BLOOD FLOW THROUGH STENOSED INCLINED TUBES WITH PERIODIC BODY ACCELERATION IN THE PRESENCE OF MAGNETIC FIELD AND ITS APPLICATIONS TO CARDIOVASCULAR DISEASES

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Abstract

This is about the mathematical model for blood flow through stenosed inclined tubes with periodic body acceleration and magnetic field and its application to cardiovascular diseases in biomedical engineering. It is observed that the velocity and volumetric flow rate decreases with increase in Hartmann number and for a particular value of phase angle, the value of shear stress increases with increase in Hartmann number .All these are studied in inclined tubes.

Keywords: *Blood flow, Stenosis, Periodic body acceleration, magnetic field, cardiovascular diseases, inclined tubes*

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1. INTRODUCTION

Pulsatile flow of blood with periodic body acceleration is studied by Chaturani. P and Palanisamy. V. [1}.Here pulsatile flow of blood through a rigid tube has been studied under the influence of body acceleration. Sud.V.K. and Sekhon .G.S.,[2] studied Arterial flow under periodic body acceleration. The study deals with the effect of externally imposed body accelarations on blood flow in arteries. Rathod and Gopichand[3] studied Pulsatile flow of blood through a stenosed tube under periodic body acceleration with magnetic field. Rathod et al [4] studied Pulsatile flow of blood under the periodic body acceleration with magnetic field. ElShahawey et al [5] studied MHD flow of an elastic-viscous fluid under periodic body acceleration. El-Shahaweyet al [6]studied Pulsatile flow of blood through a porous medium under periodic body acceleration .Coklet.G.R.[7] studied The Rheology of Human blood. Vardanyan.V.A[8]studied the Effect of magnetic field on blood flow .Bhuvan.B.C.and Hazarika.G.C.[9]studied the Effect of magnetic field on Pulsatile flow of blood in a porous channel. Chaturani.P.and Biswas[10] studied A Comparative study of two layered blood flow models with different boundary conditions.Berger.S.A.et al [11] studied Flows in stenotic vessels. Young.d.F[12] studied the Fluid mechanics of arterial stenosis. K.Das and G.C.Saha[13] studied an Arterial MHD Pulsatile flow of blood under the periodic body acceleration. D.C.Sanyal et al [14] were studied the Effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration. Further the Flow of Casson fluid through an inclined tube of non-uniform cross section with multiple stenoseshas been studied by Pelagia research library under advances in applied science[15].Gaurav Mishra et al [16] an Oscillatory blood flow through porous medium in a stenosed artery .

In this paper, using finite Hankel and Laplace transforms, analytical expressions for velocity profile, volumetric flow rate and wall shear stress have been obtained and their natures are portrayed graphically for different parameters such as Hartmann number, phase angle, time etc. in an inclined tube under stenoses.

2. MATHEMATICAL FORMULATION

Let us consider the axially symmetric and fully developed pulsatile flow of blood through a stenosed porous circular artery with body acceleration under the influence of uniform transverse magnetic field. Blood is assumed to be Newtonian and incompressible fluid. Also for mathematical model, we take the artery to be a long cylindrical tube with the axis along z-axis. The pressure gradient and body acceleration are respectively given by

$$-\frac{\partial P}{\partial z} = A_0 + A_1 \cos(\omega_P t) \tag{1}$$

$$G = a_0 \cos(\omega_b t + \phi) \tag{2}$$

where A_0 and A_1 are pressure gradient of steady flow and amplitude of oscillatory part respectively, a_0 is the amplitude of body acceleration, $\omega_P = 2\pi f_p$, $\omega_b = 2\pi f_b$

with f_p is the pulse frequency and f_b is body acceleration frequency, ϕ is the phase angle of body acceleration G with respect to pressure gradient and t is time.

The governing equation of motion for flow in cylindrical polar coordinates is given by

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial P}{\partial z}+\rho G+\mu\nabla^2 u-\frac{\mu}{k}u$$
$$-\sigma B_0 u+\rho g\sin\theta \qquad (3)$$

where u is the axial velocity of blood; P, blood pressure; $\frac{\partial P}{\partial z}$,pressure gradient; ρ, density of blood; μ ,the viscosity of blood; k ,the permeability of the isotropic porous medium; B_0 ,the external magnetic field along the radial direction and σ is the conductivity of blood.

