PRODUCT OPTIMIZATION BY SCHEDULING FORMULATION & OPTIMAL STORAGE STRATEGY FOR MULTIPRODUCT BATCH PLANTS

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Abstract

This contribution involves the formulation that addresses the minimization of both fresh water intake and waste water generation simultaneously. It optimizes the product and reactor utilization. The production optimization for this problem is based on batch product scheduling of MINLP formulation using commercial solver GAMS 24.1.1. The main objective is to enhance the profitability, minimizing waste generation and reducing fresh water intake. This case study shows that the number of batches within the same source of available resources can be increased by 12.24% and the profitability can be increased to 13.36%. On the other hand waste generation is reduced to almost 33% because of recycle and re-use opportunities. This suggests that by optimizing production, fresh water intake and waste water generation in the same scheduling formulation will minimize the operating cost and reduce the environmental impact significantly. The comparative study has been considered whether to have individual tanks of low and high COD nearer to each reactor or to have two centralized tanks, one for each low and high COD. The results of same have shown that the centralized (bigger) tanks occupies significant low space and they are highly economical. The efficacy of the results presented are demonstrated by taking a case study of pharmaceutical industry located in GIDC Vadodara, India.

Keywords: *Wastewater minimization, Scheduling, Fresh water intake, Holding tank Multiproduct batch plant*

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1. INTRODUCTION

Batch plants are getting serious attention because of their suitability and sustainability for the production of small lots and value added products. More than that their flexibility allows them to produce variety of products by sharing the same sort of equipments [1]. Various products of polymer, pharmaceutical, food and beverages and specialty chemicals are manufactured through batch process as their demand is highly influenced by seasonal variation, changing scenario of market [2]. There is no doubt about the flexibility of batch plant but they have lot of challenges towards design, synthesis and operation in comparison with continuous plants. The profitability[1][2] of these batch plants is highly dependent on the ways they synthesize, design and optimally operated so the modeling, optimum design of batch plant operation is important part of getting economic benefit[1][2]. Significant cost savings can be achieved or production efficiency can be increased through appropriate production optimization and scheduling for a production plant[2]. Because of the growing awareness in the society for the environment, increasing scarcity of water and stringent pollution norms, there is a huge opportunity for the application of recycling technology[3]. The fresh water and effluent treatment cost is moving significantly higher due to environmental sustainability. It has motivated the process industry to find new and alternate ways to curtail the fresh water consumption and waste water generation[4]. Serious measures are considered by process plants towards the minimization of fresh water consumption via creating recycle and reuse opportunity within the plant[5].The production cost can be significantly reduced by minimizing the effluent generation which ensures sustainable growth in the business environment[6][7]. So special effort is required to develop methodology that offers opportunity for both production optimization and waste water minimization. This effort would definitely provide the better production schedule which improves efficiency of batch plant when it is compared with optimization of water separatel.

2. CASE STUDY OF PHARMACEUTICAL PLANT

In this problem, an industrial case study represents a mathematical technique that addresses optimization of water usage, while simultaneously optimizing the no. of batches to be produced and profitability.

3. PROBLEM STATEMENT

This problem has been taken from Gujarat based pharmaceutical unit for single plant having 12 products and 12 reactors. Here batch size, batch length of each product, available time of each reactor, minimum no. of batches to be produced for each product is available. The reactor washing data after each batch of individual product is also available.

Table--1: Industrial Data**:** Complete data of 12 Products and Time Cycle for the same

Compound along with stage	Existing batches	Batch size(kg)	Quantity of product (kg)	Batch cycle time (hrs)	Total production time(hrs)	Profit /batch (Rs.)
a	38	230	9430	180	7380	23000
b	54	75	4500	120	7200	30000
c	4	9	36	70	280	45000
d	9	14	140	30	300	35000
e	9	20	200	100	1000	40000
f	11	9	108	20	240	45000
g	3	60	180	50	150	60000
h	4	30	150	60	300	60000
i	5	35	175	70	350	70000
j	4	15	60	20	80	45000
k	2	140	280	75	150	42000
l	4	3	12	250	1000	60000
Total	**147**					

Table- 2: Industrial Data**:** Complete data of 12 Reactors and Time Cycle for the same

Reactors	a	b	c	d	e	f	g	h	i	j	k	l	Total Time (hours)	Net available time (hours)
R-01			980										1188	980
R-02		1800											1992	1800
R-03	900	1080											2196	1980
R-04	540	360					400						1535	1303
R-05	1080	360					250						1926	1690
R-06	1260	240					100						1808	1600
R-07	1260	240					50						1742	1550
R-08		1680		180		20							2153	1880
R-09		1440		180		120							2043	1740
R-10	1260					60							1524	1320
R-11	1620												1788	1620
R-12	1620												1788	1620

Table- 3: Actual fresh water required (liters) through washing.

