$\theta_0^{(2)}(1) = \bar{\theta} \quad \text{at } y = 1$$

$$\theta_0^{(1)} = \theta_0^{(2)} \quad \text{at } y = 0$$

$$\frac{d\theta_0^{(1)}}{dy} = \frac{k}{h} \frac{d\theta_0^{(2)}}{dy} \quad \text{at } y = 0 \quad (3.16)$$

$$u_1^{(1)}(-1) = -\text{Cos}\lambda x \frac{du_0^{(1)}}{dy}; v_1^{(1)}(-1) = 0 \quad \text{at } y = -1$$

$$u_1^{(2)}(1) = 0; v_1^{(2)}(1) = 0 \quad \text{at } y = 1$$

$$u_1^{(1)} = \frac{1}{rmh} u_1^{(2)}; v_1^{(1)} = \frac{1}{rmh} v_1^{(2)} \quad \text{at } y = 0$$

$$\frac{\partial u_1^{(1)}}{\partial y} + \frac{\partial v_1^{(1)}}{\partial x} = \frac{1}{rm^2 h^2} \left(\frac{\partial u_1^{(2)}}{\partial y} + \frac{\partial v_1^{(2)}}{\partial x} \right) \quad \text{at } y = 0 \quad (3.17)$$

$$\theta_1^{(1)}(-1) = -\text{Cos}\lambda x \frac{d\theta_0^{(1)}}{dy} \quad \text{at } y = -1$$

$$\theta_1^{(2)}(1) = 0 \quad \text{at } y = 1$$

$$\theta_1^{(1)} = \theta_1^{(2)} \quad \text{at } y = 0$$

$$\frac{\partial \theta_1^{(1)}}{\partial y} + \frac{\partial \theta_1^{(1)}}{\partial x} = \frac{k}{h} \left(\frac{\partial \theta_1^{(2)}}{\partial y} + \frac{\partial \theta_1^{(2)}}{\partial x} \right) \quad \text{at } y = 0 \quad (3.18)$$

Introducing the stream function $\bar{\psi}_1$ defined by

$$u_1^{(1)} = -\frac{\partial \bar{\psi}_1^{(1)}}{\partial y}; v_1^{(1)} = \frac{\partial \bar{\psi}_1^{(1)}}{\partial x} \quad (3.19)$$

And eliminating $P_1^{(1)}$ and $P_1^{(2)}$ from equation (3.7), (3.8) and (3.11), (3.12) we get

Region-I

$$u_0^{(1)} \left(\frac{\partial^3 \bar{\psi}_1^{(1)}}{\partial x \partial y^2} + \frac{\partial^3 \bar{\psi}_1^{(1)}}{\partial x^3} \right) - \frac{\partial \bar{\psi}_1^{(1)}}{\partial x} \frac{d^2 u_0^{(1)}}{dy^2} = 2 \frac{\partial^4 \bar{\psi}_1^{(1)}}{\partial x^2 \partial y^2} + \frac{\partial^4 \bar{\psi}_1^{(1)}}{\partial x^4} + \frac{\partial^4 \bar{\psi}_1^{(1)}}{\partial y^4} - G \frac{\partial \theta_1^{(1)}}{\partial y} \quad (3.20)$$

$$\text{Pr} \left(u_0^{(1)} \frac{\partial \theta_1^{(1)}}{\partial x} + \frac{\partial \bar{\psi}_1^{(1)}}{\partial x} \frac{d\theta_0^{(1)}}{dy} \right) = \frac{\partial^2 \theta_1^{(1)}}{\partial x^2} + \frac{\partial^2 \theta_1^{(1)}}{\partial y^2} \quad (3.21)$$

Region-II

$$u_0^{(2)} \left(\frac{\partial^3 \bar{\psi}_1^{(2)}}{\partial x \partial y^2} + \frac{\partial^3 \bar{\psi}_1^{(2)}}{\partial x^3} \right) - \frac{\partial \bar{\psi}_1^{(2)}}{\partial x} \frac{d^2 u_0^{(2)}}{dy^2} = 2 \frac{\partial^4 \bar{\psi}_1^{(2)}}{\partial x^2 \partial y^2} + \frac{\partial^4 \bar{\psi}_1^{(2)}}{\partial x^4} + \frac{\partial^4 \bar{\psi}_1^{(2)}}{\partial y^4} - \beta m^2 r^2 h^3 G \frac{\partial \theta_1^{(2)}}{\partial y} - mh^2 M^2 \frac{\partial^2 \bar{\psi}_1}{\partial y^2} \quad (3.22)$$

$$u_0^{(2)} \frac{\partial \theta_1^{(2)}}{\partial x} + \frac{\partial \bar{\psi}_1^{(2)}}{\partial x} \frac{d\theta_0^{(2)}}{dy} = \frac{km}{\text{Pr}} \left(\frac{\partial^2 \theta_1^{(1)}}{\partial x^2} + \frac{\partial^2 \theta_1^{(1)}}{\partial y^2} \right) \quad (3.23)$$

Assuming

$$\bar{\psi}_1^{(i)}(x, y) = \varepsilon e^{i\lambda x} \psi^{(i)}(y), \theta_1^{(i)}(x, y) = \varepsilon e^{i\lambda x} t^{(i)}(y) \quad \text{for } i = 1, 2 \quad (3.24)$$

From which we infer

$$u_1^{(i)}(x, y) = \varepsilon e^{i\lambda x} \bar{u}_1^{(i)}(y), v_1^{(i)}(x, y) = \varepsilon e^{i\lambda x} \bar{v}_1^{(i)}(y) \quad \text{for } i = 1, 2 \quad (3.25)$$

