ANALYSIS OF THREE DIMENSIONAL COUETTE FLOW AND HEAT TRANSFER IN POROUS MEDIUM BETWEEN TWO PERMEABLE PLATES WITH SINUSOIDAL TEMPERATURE AND MAGNETIC FIELD

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Abstract

This paper constitutes the analysis of three dimensional couette flow and heat transfer through porous medium under magnetic field. There are two permeable plates in the porous medium, viz., stationery plate and moving plate. The stationery plate is maintained at sinusoidal surface temperature while the moving plate is kept at uniform injection velocity under isothermal condition. The expressions for velocity and temperature fields are obtained. With this, skin-friction components and Nusselt number at both the plates will be derived and are plotted against Reynolds number.

Keywords: *Three dimensional, Heat transfer, porous medium, Permeable plates, Sinusoidal, Magnetic field*

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1. INTRODUCTION

The unsteady laminar flow of an incompressible viscous electrically conducting second order fluid between infinite parallel plates subject to a transverse magnetic field is investigated by Lalitha Jayaraman and Ramanaiah [9]. Raptis et al, [5,4,2,13] have discussed the free convection flow through a porous medium bounded by an infinite plate, when there is no stream velocity. Raptis and Perdikis [12] have studied the effect of free convection and mass transfer flow through a porous medium bounded by infinite vertical porous plates, when there is a free stream velocity. Analysis of three dimensional couette flow and heat transfer in porous medium between two permeable plates with sinusoidal temperature is studied by Tak and Vyas [7]. Raptis and Perdikis [11] have studied the combined free and forced convective flow through a porous medium bounded by a semi-infinite vertical porous plate. Gersten and Gross [6] have studied the effect of transverse sinusoidal suction velocity on flow and heat transfer along an infinite vertical porous wall. Raptis and Takhar [1] have studied the forced flow of a viscous incompressible fluid through the forced flow of a viscous incompressible fluid through a highly porous medium bounded by a semi-infinite vertical plate in the presence of mass transfer. Oscillatory flow through a porous medium by the presence of free convective flow is studied by Raptis, Perdikis, and C.P. Magneto hydro dynamic free convective effect for an incompressible viscous fluid past an infinite limiting surface is studied by Raptis and Tzivandis[3]. The MHD flow and heat transform in a channel with porous walls of different permeability has been investigated by the method of quasi linearization by Rama Bhargava and Meena Rani [14]. Soundalgekar and Bhat[8] have studied an approximate analysis of an oscillatory MHD channel flow and heat transfer under transverse magnetic field.

In the present problem, by taking viscous dissipation in to account, the three dimensional couette flow and heat transfer in a porous medium under magnetic field are analysed .The stationery plate is subjected to both sinusoidal temperature and suction velocity while the moving plate is isothermal with uniform injection velocity.

2. MATHEMATICAL ANALYSIS

Consider the three dimensional couette flow through porous medium between two infinite porous plates under magnetic field. So that, the stationery plate is kept along x-z plane and the other plate is at a distance c from the stationery plate which is moving with velocity U along x-direction. The stationery plate is kept at sinusoidal temperature and suction velocity, varying in z-direction while the moving plate is subjected to the uniform injection velocity under isothermal condition. Taking viscous dissipation in to account, the flow and heat transfer are governed by the following equations:

$$\frac{\partial v^*}{\partial y^*}+\frac{\partial w^*}{\partial z^*}=0 \qquad (1)$$

$$v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = \gamma \left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) - \frac{\gamma u^*}{K^*} - \frac{\sigma B_0{}^2}{\rho} u^* \quad (2)$$

$$v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \gamma\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right) - \frac{\gamma v^*}{K^*} - \frac{\sigma B_0{}^2}{\rho} v^* \quad (3)$$

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} + \gamma\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) - \frac{\gamma w^*}{K^*} - \frac{\sigma B_0{}^2}{\rho} w^* \quad (4)$$

$$\rho C_p\left[v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*}\right] = K'\left[\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right] + \phi^* \quad (5)$$

Where

$$\phi^* = 2\mu\left[\left(\frac{\partial v^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial z^*}\right)^2\right] + \mu\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*}\right)^2 + \left(\frac{\partial u^*}{\partial z^*}\right)^2\right] \quad (6)$$

Where u^*, v^*, w^* are respectively the velocity components in the directions of x, y and z axes. γ, K^*, T^*, p^*, ρ, C_p, K', μ, σ and B_0 are respectively the kinematic viscosity, permeability of porous medium, temperature, pressure, density, Specific heat of the fluid at constant pressure, thermal conductivity, viscosity of the fluid, electrical conductivity, uniform magnetic field of the fluid concerned.

The boundary conditions:

$$u^* = 0,\ v^* = -V\left(1 + \psi \cos\frac{\pi z^*}{c}\right),\ w^* = 0,$$

$$T^* = T_1\left(1 + \cos\frac{\pi z^*}{c}\right),\ P^* = 0 \text{ at } y^* = 0$$

$$u^* = U,\ v^* = -V,\ w^* = 0,\ T^* = T_2,\ p^* = \frac{V\mu c}{K^*},\ T_2 > T_1 \text{ at}$$

$$y^* = c \quad (7)$$

Where $\psi << 1$, U, V are constants with dimension of velocity and c, T_1 are constants with dimensions of length and temperature respectively.

