IMAGE DENOISING USING DUAL TREE COMPLEX WAVELET TRANSFORM

Athira K. Vijay[1], M. Mathurakani[2]

[1]PG Student, [2]Formerly Scientist in DRDO, Professor, ECE, TocH Institute of Science and Technology, Kerala, India

Abstract

Image denoising is an important task which finds its application as a task itself and also as a sub task in other processes. Wavelet analyses provide high resolution and are very useful in image processing applications. But there were few factors which delayed the progress of using wavelets. Many researchers have been carried out in this field to extract the advantages of wavelets. This paper presents a survey and simulation of image processing based on DT-CWT. Here, a clearer version of an image is recovered from its noisy observation by the use of Dual Tree Complex Wavelet Transform (DT-CWT) along with Byes thresholding. Convolution based 2D processing is employed for simulation. An improvement in PSNR was observed which clearly tells about the enhancements that can happen in the field of image processing by the use of DT-CWT.

***Keywords:** Wavelets, Denoising, DT-CWT, Thresholding.*

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1. INTRODUCTION

Image denoising can be defined as the process of removing noise from an image. Noise can be the result of image acquisition process or transmissions that result in erroneous pixel values which do not reflect the actual intensities of a scene. In image analysis, removing noise without blurring the image edges is a difficult problem. This work focus on to the reduction of Gaussian noise which is a very difficult task when compared to any other noise types. The paper is organized as follows: (i)introduction (ii)Translation invariance and directional selectivity (iii) design overview and algorithm (iv) results and discussions (v) conclusion.

Fourier Transform (FT) approach does not provide simultaneous localization in both time and frequency. Thus the transform is not suitable for non-stationary signals. Another fourier based approach called Short Time Fourier Transform (STFT) can be thought as an improved version of Fourier Transform which uses narrow windows. The window size is taken to be narrow such that the part of non-stationary signal appears to be stationary. This approach provided good time resolution but poor frequency resolution. The commonly used transforms for image processing includes Discrete Cosine Transform (DCT) and Wavelet Transform (WT). DCT is a powerful tool which is widely used for image compression. But, disadvantages like low resolution, high loss of information etc makes it unsuitable for critical applications.

Introduction of wavelet transform goes back to 1967. Wavelet Transforms provided simultaneous localization in both frequency and time and found to be an efficient tool for image processing applications. Several denoising methods have been proposed based on the wavelet coefficients. The mathods proposed by Mallat and Hwang estimates the local regularity of image[1]. They calculated the Lipchits exponent which specified about the regularity conditions among coefficients. Coefficients with lower Lipchits values were diascarded and those with higher Lipchits exponent values were retained for reconstructing the image. Reconstruction was based on the interactive projection procedure which was computationally demanding. Malfait and Roose [2] developed a filtering technique that considered two measures for image filtering. First is a measure of local regularity of the image through the Hölder exponent, and the second is about the geometric constraints. These measures are combined in a Bayesian probabilistic formulation, and implemented by a Markov random field model. The method resulted in an improved SNR but lost its popularity due to its high computational demand. Another method proposed by Xu *et al*. [3] is based on the correlation of wavelet coefficients between consecutive scales. The wavelet coefficients related to noise are less correlated than coefficients associated to edges. If the correlation is smaller than a threshold, the given coefficient is made zero. Their technique needed a noise power estimate to calculate a proper threshold which served as a disadvantage for their method.

Ordinary wavelet transform had two disadvantages. They are : (i) lack of shift invariance (ii) lack of good directional selectivity. These problems delayed the progress of using wavelets. If the coefficients used are complex, we can achieve good directionality, i.e., up to six orientational directions. An improved form of DWT called Complex Discrete Wavelet Transform (CDWT) used complex filters to produce complex coeffients. The complex filter design was very difficult [4]. Thus urged the need to find an improvement which can minimize this difficulty. Dual Tree Complex Wavelet Transform (DT-CWT) produced the effect of having complex coeffients without using complex filters. This resulted in improved directional selectivity and near shift invariance.

2. TRANSLATION INVARIANCE AND DIRECTIONAL SELECTIVITY

2.1 Translation Invariance

The two-channel filter banks with analysis low-pass filter given by the z transform AL0(z), analysis highpass filter AH1(z) and with synthesis filters SL0(z) and SH1(z) is shown in figure 1. For an input signal, the analysis part of the filter bank followed by down-sampling gives the low pass and the high pass coefficients. Low frequency part $Xl^{l}(z)$ and a high frequency part $Xh^{l}(z)$ are the SL0 and SH1 filtered outputs along with upsampling. Output signal is the sum of these two components.