The geometry of stenosis is shown in figure-1.

$$R(z)=\begin{cases} a-\delta(1+\cos\frac{\pi z}{2z_0}), -2z_0\le z\le 2z_0 \\ a, otherwise\end{cases}$$

Where $R(z)$ is the radius of the stenosed artery, a is the radius of artery, $4z_0$ is the length of stenosis and 2δ is the maximum protuberance of the stenotic form of the artery wall.

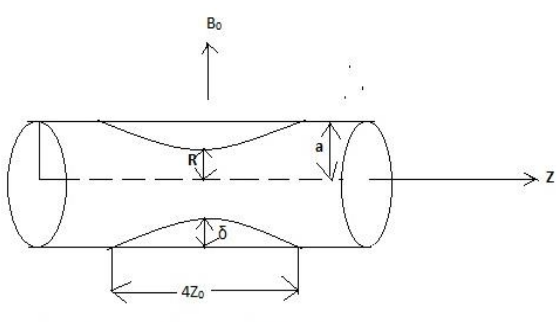

Fig.1.Geometry of artery with stenosis

$$\xi=\frac{r}{R(z)}$$

Where $R(z)$ depends on δ .

The equation (3) becomes

$$\rho\frac{\partial u}{\partial t}=A_0+A_1\cos(\omega_p t)+\rho a_0\cos(\omega_b t+\phi)$$
$$+\frac{\mu}{R^2}[\frac{\partial^2 u}{\partial\xi^2}+\frac{1}{\xi}\frac{\delta u}{\delta\xi}]-\mu C^2 u+\rho g\sin\theta \qquad (4)$$

Where

$$C=\sqrt{\frac{1}{k}+\frac{M^2}{R^2}}, \quad M=\sqrt{\frac{\sigma}{\mu}}RB_0(Hartmann\ number)$$

We assumed that t<0 only the pumping action of the heart is present and at t=0 , the flow in the artery corresponds to the instantaneous pressure gradient i.e.,

$$-\frac{\partial P}{\partial z}=A_0+A_1$$

As a result, the flow velocity at t=0 is given by

$$u(\xi,0)=\frac{A_0+A_1}{\mu C^2}\left[1-\frac{I_0(CR\xi)}{I_0(CR)}\right] \qquad (5)$$

Where I_0 is modified Bessel function of first kind of order zero

The initial and boundary conditions to the problem are

$$u(\xi,0)=\frac{A_0+A_1}{\mu C^2}\left[1-\frac{I_0(CR\xi)}{I_0(CR)}\right]$$
$$u=0 \quad at\ \xi=1$$
$$u\ is\ finite \quad at\ \xi=0 \qquad (6)$$

3. SOLUTIONS

Applying Laplace transform to equation (4) and first boundary condition of (6), We get

$$\rho s\bar{u}-\frac{\rho(A_0+A_1)}{\mu C^2}\left[1-\frac{I_0(CR\xi)}{I_0(CR)}\right]=\frac{A_0}{s}+\frac{A_1}{(s^2+\omega_P{}^2)}+\frac{\rho a_0(s\cos\phi-\omega_b\sin\phi)}{(s^2+\omega_b{}^2)}+\frac{\mu}{R^2}\left[\frac{\partial^2\bar{u}}{\partial\xi^2}+\frac{1}{\xi}\frac{\delta\bar{u}}{\delta\xi}\right]-\mu C^2\bar{u}+\frac{\rho g\sin\theta}{s} \quad (7)$$

Where

$$\bar{u}(\xi,s)=\int_0^\infty e^{-st}u(\xi,t)dt(s\succ 0)$$

Then applying the finite Hankel transform to equation (7), We obtain

$$\bar{u}^*(\lambda_n,s)=\frac{J_1(\lambda_n)R^2}{\lambda_n[\rho sR^2+\mu(C^2R^2+\lambda_n{}^2)]}\left[\frac{A_0}{s}+\frac{A_1}{(s^2+\omega_P{}^2)}+\frac{\rho a_0(s\cos\phi-\omega_b\sin\phi)}{(s^2+\omega_b{}^2)}+\frac{\rho(A_0+A_1)R^2}{\mu(C^2R^2+\lambda_n{}^2)}+\frac{\rho g\sin\theta}{s}\right. \quad (8)$$