And using (3.24) in (3.20) to (3.23), we get

Region-I

$$\psi^{iv} - i\lambda \left[u_0^{(1)} (\psi'' - \lambda^2 \psi) - \frac{d^2 u_0^{(1)}}{dy^2} \psi \right]^{(1)} - \lambda^2 (2\psi'' - \lambda^2 \psi) = G \frac{dt^{(1)}}{dy} \tag{3.26}$$

$$t'' - \lambda^2 t = \text{Pr } i\lambda \left(u_0 t + \psi \frac{d\theta_0}{dy} \right)^{(1)} \tag{3.27}$$

Region-II

$$\psi^{iv} - i\lambda \left[u_0^{(2)} (\psi'' - \lambda^2 \psi) - \frac{d^2 u_0^{(2)}}{dy^2} \psi \right]^{(2)} - mh^2 M^2 \psi'' - \lambda^2 (2\psi'' - \lambda^2 \psi) = \beta m^2 r^2 h^3 G \frac{dt^{(2)}}{dy} \tag{3.28}$$

$$t'' - \lambda^2 t = \frac{\text{Pr}}{km} i\lambda \left(u_0 t + \psi \frac{d\theta_0}{dy} \right)^{(2)} \tag{3.29}$$

Below we restrict our attention to the real parts of the solution for the perturbed quantities $\bar{\psi}_1^{(i)}, \theta_1^{(i)}, u_1^{(i)}$ and $v_1^{(i)}$ for $i=1,2$

The boundary conditions (3.17) and (3.18) can be now written in terms of

$$\frac{\partial \bar{\psi}_1^{(1)}}{\partial y} = -\text{Cos} \lambda x \frac{du_0^{(1)}}{dy}; \quad \frac{\partial \bar{\psi}_1^{(1)}}{\partial x} = 0 \quad \text{at } y = -1$$

$$\frac{\partial \bar{\psi}_1^{(2)}}{\partial y} = 0; \quad \frac{\partial \bar{\psi}_1^{(2)}}{\partial x} = 0 \quad \text{at } y = 1$$

$$\frac{\partial \bar{\psi}_1^{(1)}}{\partial y} = \frac{1}{rmh} \frac{\partial \bar{\psi}_1^{(2)}}{\partial y}; \quad \frac{\partial \bar{\psi}_1^{(1)}}{\partial x} = \frac{1}{rmh} \frac{\partial \bar{\psi}_1^{(2)}}{\partial x} \tag{3.30}$$

at $y = 0$

$$\frac{\partial^2 \bar{\psi}_1^{(1)}}{\partial y^2} + \frac{\partial^2 \bar{\psi}_1^{(1)}}{\partial x^2} = \frac{1}{rm^2 h^2} \left(\frac{\partial^2 \bar{\psi}_1^{(2)}}{\partial y^2} + \frac{\partial^2 \bar{\psi}_1^{(2)}}{\partial x^2} \right) \tag{3.30}$$

at $y = 0$

$$t^{(1)} = -\frac{d\theta_0^{(1)}}{dy} \quad \text{at } y = -1$$

$$t^{(2)} = 0 \quad \text{at } y = 1$$

$$t^{(1)} = t^{(2)} \quad \text{at } y = 0 \text{ for } i \geq 1$$

$$\frac{dt^{(1)}}{dy} = \frac{dt^{(2)}}{dy} \quad \text{at } y = 0 \tag{3.31}$$

If we consider small values of λ then substituting

$$\bar{\psi}^{(i)}(\lambda, y) = \psi_0^{(i)} + \lambda \psi_1^{(i)} + \lambda^2 \psi_2^{(i)} + \dots$$

$$t^{(i)}(\lambda, y) = t_0^{(i)} + \lambda t_1^{(i)} + \lambda^2 t_2^{(i)} + \dots$$

for $i=1,2$ (3.32) in to (3.26) to (3.31) gives, to order of λ , the following sets of ordinary differential equations and corresponding boundary and interface conditions

Region-I

$$\frac{d^4 \psi_0^{(1)}}{dy^4} = G \frac{dt_0^{(1)}}{dy} \tag{3.33}$$

$$\frac{d^2 t_0^{(1)}}{dy^2} = 0 \tag{3.34}$$

$$\frac{d^4 \psi_1^{(1)}}{dy^4} - i \left(u_0 \frac{d^2 \psi_0}{dy^2} - \frac{d^2 u_0}{dy^2} \psi_0 \right)^{(1)} = G \frac{dt_1^{(1)}}{dy}, \tag{3.35}$$

$$\frac{d^2 t_1^{(1)}}{dy^2} = \text{Pr } i \left(u_0 t_0 + \psi_0 \frac{d\theta_0}{dy} \right)^{(1)} \tag{3.36}$$

Region-II

$$\frac{d^4 \psi_0^{(2)}}{dy^4} - M^2 mh^2 \frac{d^2 \psi_0^{(2)}}{dy^2} = \beta m^2 r^2 h^3 G \frac{dt_0^{(2)}}{dy} \tag{3.37}$$

$$\frac{d^2 t_0^{(2)}}{dy^2} = 0 \tag{3.38}$$

$$\frac{d^4\psi_1^{(2)}}{dy^4} - M^2mh^2 \frac{d^2\psi_1^{(2)}}{dy^2} = i \left(u_0 \frac{d^2\psi_0}{dy^2} - \frac{d^2u_0}{dy^2} \psi_0 \right)^{(2)} + \beta m^2 r^2 h^3 G \frac{dt_1^{(2)}}{dy} \quad (3.39)$$

$$\frac{d^2t_1^{(2)}}{dy^2} = \frac{Pr}{km} i \left(u_0 t_0 + \psi_0 \frac{d\theta_0}{dy} \right)^{(2)} \quad (3.40)$$

and

$$\frac{d\psi_0^{(1)}}{dy} = \text{Cos}\lambda x \frac{du_0^{(1)}}{dy}; \quad \psi_0^{(1)} = 0 \quad \text{at } y = -1$$