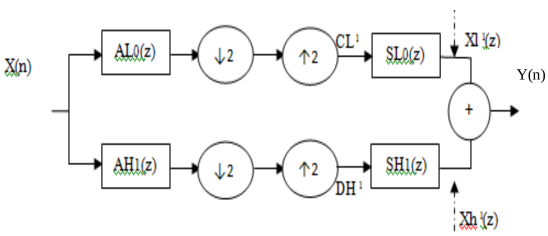


Fig -1: DWT filter bank

$$CL^{l}(z^2) = \left(\tfrac{1}{2}\right)\left(X(z)\, AL0(z) + X(-z)\, AL0(-z) \right) \quad (1)$$

$$DH^{l}(z^2) = \left(\tfrac{1}{2}\right)\left(X(z)\, AH1(z) + X(-z)\, AH1(-z) \right) \quad (2)$$

$$Y(z) = Xl^{l}(z) + Xh^{l}(z) \quad (3)$$

Where,

$$Xl^{l}(z) = CL^{l}(z^2)\, SL0(z)$$
$$= \left(\tfrac{1}{2}\right)\left(X(z)\, AL0(z)\, SL0(z) + X(-z)\, AL0(-z) SL0(z)\right)$$

$$Xh^{l}(z) = DH^{l}(z^2)\, SH1(z)$$
$$= \left(\tfrac{1}{2}\right)\left(X(z)\, AH1(z) SH1(z) + X(-z)\, AH1(-z) SH1(z) \right) \quad (5)$$

This decomposition is not shift invariant due to the terms in X(−z) of equation 4 and 5 that are introduced by the downsampling operators. If the input signal is shifted, for example $z^{-l}X(z)$, the application of the filter bank results in the following decomposition

$$z^{-l}X(z) = \tilde{X}l^{l}(z) + \tilde{X}h^{l}(z) \quad (6)$$

For an input $z^{-l}X(z)$ we have

$$C^{l}(z^2) = \left(\tfrac{1}{2}\right)\left(z^{-l}X(z)\, AL0(z) + (-z^{-l})X(-z)\, AL0(-z)\right) \quad (7)$$

And

$$\tilde{X}l^{l}(z) = \left(\tfrac{1}{2}\right)\left(X(z)\, AL0(z)\, SL0(z) - X(-z)\, AL0(-z)\, SL0(z)\right) \quad (8)$$

And similarly for the high-pass part, which of course is not the same as $z^{-l}Xl^{l}(z)$ if we substitute for z^{-l} in eqn 4. From this calculation it can be seen that the shift dependence is caused by the terms containing X(−z), the aliasing terms. One possibility to obtain a shift invariant decomposition can be achieved by the addition of a filter bank to figure 1 with shifted inputs $z^{-l}X(z)$ and subsequently taking the average of the lowpass and the highpass branches of both filter banks as shown in figure 2.

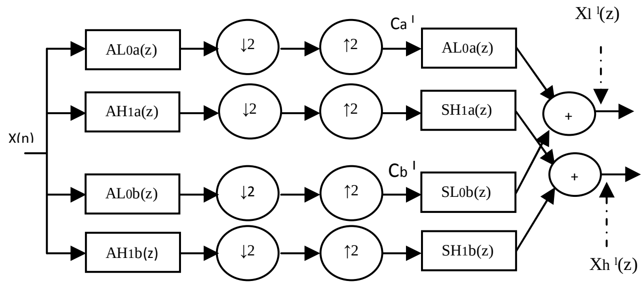


Fig -2: One level Complex dual tree

Another option is to add a filter bank with shifted inputs $z^{-l}AL0(z)$, $z^{-l}AH1(z)$ and synthesis filters zSL0(z), zSL1(z) and taking the average of the lowpass and the highpass branches of both filter banks. If we denote the first filter bank by index 'a' and the second one by index 'b' then this procedure implies the following decomposition:

$$Y(z) = Xl^{l}(z) + Xh^{l}(z) \quad (9)$$

where for lowpass channels of tree a and tree b have

$$Xl^{l}(z) = \left(\tfrac{1}{2}\right)\left(Ca^{l}(z^2)\, SL0a(z) + Cb^{l}(z^2)\, SL0b(z) \right)$$
$$= \left(\tfrac{1}{2}\right) X(z)\, H0(z)\, G0(z) \quad (10)$$

And similarly for the high-pass part . The aliasing term containing X(-z) in Xl^{l} has vanished and the decomposition becomes indeed shift invariant. Using the same principle for the design of shift invariant filter decomposition, Kingsbury suggested DT-CWT where a 'dual-tree' of two parallel filter banks are constructed and their outputs are combined. Structure of resulting analysis filter bank is shown in Figure 2. The dual-tree complex DWT of a signal X(n) is implemented using two critically-sampled DWTs in parallel on the same data.