Introducing the following non-dimensional quantities:

$$y = \frac{y^*}{c},\ z = \frac{z^*}{c},\ u = \frac{u^*}{U},\ v = \frac{v^*}{U},\ w = \frac{w^*}{U},\ p = \frac{p^*}{\rho U^2},$$

$$\theta = \frac{T^* - T_1}{T_2 - T_1}$$

$$R = \frac{Uc}{\gamma},\ P = \frac{\mu C_p}{K'},\ K = \frac{K^*}{c^2},\ \alpha = \frac{V}{U},$$

$$E = \frac{U^2}{C_p(T_2 - T_1)},$$

$$d = \frac{\alpha}{RK},\ a = \frac{T_1}{T_2 - T_1} \text{ and } M = \frac{\sigma B_0{}^2 c}{\rho U} \quad (8)$$

Where R, P, K, α and E are respectively, Reynolds number, Prandtl number, permeability parameter, suction parameter and Eckert number

With this, the equations (1) to (5) reduce to

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (9)$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{R}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{u}{RK} - Mu \quad (10)$$

Where

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial P}{\partial y} + \frac{1}{R}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{v}{RK} - Mv \quad (11)$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial P}{\partial z}+\frac{1}{R}(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}) -\frac{w}{RK}\text{-Mw} \quad (12)$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{PR}(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2})+\frac{E}{R}\phi \quad (13)$$

Where

$$\phi=2\{(\frac{\partial v}{\partial y})^2+(\frac{\partial w}{\partial z})^2\}+(\frac{\partial u}{\partial y})^2+(\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z})^2+(\frac{\partial u}{\partial z})^2$$

with boundary conditions:

y=0: u=0, $v=-\alpha(1+\psi\cos\pi z)$, w=0, p=0 and $\theta = a\psi\cos\pi z$ (14)

y=1: u=1, $v=-\alpha$, w=0, p=d, $\theta = 1$

In order to solve above equations (10) to (13), it is assumed that

$$u(y,z)=u_0(y)+\psi u_1(y,z) \quad (15)$$

$$v(y,z)=v_0(y)+\psi v_1(y,z)$$

$$w(y,z)=\psi\, w_1(y,z)$$

$$p(y,z)=p_0(y)+\psi p_1(y,z)$$

$$\theta(y,z)=\theta_0(y)+\psi\theta_1(y,z)$$

Since, amplitude $\psi(<<1)$ of sinusoidal suction velocity is small compared to its wavelength.

Employing equations (15) in equations (9) to (14) and taking $\psi=0$, the unperturbed quantities satisfies the following equations:

$$v_0{}'=0 \quad (16)$$

$$u_0''+(R\alpha)u_0'-\frac{u_0}{K}-Mu_0=0 \quad (17)$$

$$P_0'=d+M\alpha \quad (18)$$

$$\theta_0''+(PR\alpha)\theta_0'+EPu_0'^2=0 \quad (19)$$

with boundary conditions:

y=0: $u_0=0$, $v_0=-\alpha$, $\theta_0=0$, $p_0=0$,

y=1: $u_0=1$, $v_0=-\alpha$, $\theta_0=1$, $p_0=d$ (20)

Where prime denotes the derivatives with respect to y. The solutions of equations (16) to (19) satisfying the boundary conditions (20) can be expressed as follows:

$$v_0=-\alpha \quad (21)$$

$$u_0=\frac{e^{m_1 y}-e^{m_2 y}}{e^{m_1}-e^{m_2}} \quad (22)$$

$$P_0=(d+M\alpha)y \quad (23)$$

And

$$\theta_0=c_1'+c_2'e^{-(PR\alpha)y}-\frac{EP}{(e^{m_1}-e^{m_2})^2}[\frac{m_1 e^{2m_1 y}}{2(2m_1+PR\alpha)}+\frac{m_2 e^{2m_2 y}}{2(2m_2+PR\alpha)}-\frac{2m_1 m_2 e^{(m_1+m_2)y}}{(m_1+m_2)^2+PR\alpha(m_1+m_2)}] \quad \textbf{(24)}$$

Where

$$m_1=\frac{1}{2}[-R\alpha+\sqrt{R^2\alpha^2+4(\frac{1}{K}+M)}$$

$$m_2=\frac{1}{2}[-R\alpha-\sqrt{R^2\alpha^2+4(\frac{1}{K}+M)}$$

$$c_1'=\frac{1}{e^{-PR\alpha-1}}[\frac{EP}{(e^{m_1}-e^{m_2})^2}\{\frac{m_1\left(e^{-PR\alpha}-e^{2m_1}\right)}{2(2m_1+PR\alpha)}+\frac{m_2(e^{-PR\alpha}-e^{2m_2})}{2(2m_2+PR\alpha)}-\frac{2m_1 m_2(e^{-PR\alpha}-e^{m+m_2})}{(m_1+m_2)^2+PR\alpha(m_1+m_2)}\}\text{-1}]$$

$$c_2' = \frac{1}{e^{-PR\alpha}-1}[\frac{EP}{(e^{m_1}-e^{m_2})^2}\{\frac{m_1(e^{2m_1}-1)}{2(2m_1+PR\alpha)}$$
$$+\frac{m_2(e^{2m_2}-1)}{2(2m_2+PR\alpha)}-\frac{2m_1m_2(e^{m_1+m_2}-1)}{(m_1+m_2)^2+PR\alpha(m_1+m_2)}\}+1]\}$$

Also, the perturbed quantities satisfies the following.

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0 \quad (25)$$

$$v_1\frac{\partial u_0}{\partial y}-\alpha\frac{\partial u_1}{\partial y}=\frac{1}{R}[\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}]-\frac{u_1}{RK}-Mu_1 \quad (26)$$

$$-\alpha\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\frac{1}{R}[\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}]-\frac{v_1}{RK}-Mv_1 \quad (27)$$

$$-\alpha\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial z}+\frac{1}{R}[\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}]-\frac{w_1}{RK}-Mw_1 \quad (28)$$

And

$$-\alpha\frac{\partial \theta_1}{\partial y}+v_1\frac{\partial \theta_0}{\partial y}=\frac{1}{PR}[\frac{\partial^2 \theta_1}{\partial y^2}+\frac{\partial^2 \theta_1}{\partial z^2}]+\frac{2E}{R}.\frac{\partial u_0}{\partial y}.\frac{\partial u_1}{\partial y} \quad (29)$$

with boundary conditions:

y=0;
$$u_1=0\,, v_1=-\alpha\cos\pi z\,, w_1=0\,, \theta_1=a\cos\pi z\,, p_1=0$$

y=1 ; $u_1=0$ $v_1=0$ $w_1=0\,, \theta_1=0\,,$ $p_1=0$ (30)