$$\frac{d\psi_0^{(2)}}{dy} = 0; \quad \psi_0^{(2)} = 0 \quad \text{at } y = 1$$

$$\frac{d\psi_0^{(1)}}{dy} = \frac{1}{rmh} \frac{d\psi_0^{(2)}}{dy}; \quad \psi_0^{(1)} = \frac{1}{rmh} \psi_0^{(2)} \text{ at } y = 0$$

$$\frac{d^2\psi_0^{(1)}}{dy^2} = \frac{1}{rm^2h^2} \frac{d^2\psi_0^{(2)}}{dy^2}; \quad \psi_0^{(1)} = \frac{1}{rm^2h^2} \psi_0^{(2)} \text{ at } y = 0 \quad (3.41)$$

$$\frac{d\psi_1^{(1)}}{dy} = 0; \quad \psi_1^{(1)} = 0 \quad \text{at } y = -1$$

$$\frac{d\psi_1^{(2)}}{dy} = 0; \quad \psi_1^{(2)} = 0 \quad \text{at } y = 1$$

$$\frac{d\psi_1^{(1)}}{dy} = \frac{1}{rmh} \frac{d\psi_1^{(2)}}{dy}; \quad \psi_1^{(1)} = \frac{1}{rmh} \psi_1^{(2)} \text{ at } y = 0$$

$$\frac{d^2\psi_1^{(1)}}{dy^2} = \frac{1}{rm^2h^2} \frac{d^2\psi_1^{(2)}}{dy^2}; \quad \psi_1^{(1)} = \frac{1}{rm^2h^2} \psi_1^{(2)} \text{ at } y = 0 \quad (3.42)$$

$$t_0^{(1)} = -\frac{d\theta_0^{(1)}}{dy} \quad \text{at } y = -1$$

$$t_0^{(2)} = 0 \quad \text{at } y = 1$$

$$t_0^{(1)} = t_0^{(2)} \quad \text{at } y = 0$$

$$\frac{dt_0^{(1)}}{dy} = \frac{k}{h} \frac{dt_0^{(2)}}{dy} \quad \text{at } y = 0 \quad (3.43)$$

$$t_1^{(1)} = 0 \quad \text{at } y = -1$$

$$t_1^{(2)} = 0 \quad \text{at } y = 1$$

$$t_1^{(1)} = t_1^{(2)} \quad \text{at } y = 0 \quad \text{for } i \geq 1$$

$$\frac{dt_1^{(1)}}{dy} = \frac{dt_1^{(2)}}{dy} \quad \text{at } y = 0 \quad (3.44)$$

3.3 Zeroth-Order Solution (Mean Part)

The solutions to zeroth order differential Eqs. (3.3) to (3.6) using boundary and interface conditions (3.15) and (3.16) are given by

$$t_0^{(1)} = C_5 y + C_6$$

Region-I

$$u_0^{(1)} = l_1 y^4 + l_2 y^3 + l_3 y^2 + A_1 y + A_2$$

$$\theta_0^{(1)} = d_1 y^2 + C_1 y + C_2$$

Region-II

$$u_0^{(2)} = B_1 \cosh ny + B_2 \sinh ny + s_1 y^2 + s_2 y + s_3$$

$$\theta_0^{(2)} = f_2 y^2 + C_3 y + C_4$$

The solutions of zeroth and first order of λ are obtained by solving the equation (3.33) to (3.40) using boundary and interface conditions (3.41) and (3.42) and are given below

$$\psi_0^{(1)} = l_4 y^4 + \frac{A_3}{6} y^3 + \frac{A_4}{2} y^2 + A_5 y + A_6$$

$$\psi_0^{(2)} = B_3 \cosh ny + B_4 \sinh ny + B_5 y + B_6 + s_4 y^2$$

$$t_0^{(2)} = C_7 y + C_8$$

$$\psi_1^{(1)} = i \left(l_{20} y^{10} + l_{21} y^9 + l_{22} y^8 + l_{23} y^7 + l_{24} y^6 + l_{25} y^5 + l_{26} y^4 \right) + \frac{A_7}{6} y^3 + \frac{A_8}{2} y^2 + A_9 y + A_{10}$$

$$t_1^{(1)} = i \text{Pr} (d_8 y^7 + d_9 y^6 + d_{10} y^5 + d_{11} y^4 + d_{12} y^3 + d_{13} y^2) + q_6 y + q_8$$

$$\psi_1^{(2)} = B_7 \cosh ny + B_8 \sinh ny + B_7 y + B_{10} + i (s_{29} y^3 \cosh ny + s_{30} y^3 \sinh ny + s_{31} y^2 \cosh ny + s_{32} y^2 \sinh ny + s_{33} y \cosh ny + s_{34} y \sinh ny + s_{35} \cosh ny + s_{36} \sinh ny + S_{37} y^6 + S_{38} y^5 + S_{39} y^4 + S_{40} y^3 + S_{41} y^2)$$

$$t_1^{(2)} = i \frac{\text{Pr}}{km} (f_{11} y \cosh ny + f_{12} y \sinh ny + f_{13} \cosh ny + f_{14} \sinh ny + f_{15} y^5 + f_{16} y^4 + f_{17} y^3 + f_{18} y^2) + q_5 y + q_7$$

The first order quantities can be put in the forms

$$u_1 = \psi'_i \sin \lambda x - \psi'_r \cos \lambda x$$

$$v_1 = \lambda \psi'_r \sin \lambda x - \lambda \psi'_i \cos \lambda x$$

$$\theta_1 = t_r \cos \lambda x - t_i \sin \lambda x$$

Region-I

$$u_1^{(1)} = -\cos \lambda x (\psi'_{0r} + \lambda \psi'_{1r})^{(1)} + \sin \lambda x (\psi'_{0i} + \lambda \psi'_{1i})^{(1)}$$

$$v_1^{(1)} = -\lambda \cos \lambda x (\psi_{0i} + \lambda \psi_{1i})^{(1)} - \lambda \sin \lambda x (\psi_{0r} + \lambda \psi_{1r})^{(1)}$$

$$\theta_1^{(1)} = \cos \lambda x (t_{0r} + \lambda t_{1r})^{(1)} - \sin \lambda x (t_{0i} + \lambda t_{1i})^{(1)}$$