2.2 Directional Selectivity

Ordinary DWT gives information about three orientational directions of a pixel where as DT-CWT gives information about six orientational directions of a pixel.

Fig -3: Orientational directions in DT-CWT

In DT-CWT, there are six sub bands that give information about the details of an image. The six subband filters are oriented at angles ±15 , ±45 , ±75 degrees.

2.3 Byes Thresholding

Let the observation model be Y = X+N ,where Y, X and N are wavelet coefficients of noisy, original and noise images respectively with X and N independent of each other.

$$\sigma Y^2 = \sigma X^2 + \sigma N^2$$

Where σY^2 is the variance of noisy image. An estimate of σY^2 can be found by

$$\sigma Y^2 = \left(\frac{1}{n^2}\right) \sum_{i,j=0}^{n} Yij^2$$

Where n×n is the size of the subband under consideration

Variance of noise can be estimated by the formula as given below:

$$\sigma N = \frac{\text{Median}(|Yij|)}{0.6745}, \quad Yij^2 \in \text{ subband HH.}$$

Where HH is the subband containing finest level diagonal details. Now the threshold value is given by

$$\hat{T} = \frac{\hat{\sigma}_N^2}{\hat{\sigma}_X}$$

Where

$$\sigma X = \sqrt{\max(\sigma Y^2 - \sigma N^2), 0}$$

In the case that $\sigma N^2 \geq \sigma Y^2$, σX is taken to be 0. That is, $\hat{T}$ is 1, or, in practice, $\hat{T} = \max(|Yij|)$, and all coefficients are set to 0. This happens at times when σN is large [8].

3. DESIGN OVERVIEW AND ALGORITHM

3.1 Design Overview

The analysis and synthesis filter banks are as shown in figure 4 and 5 respectively. The dual-tree CDWT of a signal x(n) is implemented using two critically-sampled DWTs in parallel on the same data.

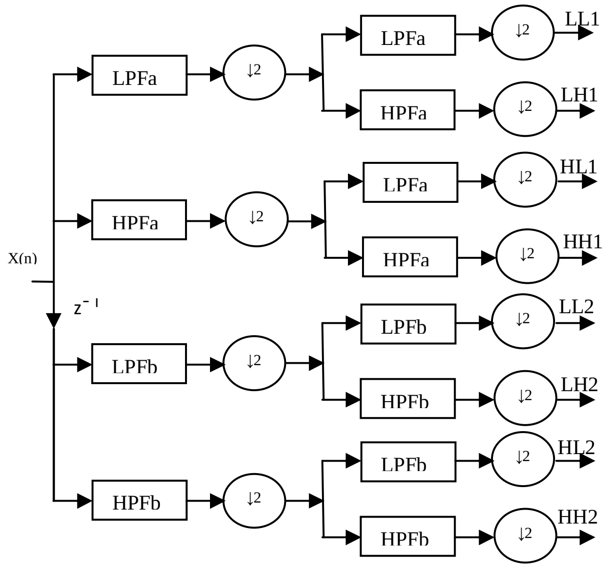


Fig -4: 2D- DWT analysis filter bank

The transform is two times expansive because for an N-point signal it gives 2N DWT coefficients. The analysis and synthesis bank filters are biorthogonal filters.

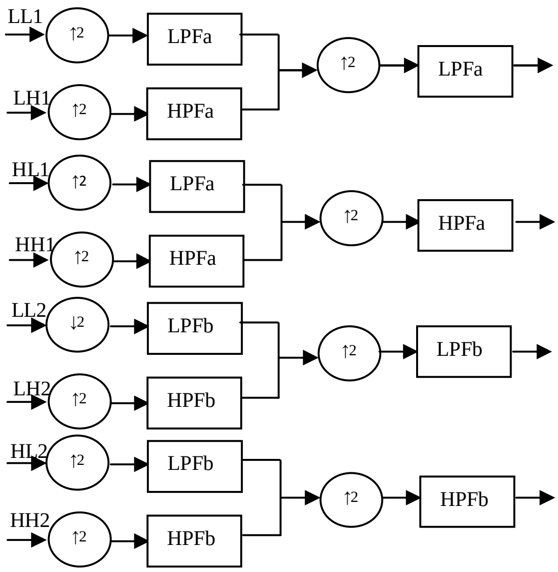


Fig – 5: 2D- DWT synthesis filter bank

3.2 Algorithm

1. Observed image that contains Gaussian noise is fed to the first tree and its shifted version is given to the second tree.
2. One level decomposition is done in each tree by the use of low pass and high pass filtering along with down sampling which results in four sub bands for each tree.
3. LH, HL and HH bands of each tree are thresholded separately. The threshold value is found by using Byes thresholding .
4. The corresponding sub bands of both tree are averaged together to produce one LL, HL, LH, HH sub band each.
5. Filtered output is obtained by taking the inverse transform of the resultant sub bands.