Equations (25) to (29) are linear partial differential equations, which describes perturbed three dimentional flow due to variation of suction velocity stationery surface temperature along z-direction. The form of suction velocity and surface temperature suggests the following forms of u_1, v_1, w_1, P_1 and θ_1:

$$u_1=u_2(y)\cos\pi z \quad (31)$$

$$v_1=v_2(y)cos\pi z \quad (32)$$

$$w_1=-\frac{1}{\pi}v_2'(y)\sin\pi z \quad (33)$$

$$p_1=p_2(y)\cos\pi z \quad (34)$$

$$\theta_1=\theta_2(y)\cos\pi z \quad (35)$$

The expression for v_1 and w_1 have been chosen so that the equation of continuity (25) is satisfied. The equations (27) and (28) , being independent of the main flow and the temperature field, can be solved first. Therefore, substituting (32), (33) and (34) in equations (27) and (28), the following ordinary simultaneous differential equations are obtained:

$$kv_2''' + (RK\alpha)v_2'' - (k\pi^2+1+RKM)v_2' = (\pi^2 RK)P_2 \quad (36)$$

$$kv_2'' + (RK\alpha)v_2' - (k\pi^2+1+RKM)v_2 = RKP_2' \quad (37)$$

With corresponding boundary conditions:

$$y=0: v_2=-\alpha, v_2'=0,$$
$$y=1: v_2=0, v_2'=0 \quad (38)$$

Where a prime denotes derivative with respect to y.

The solution of system of equations (36) and (37) is substituted in equations (32) to (34) to obtain the values of v_1, w_1 and p_1:

$$v_1=[c_1e^{m_1'y}+c_2e^{m_2'y}+c_3e^{\pi y}+c_4e^{-\pi y}]\cos\pi z \quad (39)$$

$$w_1=-\frac{1}{\pi}[c_1m_1'e^{m_1'y}+c_2m_2'e^{m_2'y}+c_3\pi e^{\pi y}-c_4\pi e^{-\pi y}]\sin\pi z \quad (40)$$

$$P_1=\frac{1}{\pi^2 RK}[A_1'e^{m_1'y}+A_2'e^{m_2'y}$$
$$+A_3'e^{\pi y}+A_4'e^{-\pi y}]\cos\pi z \quad (41)$$

Where

$$m_1'=\frac{-R\alpha+\sqrt{R^2\alpha^2+4(\pi^2+RM+\frac{1}{K})}}{2}$$

And

$$m_2' = \frac{-R\alpha - \sqrt{R^2\alpha^2 + 4(\pi^2 + RM + \frac{1}{K})}}{2}$$

$$c_1 = \frac{d_1}{d}, c_2 = \frac{d_2}{d}, c_3 = \frac{d_3}{d}, c_4 = \frac{d_4}{d}$$

$$d = \begin{vmatrix} 1 & 1 & 1 & 1 \\ e^{m_1'} & e^{m_2'} & e^{\pi} & e^{-\pi} \\ m_1' & m_2' & \pi & -\pi \\ m_1' e^{m_1'} & m_2' e^{m_2'} & \pi e^{\pi} & -\pi e^{-\pi} \end{vmatrix};$$

$$d_1 = \begin{vmatrix} -\alpha & 1 & 1 & 1 \\ 0 & e^{m_2'} & e^{\pi} & e^{-\pi} \\ 0 & m_2' & \pi & -\pi \\ 0 & m_2' e^{m_2'} & \pi e^{\pi} & -\pi e^{-\pi} \end{vmatrix};$$

$$d_2 = \begin{vmatrix} 1 & -\alpha & 1 & 1 \\ e^{m_1'} & 0 & e^{\pi} & e^{-\pi} \\ m_1' & 0 & \pi & -\pi \\ m_1' e^{m_1'} & 0 & \pi e^{\pi} & -\pi e^{-\pi} \end{vmatrix};$$

$$d_3 = \begin{vmatrix} 1 & 1 & -\alpha & 1 \\ e^{m_1'} & e^{m_2'} & 0 & e^{-\pi} \\ m_1' & m_2' & 0 & -\pi \\ m_1' e^{m_1'} & m_2' e^{m_2'} & 0 & -\pi e^{-\pi} \end{vmatrix};$$

$$d_4 = \begin{vmatrix} 1 & 1 & 1 & -\alpha \\ e^{m_1'} & e^{m_2'} & e^{\pi} & 0 \\ m_1' & m_2' & \pi & 0 \\ m_1' e^{m_1'} & m_2' e^{m_2'} & \pi e^{\pi} & 0 \end{vmatrix}$$

$$A_1' = Kc_1 m_1'^3 + RK\alpha c_1 m_1'^2 - (k\pi^2 + 1 + RKM)c_1 m_1',$$

$$A_2' = Kc_2 m_2'^3 + RK\alpha c_2 m_2'^2 - (k\pi^2 + 1 + RKM)c_2 m_2',$$

$$A_3' = Kc_3\pi^3 + RK\alpha c_3\pi^2 - (k\pi^2 + 1 + RKM)c_3\pi,$$

$$A_4' = -Kc_4\pi^3 + RK\alpha c_4\pi^2 + (k\pi^2 + 1 + RKM)c_4\pi$$

Now, for the main flow and the temperature field, Substitute the expressions (31) and (35) in equations (26) & (29), we get,

$$Ku_2'' + (RK\alpha)u_2' - (k\pi^2 + 1 + RKM)u_2 = RKv_2 u_0' \qquad (42)$$

$$\theta_2'' + (PR\alpha)\theta_2' - \pi^2\theta_2 = v_2 PR\theta_0' - 2PEu_0' u_2' \qquad (43)$$

With boundary conditions.

$$\begin{aligned} y = 0 &: u_2 = 0, \theta_2 = a \\ y = 1 &: u_2 = 0, \theta_2 = 0 \end{aligned} \qquad (44)$$

Solving (42) & (43) with boundary conditions (44) & substituting the solutions in equations (31) & (35), the expressions for $u_1 \, and \, \theta_1$ can be given as:

$$\begin{aligned} u_1 = [&D_5 e^{m_1^* y} + D_6 e^{m_2^* y} + \frac{R}{e^{m_1} - e^{m_2}}\{B_1 e^{(m_1+m_1')y} \\ &+ B_2 e^{(m_1+m_2')y} + B_3 e^{(m_1+\pi)y} + B_4 e^{(m_1-\pi)y} \\ &- B_5 e^{(m_2+m_1')y} - B_6 e^{(m_2+m_2')y} - B_7 e^{(m_2+\pi)y} \\ &- B_8 e^{(m_2-\pi)y}\}]\cos\pi z \end{aligned} \qquad (45)$$

$$\begin{aligned} \theta_1 = [&(\frac{G - e^{a_2}(F-a)}{e^{a_2} - e^{a_1}})e^{a_1 y} \\ &+ (\frac{G - e^{a_1}(F-a)}{e^{a_1} - e^{a_2}})e^{a_2 y} + \text{f(y)}]\cos\pi z \end{aligned} \qquad (46)$$