Region-II

$$u_1^{(2)} = -\cos \lambda x (\psi'_{0r} + \lambda \psi'_{1r})^{(2)} + \sin \lambda x (\psi'_{0i} + \lambda \psi'_{1i})^{(2)}$$

$$v_1^{(2)} = -\lambda \cos \lambda x (\psi_{0i} + \lambda \psi_{1i})^{(2)} - \lambda \sin \lambda x (\psi_{0r} + \lambda \psi_{1r})^{(2)}$$

$$\theta_1^{(2)} = \cos \lambda x (t_{0r} + \lambda t_{1r})^{(2)} - \sin \lambda x (t_{0i} + \lambda t_{1i})^{(2)}$$

4. RESULTS AND DISCUSSION

4.1 Discussion of the Zeroth Order Solution:

The effect of Hartmann number M on zeroth order velocity is to decrease the velocity for $\bar{\theta} = 0, \pm 1$, but the suppression is more effective near the flat wall as M increases as shown in figure 2.

The effect of heat source ($\alpha > 0$) or sink ($\alpha < 0$) and in the absence of heat source or sink ($\alpha = 0$), on zeroth order velocity is shown in figure 3 for $\bar{\theta} = 0, \pm 1$. It is observed that heat source promote the flow, sink suppress the flow, and the velocity profiles lie in between source or sink for $\alpha = 0$. We also observe that the magnitude of zeroth order velocity is optimum for $\bar{\theta} = 1$ and minimal for $\bar{\theta} = -1$, and profiles lies between $\bar{\theta} = \pm 1$ for $\bar{\theta} = 0$. The effect of source or sink parameter α on zeroth order temperature is similar to that on zeroth order velocity as shown in figure 4. The effect of free convection parameter G , viscosity ratio m , width ratio h , on zeroth order velocity and the effect of width ratio h , conductivity ratio k , on zeroth order temperature remain the same as explained in chapter-III

The effect of free convection parameter G on first order velocity is shown in figure.5. As G increases u_1 increases near the wavy and flat wall where as it decreases at the interface and the suppression is effective near the flat wall for $\bar{\theta} = 0, \pm 1$. The effect of viscosity ratio m on u_1 shows that as m increases first order velocity increases near the wavy and flat wall, but the magnitude is very large near flat wall. At the interface velocity decreases as m increases and the suppression is significant towards the flat wall as seen in figure 6 for $\bar{\theta} = 0, \pm 1$. The effect of width ratio h on first order velocity u_1 shows that u_1 remains almost same for $h < 1$ but is more effective for $h > 1$. For $h = 2$, u_1 increases near the wavy and flat wall and drops at the interface, for $\bar{\theta} = 0, \pm 1$ as seen in figure 7.

The effect of Hartmann number M on u_1 shows that as M increases velocity decreases at the wavy wall and the flat wall but the suppression near the flat wall compared to wavy wall is insignificant and as M increases u_1 increases in magnitude at the interface for $\bar{\theta} = 0, \pm 1$ as seen in figure 8. The effect of α on u_1 is shown in figure 9, which shows that velocity is large near the wavy and flat wall for heat source $\alpha = 5$ and is less for heat sink $\alpha = -5$. Similar result is obtained at the interface but for negative values of u_1 . Here also we observe that the magnitude is very large near the wavy wall compared to flat wall.

The effect of convection parameter G , viscosity ratio m , width ratio h , thermal conductivity ratio k on first order velocity v_1 is similar as explained in chapter III. The effect of Hartmann number M is to increase the velocity near the wavy wall and decrease velocity near the flat wall for $\bar{\theta} = 0, \pm 1$, whose results are applicable to flow reversal problems as shown in figure.10. The effect of source or sink on first order velocity v_1 is shown in figure 11. For heat source, v_1 is less near wavy wall and more for flat wall where as we obtained the opposite result for sink i.e. v_1 is maximum near wavy wall and minimum near the flat wall, for $\alpha = 0$ the profiles lie in between $\alpha = \pm 5$

The effect of convection parameter G , viscosity ratio m , width ratio h , thermal conductivity ratio k , Hartmann number and source and sink parameter α on first order temperature are shown in figures 12 to 17. It is seen that G , m , h , and k increases in magnitude for values of $\bar{\theta} = 0, \pm 1$. It is seen that from figure 16 that as M increases the magnitude of θ_1 decreases for $\bar{\theta} = 0, \pm 1$. Figure 17 shows that the magnitude of α is large for heat source and is less for sink whereas θ_1 remains invariant for $\alpha = 0$.

The effect of convection parameter G , viscosity ratio m , width ratio h , and thermal conductivity ratio k , on total velocity remains the same as explained in chapter III. The effect of heat source or sink on total velocity shows that U is very large for $\alpha = 5$ compared to $\alpha = -5$ and is almost invariant for $\alpha = 0$ as seen figure 18, for all values of $\bar{\theta}$.

The effect of Grashof number G , viscosity ratio m , shows that increasing G and m suppress the total temperature but the suppression for m is negligible as seen in figures.19 and 20. The effects of width ratio h and conductivity ratio k is same as explained in chapter-III. The effect of source or sink parameter on total temperature remains the same as that on total velocity as seen figure.21.