Where

$$\bar{u}^*(\lambda_n,s)=\int_0^1 ru(r,s)J_0(r\lambda_n)dr$$

and

λ_n are zeros of J_0,Bessel function of first kind and $\nu=\dfrac{\mu}{\rho}$

The Laplace and Hankel inversions of equation (8)give the final solution for blood velocity as

$$\begin{aligned}u(\xi,t)=2\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{J_1(\lambda_n)}[\{&\frac{(A_0+g\sin\theta)R^2}{\mu(\lambda_n{}^2+C^2R^2)}\\&+\frac{A_1R^2[\nu(\lambda_n{}^2+C^2R^2)\cos\omega_Pt+\omega_PR^2\sin\omega_Pt]}{\rho[R^4\omega_P{}^2+\nu^2(\lambda_n{}^2+C^2R^2)^2]}\\&+\frac{a_0R^2[\nu(\lambda_n{}^2+C^2R^2)\cos(\omega_bt+\phi)+\omega_bR^2\sin(\omega_bt+\phi)]}{R^4\omega_P{}^2+\nu^2(\lambda_n{}^2+C^2R^2)^2}\\&-e^{-(\frac{\nu}{R^2})(\lambda_n{}^2+C^2R^2)t}\{\frac{-A_1\omega_P{}^2R^6}{\mu(\lambda_n{}^2+C^2R^2)[R^4\omega_P{}^2+\nu^2(\lambda_n{}^2+C^2R^2)^2]}\\&+\frac{a_0R^2[\nu(\lambda_n{}^2+C^2R^2)\cos\phi+\omega_bR^2\sin\phi]}{R^4\omega_b{}^2+\nu^2(\lambda_n{}^2+C^2R^2)^2}\\&+\frac{g\sin\theta}{(\frac{\nu}{R^2})(\lambda_n{}^2+C^2R^2)}\}]\end{aligned} \quad (9)$$

which can be written in the form

$$\begin{aligned}u(\xi,t)=\frac{2A_0R^2}{\mu}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{J_1(\lambda_n)}[\{&\frac{A_0+g\sin\theta}{A_0\,(\lambda_n{}^2+C^2R^2)}\\&+\frac{\varepsilon(\lambda_n{}^2+C^2R^2)\cos\omega_Pt+\alpha^2\sin\omega_Pt}{(\lambda_n{}^2+C^2R^2)^2+\alpha^4}\}\\&+\frac{\rho a_0}{A_0}\{\frac{(\lambda_n{}^2+C^2R^2)\cos(\omega_bt+\phi)+\beta^2\sin(\omega_bt+\phi)}{(\lambda_n{}^2+C^2R^2)^2+\beta^4}\}\\&-e^{-(\frac{\nu}{R^2})(\lambda_n{}^2+C^2R^2)t}\{\frac{\varepsilon\alpha^4}{(\lambda_n{}^2+C^2R^2)[\alpha^4+(\lambda_n{}^2+C^2R^2)^2]}\\&+\frac{\frac{\rho a_0}{A_0}\{(\lambda_n{}^2+C^2R^2)\cos\phi+\beta^2\sin\phi\}}{(\lambda_n{}^2+C^2R^2)^2+\beta^4}\\&+\frac{g\sin\theta}{(\frac{\nu}{R^2})(\lambda_n{}^2+C^2R^2)}\frac{\mu}{A_0R^2}\}]\end{aligned} \quad (10)$$

Where

$$\alpha^2=\frac{\omega_PR^2}{\nu}=\mathrm{Re}_P,\quad \beta^2=\frac{\omega_bR^2}{\nu}=\mathrm{Re}_b,\quad \varepsilon=\frac{A_1}{A_0}$$