Where

$$B_1 = \frac{Kc_1 m_1}{K(m_1 + m_1')^2 + RK\alpha(m_1 + m_1') - (K\pi^2 + 1 + RKM)}$$

$$B_2 = \frac{Kc_2 m_1}{K(m_1 + m_2')^2 + RK\alpha(m_1 + m_2') - (K\pi^2 + 1 + RKM)}$$

$$B_3 = \frac{Kc_3 m_1}{K(m_1+\pi)^2 + RK\alpha(m_1+\pi) - (K\pi^2+1+RKM)}$$

$$B_4 = \frac{Kc_4 m_1}{K(m_1-\pi)^2 + RK\alpha(m_1-\pi) - (K\pi^2+1+RKM)}$$

$$B_5 = \frac{Kc_1 m_2}{K(m_2+m_1')^2 + RK\alpha(m_2+m_1') - (K\pi^2+1+RKM)}$$

$$B_6 = \frac{Kc_2 m_2}{K(m_2+m_2')^2 + RK\alpha(m_2+m_2') - (K\pi^2+1+RKM)}$$

$$B_7 = \frac{Kc_3 m_2}{K(m_2+\pi)^2 + RK\alpha(m_2+\pi) - (K\pi^2+1+RKM)}$$

$$B_8 = \frac{Kc_4 m_2}{K(m_2-\pi)^2 + RK\alpha(m_2-\pi) - (K\pi^2+1+RKM)}$$

$$D_5 = \frac{R(B - Ae^{m_2*})}{(e^{m_2*} - e^{m_1*})(e^{m_1} - e^{m_2})};$$

$$D_6 = \frac{R(B - Ae^{m_1*})}{(e^{m_1*} - e^{m_2*})(e^{m_1} - e^{m_2})}$$

$$A = B_1 + B_2 + B_3 + B_4 - B_5 - B_6 - B_7 - B_8$$

$$B = B_1 e^{(m_1+m_1')} + B_2 e^{(m_1+m_2')} + B_3 e^{(m_1+\pi)} + B_4 e^{(m_1-\pi)} - B_5 e^{(m_2+m_1')} - B_6 e^{(m_2+m_2')} + B_7 e^{(m_2+\pi)} + B_8 e^{(m_2-\pi)}$$

$$m_1^* = -(\frac{R\alpha}{2}) + \sqrt{\frac{R^2\alpha^2}{4} + (\pi^2 + \frac{1}{K} + RM)},$$

$$m_2^* = -(\frac{R\alpha}{2}) - \sqrt{\frac{R^2\alpha^2}{4} + (\pi^2 + \frac{1}{K} + RM)}$$

$$F = -A_1 - A_2 - A_3 - A_4 - A_5 - A_6 - A_7 - A_8 - A_9 - A_{10} - A_{11} - A_{12} + A_{13} + A_{14} + A_{15} + A_{16} - A_{17} - A_{18} + A_{19} + A_{20}$$

$$f(y) = -A_1 e^{(m_1'-PR\alpha)y} - A_2 e^{(m_2'-PR\alpha)y} - A_3 e^{(\pi-PR\alpha)y} - A_4 e^{(-\pi-PR\alpha)y} - A_5 e^{(m_1'+2m_1)y} - A_6 e^{(m_1'+2m_2)y} - A_7 e^{(m_2'+2m_1)y} - A_8 e^{(m_2'+2m_2)y} - A_9 e^{(\pi+2m_1)y} - A_{10} e^{(\pi+2m_2)y} - A_{11} e^{(-\pi+2m_1)y} - A_{12} e^{(-\pi+2m_2)y} + A_{13} e^{(m_1+m_2+m_1')y} + A_{14} e^{(m_1+m_2+m_2')y} + A_{15} e^{(m_1+m_2+\pi)y} + A_{16} e^{(m_1+m_2-\pi)y} - A_{17} e^{(m_1+m_1*)y} - A_{18} e^{(m_1+m_2*)y} + A_{19} e^{(m_2+m_1*)y} + A_{20} e^{(m_2+m_2*)y}$$

$$G = -A_1 e^{(m_1'-PR\alpha)} - A_2 e^{(m_2'-PR\alpha)} - A_3 e^{(\pi-PR\alpha)} - A_4 e^{(-\pi-PR\alpha)} - A_5 e^{(m_1'+2m_1)} - A_6 e^{(m_1'+2m_2)} - A_7 e^{(m_2'+2m_1)} - A_8 e^{(m_2'+2m_2)} - A_9 e^{(\pi+2m_1)} - A_{10} e^{(\pi+2m_2)} - A_{11} e^{(-\pi+2m_1)} - A_{12} e^{(-\pi+2m_2)} + A_{13} e^{(m_1+m_2+m_1')} + A_{14} e^{(m_1+m_2+m_2')} + A_{15} e^{(m_1+m_2+\pi)} + A_{16} e^{(m_1+m_2-\pi)} - A_{17} e^{(m_1+m_1*)} - A_{18} e^{(m_1+m_2*)} + A_{19} e^{(m_2+m_1*)} + A_{20} e^{(m_2+m_2*)}$$

$$a_1 = \frac{-(PR\alpha) + \sqrt{P^2R^2\alpha^2 + 4\pi^2}}{2};$$

$$a_2 = \frac{-(PR\alpha) - \sqrt{P^2R^2\alpha^2 + 4\pi^2}}{2}$$

$$A_1 = \frac{P^2R^2 c_1 c_2' \alpha}{(m_1' - PR\alpha)^2 + PR\alpha(m_1' - PR\alpha) - \pi^2};$$

$$A_2 = \frac{P^2R^2 c_2 c_2' \alpha}{(m_2' - PR\alpha)^2 + PR\alpha(m_2' - PR\alpha) - \pi^2};$$