Where

$$d_1 = -\alpha/2; \quad f_1 = -\alpha Qh^2/k; \quad f_2 = f_1/2$$

$$C_3 = -\frac{(1-\theta-d_1+f_2)h}{(k+h)}; \quad C_4 = \theta - C_3 - f_2;$$

$$C_2 = C_4 \quad C_1 = C_2 - 1 + d_1; \quad l_1 = -\frac{Gd_1}{12};$$

$$l_2 = -\frac{GC_1}{6}; \quad l_3 = -\frac{GC_2}{2} \quad s_1 = \frac{\beta m^2 r^2 h^3 G f_2}{n^2};$$

$$s_2 = \frac{\beta m^2 r^2 h^3 G C_3}{n^2} \quad s_3 = \frac{2\beta m^2 r^2 h^3 G f_2}{n^4} + \frac{\beta m^2 r^2 h^3 G C_4}{n^2}$$

$$l_4 = \frac{G C_5}{24}; \quad s_4 = -\frac{\beta m^2 r^2 h^3 G_7}{n^2} \quad l_5 = \frac{6+n^2}{rm^2 h^2}; \quad l_6 = -\frac{4n}{rmh};$$

$$l_7 = -\frac{4}{rmh}; \quad l_8 = \frac{6}{rm^2 h^2}$$

$$l_9 = 2A_1 + 2l_4 - \frac{6s_4}{rm^2 h^2} - 8l_1 + 6l_2 - 4l_3$$

$$l_{10} = (\cosh n - n \sinh n)(l_7 n \cosh n - l_6) - (l_7 n \sinh n - l_5)(\sinh n - n \cosh n)$$

$$l_{11} = (l_7 n \cosh n - l_6) + l_8 (\sinh n - n \cosh n)$$

$$l_{12} = l_9 (\sinh n - n \cosh n) - s_4 (l_7 n \cosh n - l_6)$$

$$d_2 = l_1 C_5 - \alpha l_4; \quad d_3 = l_2 C_5 + l_1 C_6 - \frac{\alpha A_3}{6} + C_1 l_4$$

$$d_4 = l_3 C_5 + l_2 C_8 - \frac{\alpha A_4}{2} + \frac{C_1 A_3}{6};$$

$$d_5 = A_1 C_5 + l_3 C_6 - \alpha A_4 + \frac{C_1 A_4}{2}$$

$$d_6 = A_2 C_5 + A_1 C_6 - \alpha A_6 + C_1 A_3;$$

$$d_7 = A_2 C_6 + C_1 A_6 \quad d_8 = d_2 / 42;$$

$$d_9 = d_3 / 30; \quad d_{10} = d_4 / 20$$

$$d_{11} = d_5 / 12; \quad d_{12} = d_6 / 6;$$

$$d_{13} = d_7 / 2 \quad f_3 = B_1 C_7 + B_3 f_1;$$

$$f_4 = B_2 C_7 + B_4 f_1; \quad f_5 = B_1 C_8 + B_3 C_3$$

$$f_6 = B_2 C_8 + B_4 C_3; \quad f_7 = s_1 C_7 + s_4 C_1$$

$$f_8 = s_2 C_7 + s_1 C_8 + B_5 f_1 + s_4 C_3;$$

$$f_9 = s_3 C_7 + s_2 C_8 + B_6 f_1 + B_5 C_3$$

$$f_{10} = s_3 C_8 + C_3 B_6; \quad f_{11} = f_3 / n^2;$$

$$f_{12} = f_4 / n^2 \quad f_{13} = -\left(2 \frac{f_4}{n^3} - \frac{f_5}{n^2}\right);$$

$$f_{14} = -\left(2 \frac{f_3}{n^3} - \frac{f_6}{n^2}\right) \quad f_{15} = f_7 / 20$$

$$f_{16} = f_8 / 12; \quad f_{17} = f_9 / 6;$$

$$f_{18} = f_{10} / 2$$

$$q_1 = \Pr(-d_8 + d_9 - d_{10} + d_{11} - d_{12} + d_3)$$

$$q_2 = \frac{\Pr}{km} \left((f_{11} + f_{13}) \cosh n + (f_{12} + f_{14}) \sinh n + \right. \\ \left. f_{15} + f_{16} + f_{17} + f_{18} \right)$$

$$q_3 = \frac{\Pr}{km} (f_{13}); \quad q_4 = \frac{k \Pr}{h km} (f_{14} n + f_{11})$$

$$q_5 = \frac{(q_1 - q_2 + q_3 - q_4) h}{k + h}; \quad q_6 = \frac{k q_5}{h} + q_4;$$

$$q_7 = -(q_5 + q_2); \quad q_8 = (q_6 - q_1); \quad l_{13} = G \Pr \frac{d_2}{6};$$

$$l_{14} = l_1 A_3 + 12 l_2 l_4 - 2 l_1 A_3 - 6 l_2 l_4 + G \Pr \frac{d_3}{5}$$

$$l_{15} = l_1 A_4 + l_2 A_3 + 12 l_3 l_4 - 6 l_1 A_4 - l_2 A_3 - 2 l_3 l_4 + G \Pr \frac{d_4}{4}$$

$$l_{16} = l_2 A_4 + l_3 A_3 + 12 A_1 l_4 - 12 l_1 A_5 -$$

$$3 l_2 A_4 - l_3 \frac{A_3}{3} + G \Pr \frac{d_5}{3}$$

$$l_{17} = l_3 A_4 + A_1 A_3 + 12 A_2 l_4 - 12 l_1 A_6 -$$

$$6 l_2 A_5 - l_3 A_4 + G \Pr \frac{d_6}{2}$$

$$l_{18} = A_1 A_4 + A_2 A_3 - 6 l_2 A_6 - 2 l_3 A_5 + G \Pr d_7;$$

$$l_{19} = A_2 A_4 - 2 l_3 A_6 + G q_6$$

$$l_{20} = \frac{l_{13}}{5040}; \quad l_{21} = \frac{l_{14}}{3024}; \quad l_{22} = \frac{l_{15}}{1680};$$

$$l_{23} = \frac{l_{16}}{840}; \quad l_{24} = \frac{l_{17}}{360}; \quad l_{25} = \frac{l_{18}}{120};$$