The analytical expression of u consists of four parts. The first and second parts correspond to steady and oscillatory parts of pressure gradient, the third term indicates body acceleration and the last term is the transient term .As $t\to\infty$, the transient term approaches to zero. Then from equation (10),we get

$$u(\xi,t)=\frac{2A_0R^2}{\mu}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{J_1(\lambda_n)}[\{\frac{A_0+g\sin\theta}{A_0\,(\lambda_n^2+C^2R^2)} +\frac{\varepsilon(\lambda_n^2+C^2R^2)\cos\omega_p t+\alpha^2\sin\omega_p t}{(\lambda_n^2+C^2R^2)^2+\alpha^4}\} +\frac{\rho a_0}{A_0}\{\frac{(\lambda_n^2+C^2R^2)\cos(\omega_b t+\phi)+\beta^2\sin(\omega_b t+\phi)}{(\lambda_n^2+C^2R^2)^2+\beta^4}\}] \quad (11)$$

The volumetric flow rate Q is given by

$$Q(\xi,t)=2\pi\int_0^R ru\,dr$$

$$Q(\xi,t)=\frac{4\pi A_0R^4}{\mu}\sum_{n=0}^{\infty}\frac{1}{\lambda_n^2}[\{\frac{A_0+g\sin\theta}{A_0\,(\lambda_n^2+C^2R^2)} +\frac{\varepsilon(\lambda_n^2+C^2R^2)\cos\omega_p t+\alpha^2\sin\omega_p t}{(\lambda_n^2+C^2R^2)^2+\alpha^4}\} +\frac{\rho a_0}{A_0}\{\frac{(\lambda_n^2+C^2R^2)\cos(\omega_b t+\phi)+\beta^2\sin(\omega_b t+\phi)}{(\lambda_n^2+C^2R^2)^2+\beta^4}\}] \quad (12)$$

The fluid acceleration F is given by

$$F(\xi,t)=\frac{\partial u}{\partial t}$$

$$F(\xi,t)=\frac{2a_0}{\rho}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{J_1(\lambda_n)}[\{\frac{\alpha^2\{-\varepsilon(\lambda_n^2+C^2R^2)\sin\omega_p t+\alpha^2\cos\omega_p t\}}{(\lambda_n^2+C^2R^2)^2+\alpha^4}\} +\frac{\rho a_0\beta^2}{A_0}\{\frac{-(\lambda_n^2+C^2R^2)\sin(\omega_b t+\phi)+\beta^2\cos(\omega_b t+\phi)}{(\lambda_n^2+C^2R^2)^2+\beta^4}\}] \quad (13)$$

The expression for the wall shear stress τ_w can be obtained from

$$\tau_w=\mu\left(\frac{\partial u}{\partial r}\right)_{r=R}$$

$$\tau_w(\xi,t)=-2A_0R\sum_{n=1}^{\infty}[\{\frac{A_0+g\sin\theta}{A_0\,(\lambda_n^2+C^2R^2)} +\frac{\varepsilon(\lambda_n^2+C^2R^2)\cos\omega_p t+\alpha^2\sin\omega_p t}{(\lambda_n^2+C^2R^2)^2+\alpha^4}\} +\frac{\rho a_0}{A_0}\{\frac{(\lambda_n^2+C^2R^2)\cos(\omega_b t+\phi)+\beta^2\sin(\omega_b t+\phi)}{(\lambda_n^2+C^2R^2)^2+\beta^4}\}] \quad (14)$$

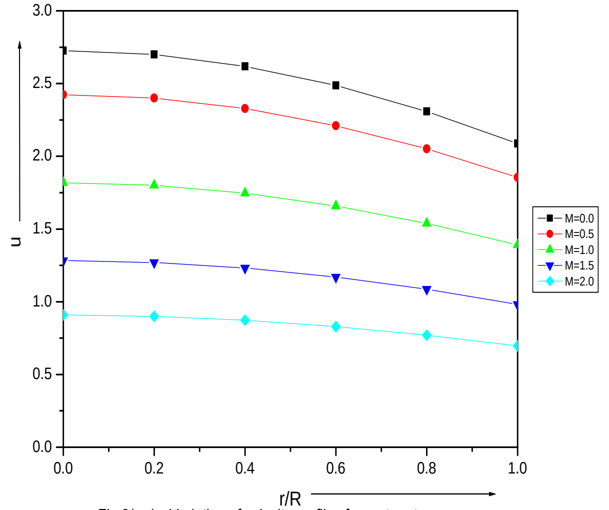

Fig.2(a) .Variation of velocity profiles for aorta artery against r/R with Phi=0,t=0