$$A_3 = \frac{P^2R^2 c_3 c_2' \alpha}{(\pi - PR\alpha)^2 + PR\alpha(\pi - PR\alpha) - \pi^2};$$

$$A_4 = \frac{P^2R^2 c_4 c_2' \alpha}{(\pi + PR\alpha)^2 - PR\alpha(\pi + PR\alpha) - \pi^2};$$

$$A_5=\frac{EPRm_1}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_1m_1}{2m_1+PR\alpha}$$

$$+2B_1(m_1+m_1^{'}))(\frac{1}{(m_1^{'}+2m_1)^2+PR\alpha(m_1^{'}+2m_1)-\pi^2})$$

;

$$A_6=\frac{EPRm_2}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_1m_2}{2m_2+PR\alpha}$$

$$+2B_5(m_2+m_1^{'}))(\frac{1}{(m_1^{'}+2m_2)^2+PR\alpha(m_1^{'}+2m_2)-\pi^2})$$

;

$$A_7=\frac{EPRm_1}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_2m_1}{2m_1+PR\alpha}$$

$$+2B_2(m_1+m_2^{'}))(\frac{1}{(m_2^{'}+2m_1)^2+PR\alpha(m_2^{'}+2m_1)-\pi^2}$$

$$A_8=\frac{EPRm_2}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_2m_2}{2m_2+PR\alpha}$$

$$+2B_6(m_2+m_2^{'}))(\frac{1}{(m_2^{'}+2m_2)^2+PR\alpha(m_2^{'}+2m_2)-\pi^2})$$

$$A_9=\frac{EPRm_1}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_3m_1}{2m_1+PR\alpha}$$

$$+2B_3(m_1+\pi))(\frac{1}{(\pi+2m_1)^2+PR\alpha(\pi+2m_1)-\pi^2})$$

$$A_{10}=\frac{EPRm_2}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_3m_2}{2m_2+PR\alpha}$$

$$+2B_7(m_2+\pi))(\frac{1}{(\pi+2m_2)^2+PR\alpha(\pi+2m_2)-\pi^2})$$

$$A_{11}=\frac{EPRm_1}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_4m_1}{2m_1+PR\alpha}$$

$$+2B_4(m_1-\pi))(\frac{1}{(-\pi+2m_1)^2+PR\alpha(-\pi+2m_1)-\pi^2})$$

$$A_{12}=\frac{EPRm_2}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_4m_2}{2m_2+PR\alpha}$$

$$+2B_8(m_2-\pi))(\frac{1}{(-\pi+2m_2)^2+PR\alpha(-\pi+2m_2)-\pi^2})$$

$$A_{13}=\frac{2EPR}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_1m_1m_2}{(m_1+m_2)+PR\alpha}+m_1B_5(m_2+m_1^{'})$$

$$+m_2B_1(m_1+m_1^{'}))$$

$$(\frac{1}{(m_1+m_2+m_1^{'})^2+PR\alpha(m_1+m_2+m_1^{'})-\pi^2})$$

$$A_{14}=\frac{2EPR}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_2m_1m_2}{(m_1+m_2)+PR\alpha}-m_1B_6(m_2+m_2^{'})$$

$$+m_2B_2(m_1+m_2^{'}))$$

$$(\frac{1}{(m_1+m_2+m_2^{'})^2+PR\alpha(m_1+m_2+m_2^{'})-\pi^2})$$

$$A_{15}=\frac{2EPR}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_3m_1m_2}{(m_1+m_2)+PR\alpha}+m_1B_7(m_2+\pi)$$

$$+m_2B_3(m_1+\pi))$$

$$(\frac{1}{(m_1+m_2+\pi)^2+PR\alpha(m_1+m_2+\pi)-\pi^2})$$

$$A_{16}=\frac{2EPR}{(e^{m_1}-e^{m_2})^2}(\frac{Pc_4m_1m_2}{(m_1+m_2)+PR\alpha}+m_1B_8(m_2-\pi)$$

$$+m_2B_4(m_1-\pi))$$

$$(\frac{1}{(m_1+m_2-\pi)^2+PR\alpha(m_1+m_2-\pi)-\pi^2})$$

$$A_{17}=\frac{2EPm_1m_1^{*}D_5}{e^{m_1}-e^{m_2}}(\frac{1}{(m_1+m_1^{*})^2+PR\alpha(m_1+m_1^{*})-\pi^2})$$

$$A_{18}=\frac{2EPm_1m_2^{*}D_6}{e^{m_1}-e^{m_2}}(\frac{1}{(m_1+m_2^{*})^2+PR\alpha(m_1+m_2^{*})-\pi^2})$$

$$A_{19}=\frac{2EPm_2m_1^{*}D_5}{e^{m_1}-e^{m_2}}(\frac{1}{(m_2+m_1^{*})^2+PR\alpha(m_2+m_1^{*})-\pi^2})$$

$$A_{20}=\frac{2EPm_2m_2^{*}D_6}{e^{m_1}-e^{m_2}}(\frac{1}{(m_2+m_2^{*})^2+PR\alpha(m_2+m_2^{*})-\pi^2})$$

3. RESULTS AND DISCUSSION

Knowing velocity and temperature fields, the important characteristic parameters namely, skin-friction components τ_x and τ_z along main flow and transverse direction, non-dimensional rate of heat transfer (Nusselt number) at both the plates can be calculated.

$$\tau_x = \frac{d\tau_x^*}{\mu u} = (\frac{du_0}{dy})_0 + \psi(\frac{du_2}{dy})_0 \cos \pi z$$

$$\begin{aligned}\tau_x &= \frac{m_1 - m_2}{e^{m_1} - e^{m_2}} + \psi[m_1^* D_5 + m_2^* D_6 \\ &+ \frac{R}{e^{m_1} - e^{m_2}}\{(m_1 + m_1^{'})B_1 + (m_1 + m_2^{'})B_2 \\ &+ (m_1 + \pi)B_3 + (m_1 - \pi)B_4 - (m_2 + m_1^{'})B_5 \\ &- (m_2 + m_2^{'})B_6 - (m_2 + \pi)B_7 - (m_2 - \pi)B_8\}]\cos \pi z\end{aligned} \tag{47}$$

$$\tau_z = \frac{d\tau_z^*}{\mu v} = \psi(\frac{dw_1}{dy}) = -\frac{\psi}{\pi}[c_1 m_1'^2 + c_2 m_2'^2 + c_3\pi^2 + c_4\pi^2]\sin \pi z \tag{48}$$