$$l_{26} = \frac{l_{19}}{24}; \quad s_5 = s_1 B_3 n^2 - B_1 s_4 n^2;$$

$$s_6 = s_1 B_4 n^2 - B_2 s_4 n^2;$$

$$s_7 = \left(s_2 B_3 n^2 - B_1 B_5 n^2 + \beta m^2 r^2 h^3 \frac{\Pr}{km} f_{12} \right)$$

$$s_8 = \left(s_2 B_4 n^2 - B_2 B_5 n^2 + \beta m^2 r^2 h^3 \frac{\Pr}{km} f_{11} \right)$$

$$s_9 = (s_3 B_3 n^2 - B_1 B_6 n^2 - B_1 s_4 n^2 - 2 s_1 B_3 +$$

$$\beta m^2 r^2 h^3 \frac{\Pr}{km} (f_{11} + f_{14} n)) + 2 B_1 s_4$$

$$s_{10} = (s_3 B_4 n^2 - B_2 B_6 n^2 - B_2 s_4 n^2 - 2 s_1 B_4 +$$

$$\beta m^2 r^2 h^3 \frac{\Pr}{km} (f_{12} + f_{13} n)) + 2 B_2 s_4$$

$$s_{11} = 5 f_{15} \beta m^2 r^2 h^3 G \frac{\Pr}{km}; \quad s_{12} = 4 f_{16} \beta m^2 r^2 h^3 G \frac{\Pr}{km}$$

$$s_{13} = 3 f_{17} \beta m^2 r^2 h^3 G \frac{\Pr}{km};$$

$$s_{14} = 2 f_{18} \beta m^2 r^2 h^3 G \frac{\Pr}{km} - 2 s_1 B_5 + 2 s_2 s_4$$

$$s_{15} = -2 s_1 B_6 - 2 s_1 s_4 + \beta m^2 r^2 h^3 G q_5 + 2 s_3 s_4; \quad s_{16} = \frac{s_6}{6n};$$

$$s_{17} = \frac{s_5}{6n}; \quad s_{18} = -\frac{s_5}{4n^2} + \frac{s_8}{4n}; \quad s_{19} = -\frac{s_6}{4n^2} + \frac{s_7}{4n};$$

$$s_{20} = \frac{s_6}{4n^3} - \frac{s_7}{4n^2} + \frac{s_{10}}{2n}$$

$$s_{21} = \frac{s_5}{4n^3} - \frac{s_8}{4n^2} + \frac{s_9}{2n}; \quad s_{22} = -\frac{s_5}{8n^4} + \frac{s_8}{8n^3};$$

$$s_{23} = -\frac{s_6}{8n^4} + \frac{s_7}{8n^3}; \quad s_{24} = -\frac{s_{11}}{n^2}; \quad s_{25} = -\frac{s_{12}}{n^2};$$

$$s_{26} = -\frac{12s_{11}}{n^4} - \frac{s_{13}}{n^2}; \quad s_{27} = -\frac{6s_{12}}{n^4} - \frac{s_{14}}{n^2}$$

$$s_{28} = -\frac{24s_{11}}{n^6} - \frac{2s_{13}}{n^4} - \frac{s_{15}}{n^2}; \quad s_{29} = \frac{s_{16}}{n^2};$$

$$s_{30} = \frac{s_{17}}{n^2}; \quad s_{31} = -\frac{3s_{17}}{n^3} - \frac{3s_{17}}{n^4} + \frac{s_{18}}{n^2};$$

$$s_{32} = -\frac{3s_{16}}{n^3} - \frac{3s_{16}}{n^4} + \frac{s_{19}}{n^2}$$

$$s_{33} = \frac{6s_{16}}{n^4} + \frac{12s_{16}}{n^5} + \frac{6s_{17}}{n^4} - \frac{4s_{19}}{n^3} - \frac{s_{20}}{n^2};$$

$$s_{34} = \frac{6s_{16}}{n^4} + \frac{12s_{17}}{n^5} + \frac{6s_{17}}{n^4} - \frac{4s_{18}}{n^3} - \frac{s_{21}}{n^2}$$

$$s_{35} = \frac{-12s_{16}}{n^5} - \frac{6s_{17}}{n^5} - \frac{18s_{17}}{n^6} + \frac{6s_{18}}{n^4} - \frac{2s_{21}}{n^3} + \frac{s_{22}}{n^2}$$

$$s_{36} = \frac{-12s_{17}}{n^5} - \frac{6s_{16}}{n^5} - \frac{18s_{16}}{n^6} + \frac{6s_{19}}{n^4} - \frac{2s_{20}}{n^3} + \frac{s_{23}}{n^2}$$

$$s_{37} = \frac{s_{24}}{30}; \quad s_{38} = \frac{s_{25}}{20}; \quad s_{39} = \frac{s_{26}}{12}; \quad s_{40} = \frac{s_{27}}{6}$$