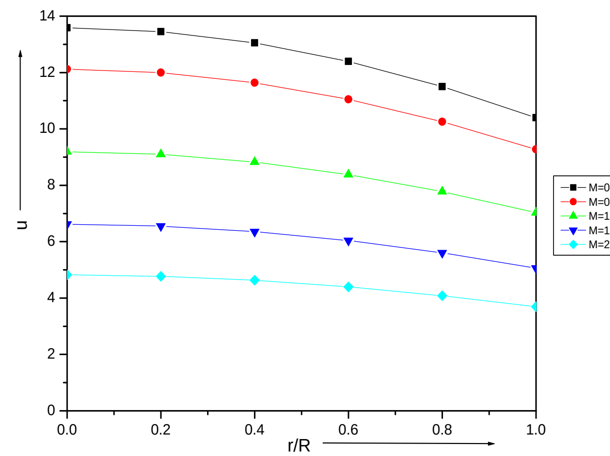

Fig.3(a).Variation of velocity profiles for aorta artery against r/R with Phi=0.0,t=45

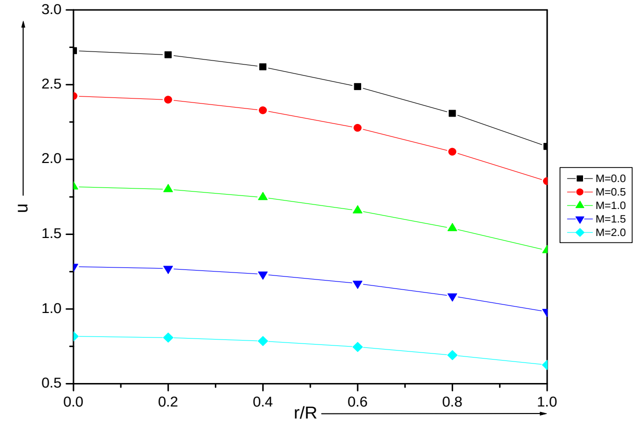

Fig.4(a).Variation of velocity profiles for aorta artery against r/R with Phi=45^0,t=0.0

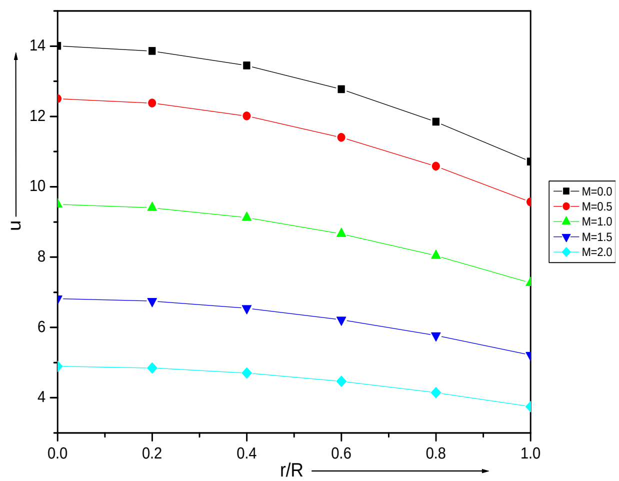

Fig.5(a).Variation of velocity profiles for aorta artery against r/R with Phi=90,t=45