3.1 Nusselts Number at the Stationary Plate:

$$Nu = \frac{-c}{T_2 - T_1}(\frac{\delta T^*}{\delta y^*})_{y^*=0}$$

$$\begin{aligned}Nu=-[&-(PR\alpha)c_2' - \frac{EP}{(e^{m_1} - e^{m_2})^2}(\frac{m_1^2}{2m_1 + PR\alpha} + \frac{m_2^2}{2m_2 + PR\alpha} \\ &- \frac{2m_1 m_2}{(m_1 + m_2) + PR\alpha} + \psi \\ &\{(\frac{G - e^{a_2}(F-a)}{e^{a_2} - e^{a_1}})a_1 + (\frac{G - e^{a_1}(F-a)}{e^{a_1} - e^{a_2}})a_2 + f'(0)\}\cos \pi z]\end{aligned} \tag{49}$$

3.2 Nusselt Number at the Moving Plate

$$Nu = -\frac{c}{T_2 - T_1}(\frac{\partial T^*}{\partial y*})_{y^*=c}$$

$$\begin{aligned}Nu = -[&-(PR\alpha)c_2' e^{-(PR\alpha)} - \frac{EP}{(e^{m_1} - e^{m_2})^2}(\frac{m_1^2 e^{2m_1}}{2m_1 + PR\alpha} \\ &+ \frac{m_2^2 e^{2m_2}}{2m_2 + PR\alpha} - \frac{2m_1 m_2 e^{(m_1+m_2)}}{(m_1 + m_2) + PR\alpha} \\ &+ \psi\{(\frac{G - e^{a_2}(F-a)}{e^{a_2} - e^{a_1}})a_1 e^{a_1} + (\frac{G - e^{a_1}(F-a)}{e^{a_1} - e^{a_2}})a_2 e^{a_2} \\ &+ f'(1)\}\cos \pi z]\end{aligned} \tag{50}$$

The velocity components u, v and w in representative plane z=0 and z=1/2,are plotted against y in figures 1,2 and 3 respectively, for various values of Reynolds number R, Suction parameters α and permeability parameters K at constant magnetic field M. It is observed from these figures, that u and w both increase as R, α or K. Also , when R or K increase v increases.

In figure4, the temperature distribution function θ is plotted against y, in representative plane z=0 at constant magnetic field. From the fig, temperature increases as Eckert number E or Prandtl number P increases but the same decreases as ψ increases.

The absolute values of the skin friction components τ_x in plane z=0 and τ_z in plane z=1/2are plotted against Reynolds number R at constant magnetic field M in figures 5 and 6 respectively, for various values of α and K. It is noted that both τ_x and τ_z increase as Reynolds number R increases or α increases. Further, permeability parameter K increases τ_x increases whereas τ_z decreases.

Fig7.shows the variation of Nusselt number lNul, at the plate y=1, against R at constant magnetic field M. Here, lNul decreases significantly with R. Also when α increases lNul increases.

The velocity components u, v and w in representative plane z=0 and z=1/2,are plotted against y in figures 8,9 and 10 respectively, for the fixed values of Reynolds number R, Suction parameters α and permeability parameters K at different magnetic fields. It is observed from these figures, that if Magnetic field parameter M increases, both u and v decreases, but w increases. As R increases, u, v and w increases.

In figure11, the temperature distribution function θ is plotted against y, in representative plane z=0 for different magnetic fields. As y and M increases θ also increases for fixed Eckert number E, Prandtl number P, Reynolds number R, Suction parameters α and permeability parameters K.

The absolute values of the skin friction components τ_x in plane z=0 and τ_z in plane z=1/2 are plotted against Reynolds number R in figures 12 and 13 respectively, for fixed values of α and K at different magnetic fields. It is noted that τ_x increases and τ_z decreases as Reynolds number R increases, while τ_x and τ_z decreases, as M increases for fixed α , K.

Figure14.shows the variation of Nusselt number |Nu|, at the plate y=1, against R. Here, |Nu| increases significantly with R. Also, |Nu| increases as M increases.