$$s_{41} = \frac{s_{28}}{2}$$

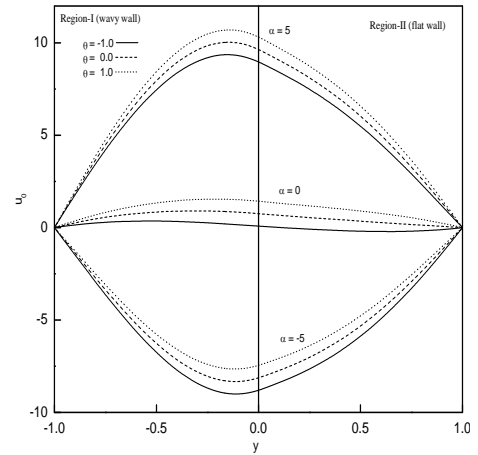


Fig. 3 Zeroth order velocity profiles for different values of α

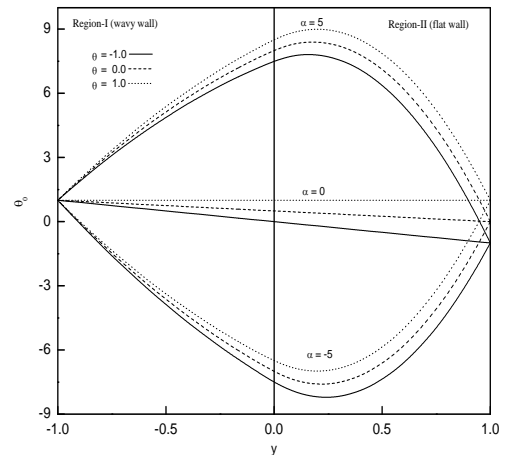


Fig. 4 Zeroth order temperature profiles for different values of α

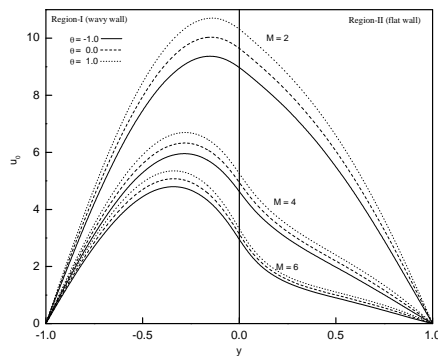


Fig. 2 Zeroth order velocity profiles for different values of Hartmann number M

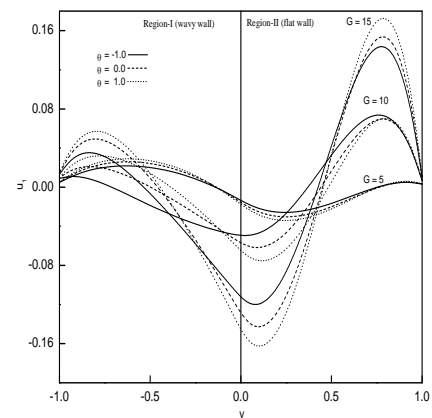


Fig. 5 First order velocity profiles for different values of Grashof number G

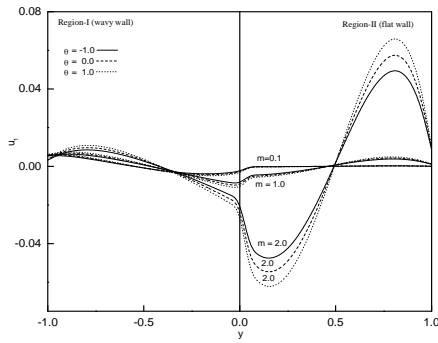


Fig. 6 First order velocity profiles for different values of viscosity ratio m

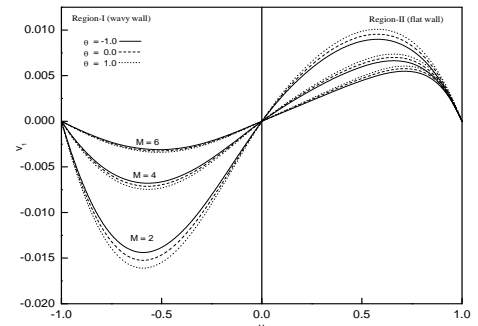


Fig. 10 First order velocity profiles for different values of M

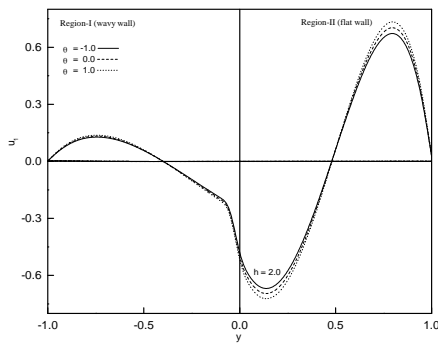


Fig. 7 First order velocity profiles for different values of width ratio h

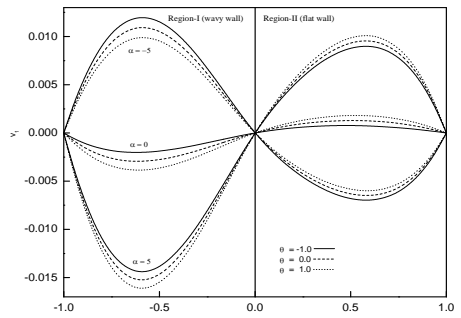


Fig. 11 First order velocity profiles for different values of α

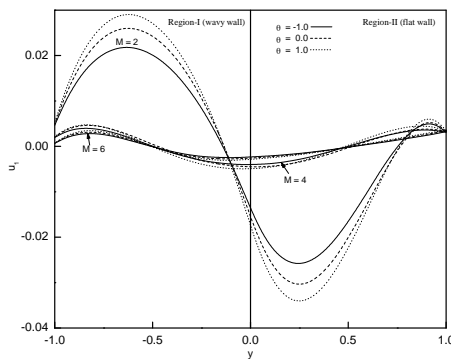


Fig. 8 First order velocity profiles for different values of M

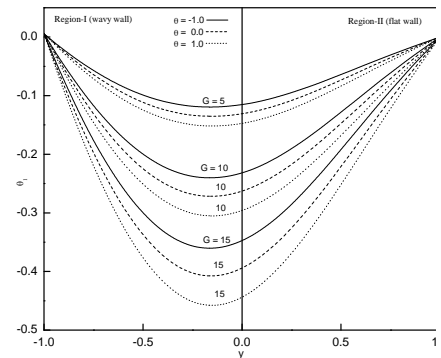


Fig. 12 First order temperature profiles for different values of G