4. RESULTS AND DISCUSSIONS

Results are analysed based on performance metrics like MSE (Mean Squared Error) and PSNR (Peak Signal to Noise Ratio). MSE represents the squared error between the denoised and the original image, whereas PSNR is the ratio between the maximum possible value (power) of a signal and the power of distorting noise that affects the quality of its representation. Many of the signals have a very wide dynamic range. Therefore, the **PSNR** is usually expressed in terms of the logarithmic decibel scale. A low value for MSE denotes less error and an improvement in PSNR denotes how effectively the denoising is performed. It was found that the use DT-CWT filtering resulted in improved PSNR when compared to ordinary DWT. DT-CWT retains the original picture information but reduces the noise. Results obtained using various images are shown below.

(a)

(b) (c)

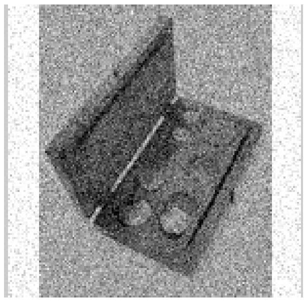
Fig – 6: Lena image: (a) noisy input image (b) DWT filtered output (c) DT-CWT filtered output.

(a)

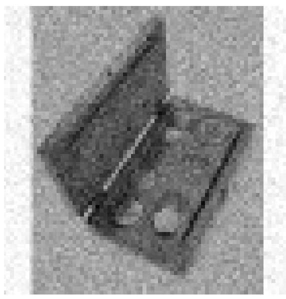
(b) (c)

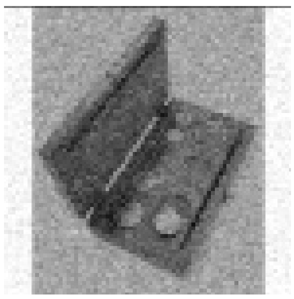
Fig – 7: Cameraman image : (a) noisy input image (b)DWT filtered output (c) DT-CWT filtered output.

Results obtained using ‘Lena’ image of size 512*512 pixels is shown in figure 6. Figure 7 shows the results obtained using a ‘Cameraman’ image of size 204*204 pixels and figure 8 is for a ‘Grain weight’ image of size 150*150 pixels.



(a)





(b) (c)

Fig – 8: Grain weights image: (a) noisy input image (b) DWT filtered output (c) DT-CWT filtered output.

Table 1: Results

Images	DWT		DT-CWT	
	MSE	PSNR(db)	MSE	PSNR(db)
Lena (size:512*512)	0.0050	52.979	0.0042	56.3923
Cameraman (size: 204*204)	0.0229	37.7504	0.0158	41.4799
Grain weight (size :150*150)	0.0111	45.0200	0.0091	47.0003

MSE and PSNR for these test images have been shown in table 1. The difference in size and image contents results in varying PSNR for different images.

5. CONCLUSIONS

It was found that the use of DT-CWT resulted in improved PSNR when compared to that of ordinary DWT. This improvement is due to near shift invariance and good directionality of wavelet coefficients. This can be thought as an efficient method which will be very useful in critical applications like satellite imaging, remote explorations, medical imaging etc.

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BIOGRAPHIES

Athira K Vijay received Bachelor of Engineering degree from the Department of Electronics and Communications, Karavali Institute of Technology, Visvesvaraya Technological university, Belgaum, India in 2011. Since 2012, she has been an M.Tech student in the Department of Electronics and Communication with specialization in VLSI and Embedded systems, TocH Institute of Science and Technology, India. Her research interests include image and video processing, robotics.

Prof. M. Mathurakani has graduated from Alagappa Chettiar College of Engineering and Technology of Madurai University and completed his masters from PSG college of Technology, Madras University. He has worked as a Scientist in Defence Research and development organization (DRDO) in the area of signal processing and embedded system design. He was honoured with the DRDO Scientist of the year award in 2003.Currently he is a professor in Toc H Institute of Science and Technology, Arakunnam. His area of research interest includes signal processing algorithms, embedded system implementations, reusable architecture and communication systems.