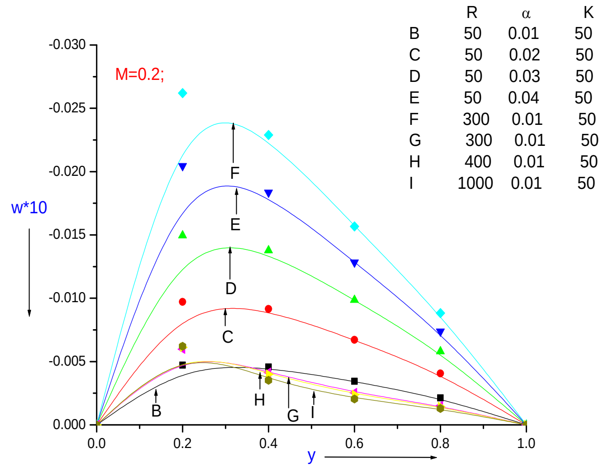

Fig.3.Velocity component w against y in plane z=1/2 for ψ=0.1

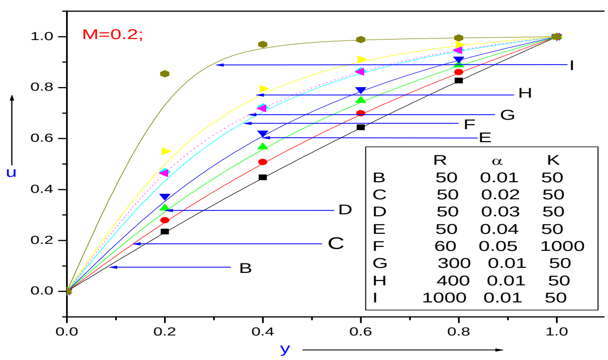

Fig1.velocity component u against y in plane z=0 for ψ=0.1

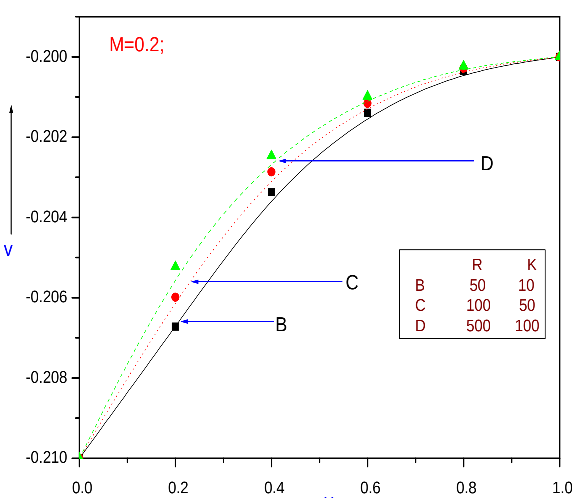

Fig2.Velocity component v against y in plane z=0 for α=0.2,ψ=0.05

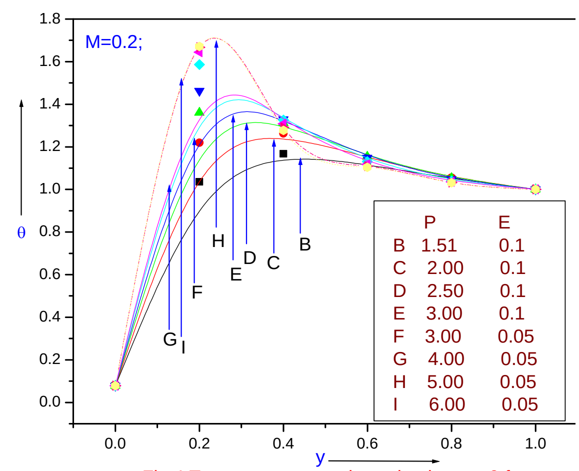

Fig.4.Temperature θ against y in plane z=0 for R=50,α=0.05,K=10,ψ=0.1 and a=0.78

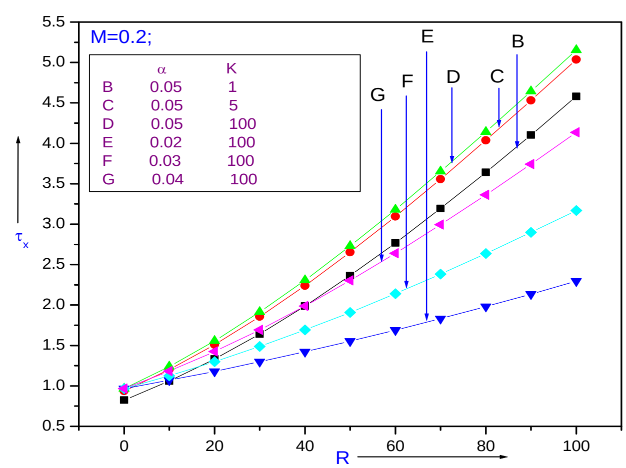

Fig.5.Skin-friction coeffecient τ_x against Reynolds-number R in plane z=0 for ψ=0.1