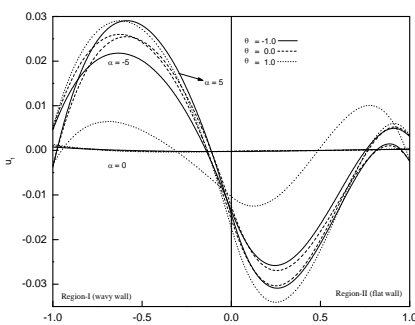


Fig. 9 First order velocity profiles for different values of α

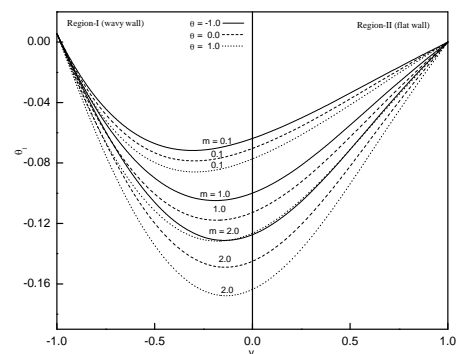


Fig. 13 First order temperature profiles for different values of m

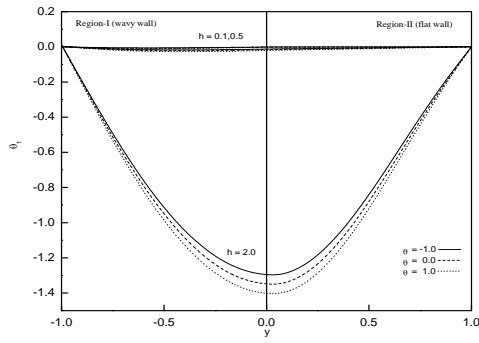


Fig. 14 First order temperature profiles for different values of h

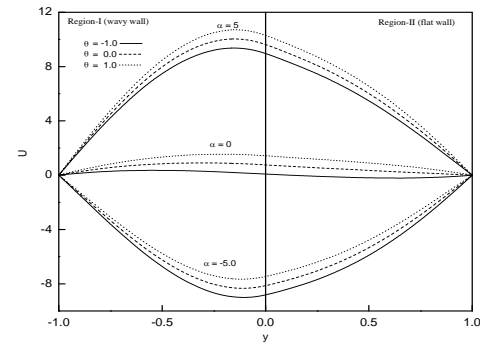


Fig. 18 Total solution of velocity profiles for different values of α

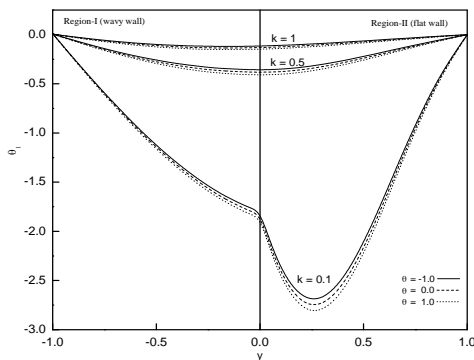


Fig. 15 First order temperature profiles for different values of k

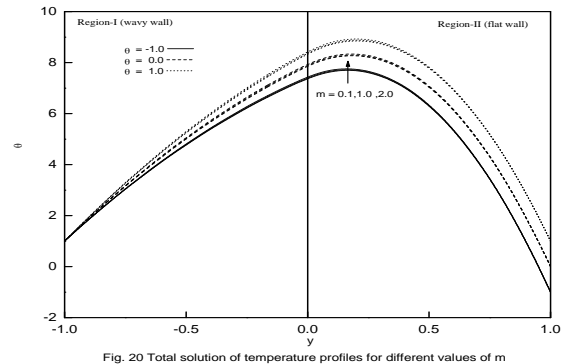


Fig. 20 Total solution of temperature profiles for different values of m

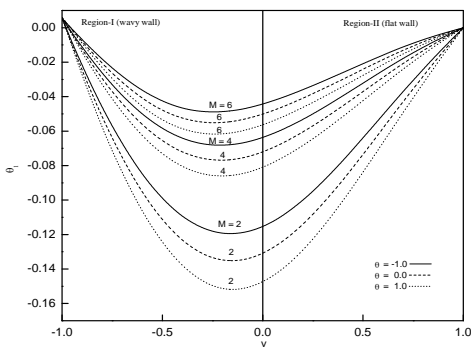


Fig. 16 First order temperature profiles for different values of M

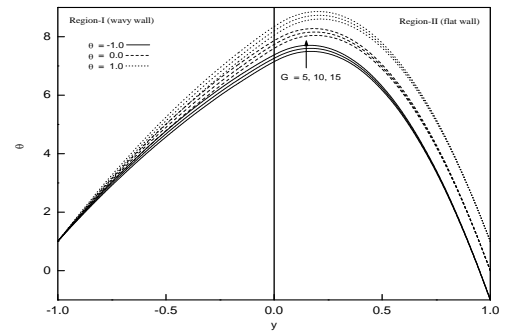


Fig. 19 Total solution of temperature profiles for different values of Grashof number G

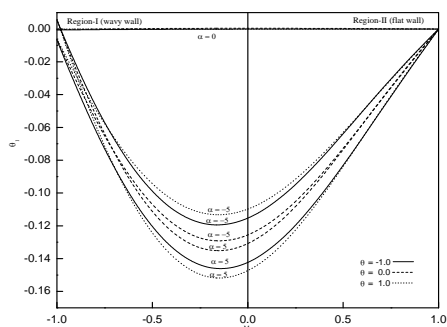


Fig. 17 First order temperature profiles for different values of α

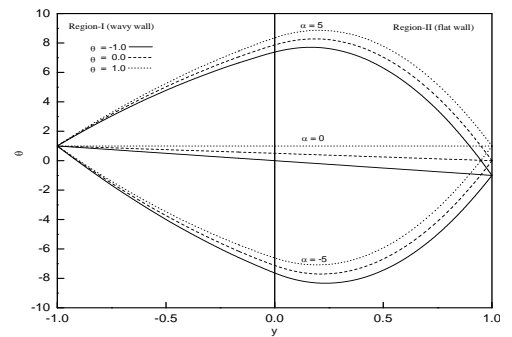


Fig. 21 Total solution of temperature profiles for different values of α

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