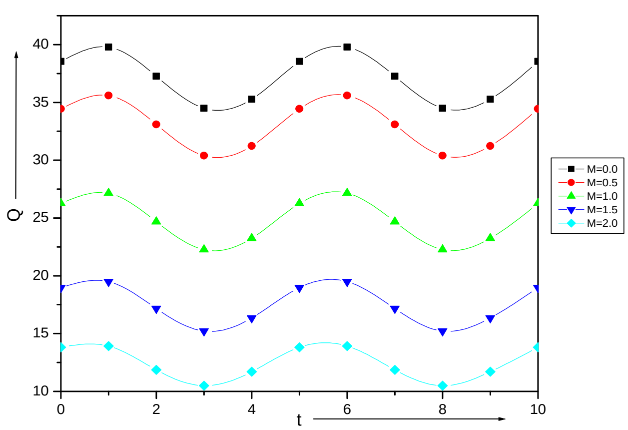

Fig.6(a).Variation of flow rate for aorta artery against t when Phi=45

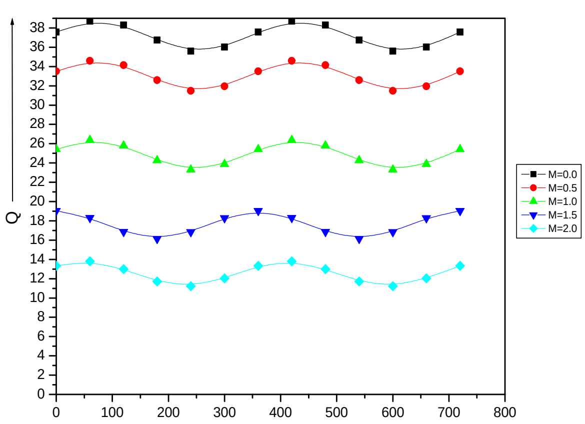

Fig.7(a).Variation of flow rate for aorta artery against Phi when t=45

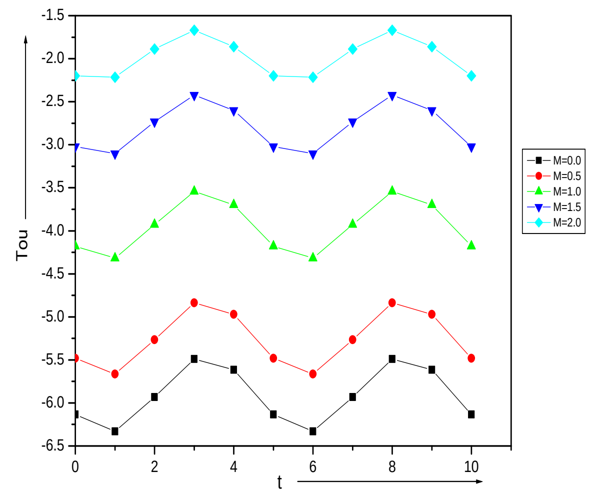

Fig.8(a).Variation of wall shear stress for aorta artery against t when Phi=45

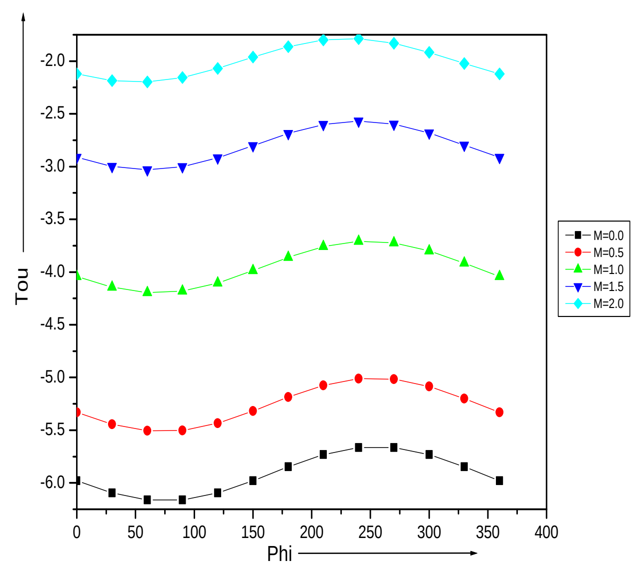

Fig.9(a).Variation of wall shear atress for aorta artery against Phi when t=45

The expression for velocity profile computed in equation (10)has been depicted in figures2(a)to 5(c) by plotting r/R versus u in presence/absence of Hartmann number(M),for different values of phase angle (ϕ)and time t. It is observed that velocity decreases with increasing Hartmann number (M). For fixed value of ϕ,it is observed that increase in M decreases the maximum value of flow rate Q and the oscillatory nature of the curves with time if different for different values of M[Figure(6)].

In figure (7), the flow rate Q decreases with increase in Hartmann number(M)at the particular time for different values of phase angle.

For fixed value of ϕ, it is found from figure(8) that the maximum value of the wall shear stress decreases with increase in M whereas in figure(9),it is observed that for fixed value of t, the maximum value of τ_w increases with increase in M.

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