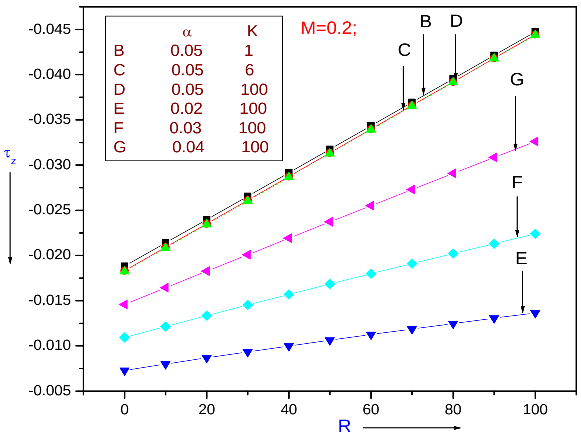

Fig.6.Skin-friction coeffecient τ_z against Reynolds-number R in plane z

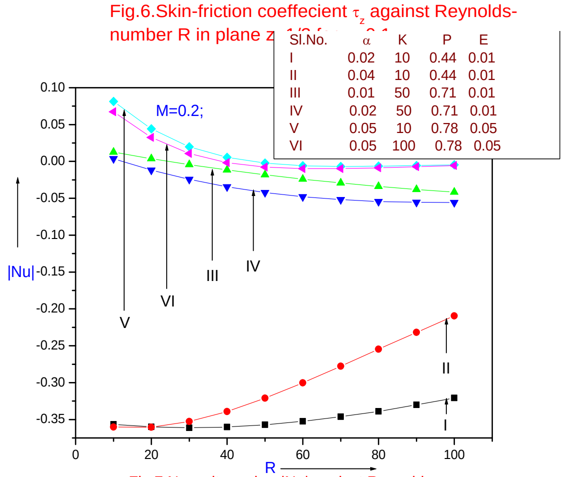

Fig.7.Nusselt number |Nu| against Reynolds-number R inplane z=0 for ψ=0.1

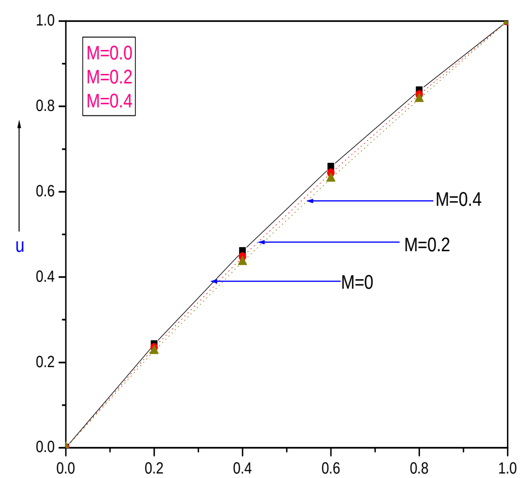

Fig.8.Velocity component u against y in plane z=0 for ψ=0.1,R=50,α=0.01,k=50

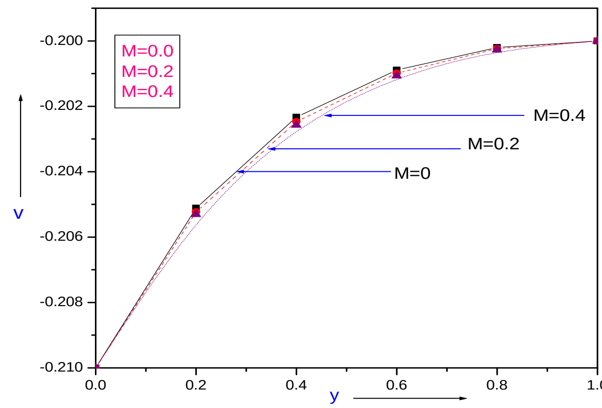

Fig.9.velocity component v against y in plane z=0 for α=0.2,ψ=0.05,R=500,K=100

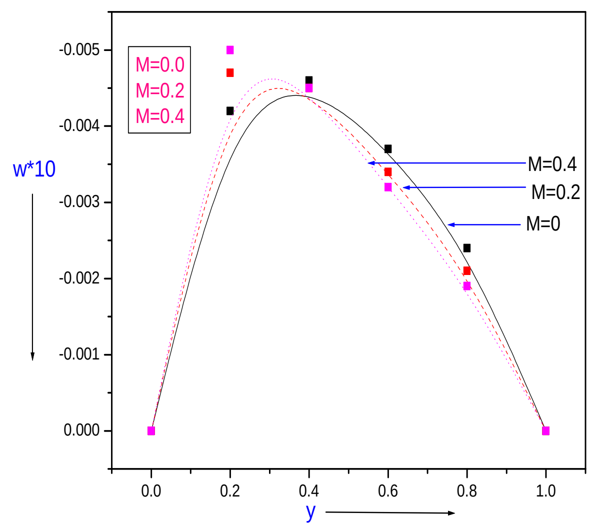

Fig.10.Velocity component w against y in plae z=1/2 for Ψ=0.1,R=50,α=0.02,K=50

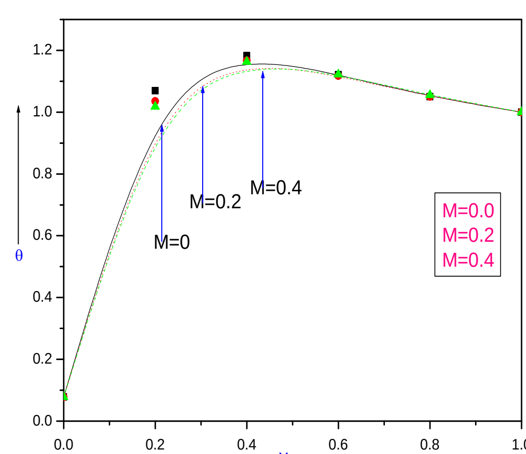

Fig.11.Temperature function θagainst y in plane z=0 R=50,α=0.05,K=10,ψ=0.1,a=0.78, P=1.51,E=0.1

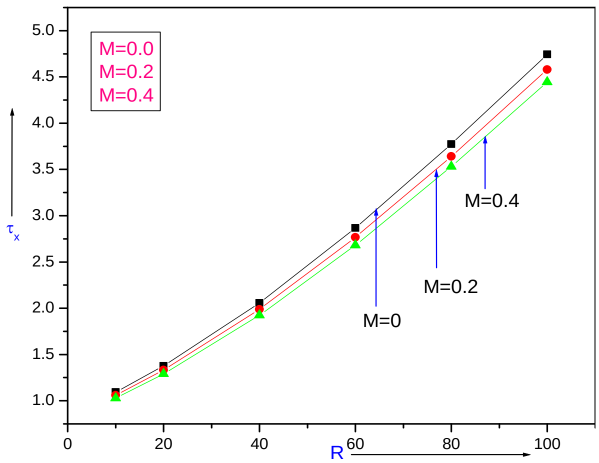

Fig.12.Skin-friction coefficient τ_x against Reynolds number R in plane z=0 for ψ=0.1,α=0.05,K=1

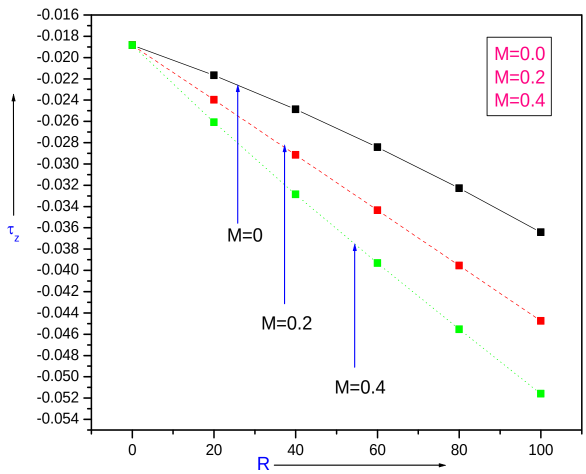

Fig.13.Skin-friction coefficient τ_z against Reynolds number in plane z=1/2 for ψ=0.1,α=0.05,K=100

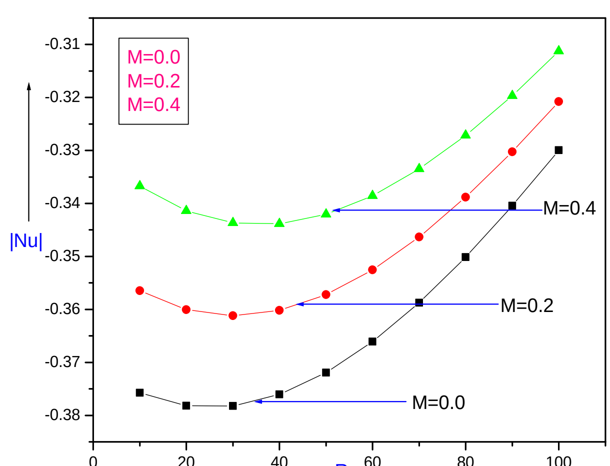

Fig.14.Nusselt number |Nu| against Reynolds number R in plane z=0 for ψ=0.1,α=0.02, K=10,P=0.44,E=0.01,y=1,a=0.78

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