Reactors	Size of Reactor in [Lit]	a	b	c	d	e	F	G	h	i	j	k	l
No of Washes Required		7	5	3	3	3	3	5	4	4	3	5	3
R-01	750	5250	3750	2250	2250	2250	2250	3750	3000	3000	2250	3750	2250
R-02	1200	8400	6000	3600	3600	3600	3600	6000	4800	4800	3600	6000	3600
R-03	2000	14000	10000	6000	6000	6000	6000	10000	8000	8000	6000	10000	6000

R-04	2500	17500	12500	7500	7500	7500	7500	12500	10000	10000	7500	12500	7500
R-05	3000	21000	15000	9000	9000	9000	9000	15000	12000	12000	9000	15000	9000
R-06	3000	21000	15000	9000	9000	9000	9000	15000	12000	12000	9000	15000	9000
R-07	2000	14000	10000	6000	6000	6000	6000	10000	8000	8000	6000	10000	6000
R-08	630	4410	3150	1890	1890	1890	1890	3150	2520	2520	1890	3150	1890
R-09	1000	7000	5000	3000	3000	3000	3000	5000	4000	4000	3000	5000	3000
R-10	2000	14000	10000	6000	6000	6000	6000	10000	8000	8000	6000	10000	6000
R-11	2500	17500	12500	7500	7500	7500	7500	12500	10000	10000	7500	12500	7500
R-12	3000	21000	15000	9000	9000	9000	9000	15000	12000	12000	9000	15000	9000

Table- 4: High COD waste water Produced Through Washing

Reactors	Size of Reactor in [Liters]	a	b	c	d	e	f	g	h	i	j	K	l
No of Washes Required		4	3	2	2	2	2	3	2	2	2	3	2
R-01	750	3000	2250	1500	1500	1500	1500	2250	1500	1500	1500	2250	1500
R-02	1200	4800	3600	2400	2400	2400	2400	3600	2400	2400	2400	3600	2400
R-03	2000	8000	6000	4000	4000	4000	4000	6000	4000	4000	4000	6000	4000
R-04	2500	10000	7500	5000	5000	5000	5000	7500	5000	5000	5000	7500	5000
R-05	3000	12000	9000	6000	6000	6000	6000	9000	6000	6000	6000	9000	6000
R-06	3000	12000	9000	6000	6000	6000	6000	9000	6000	6000	6000	9000	6000
R-07	2000	8000	6000	4000	4000	4000	4000	6000	4000	4000	4000	6000	4000
R-08	630	2520	1890	1260	1260	1260	1260	1890	1260	1260	1260	1890	1260
R-09	1000	4000	3000	2000	2000	2000	2000	3000	2000	2000	2000	3000	2000
R-10	2000	8000	6000	4000	4000	4000	4000	6000	4000	4000	4000	6000	4000
R-11	2500	10000	7500	5000	5000	5000	5000	7500	5000	5000	5000	7500	5000
R-12	3000	12000	9000	6000	6000	6000	6000	9000	6000	6000	6000	9000	6000

Table- 5: Waste water(Liters) which can be reused for further washing:

React-ors	Size of Reactor in [Lit]	a	b	c	d	e	f	G	h	i	j	K	l
No of Washes Reused		3	2	1	1	1	1	2	2	2	1	2	1
R-01	750	2250	1500	750	750	750	750	1500	1500	1500	750	1500	750

R-02	1200	3600	2400	1200	1200	1200	1200	2400	2400	2400	1200	2400	1200
R-03	2000	6000	4000	2000	2000	2000	2000	4000	4000	4000	2000	4000	2000
R-04	2500	7500	5000	2500	2500	2500	2500	5000	5000	5000	2500	5000	2500
R-05	3000	9000	6000	3000	3000	3000	3000	6000	6000	6000	3000	6000	3000
R-06	3000	9000	6000	3000	3000	3000	3000	6000	6000	6000	3000	6000	3000
R-07	2000	6000	4000	2000	2000	2000	2000	4000	4000	4000	2000	4000	2000
R-08	630	1890	1260	630	630	630	630	1260	1260	1260	630	1260	630
R-09	1000	3000	2000	1000	1000	1000	1000	2000	2000	2000	1000	2000	1000
R-10	2000	6000	4000	2000	2000	2000	2000	4000	4000	4000	2000	4000	2000
R-11	2500	7500	5000	2500	2500	2500	2500	5000	5000	5000	2500	5000	2500
R-12	3000	9000	6000	3000	3000	3000	3000	6000	6000	6000	3000	6000	3000

4. PROBLEM SOLUTION

The main objectives of the problem are to develop a mathematical program to optimize the no of batches of product produced, minimize the waste water generation by optimizing the maximum recovery of water which can reduce the usage of fresh water. Hence, it can reduce the production cost and increase the profitability

Four objectives equation considered separately are

$$Zprod = \Sigma((i,j),x(j))$$

$$Zprofit = \Sigma((i,j),f(j)*x(j))$$

$$Zwaste = \Sigma((i,j),d(I,j)*x(j))$$

$$Zrecycle = \Sigma((i,j),e(I,j)*x(j))$$

With 144 decision variables X(i,j) where i is for reactor and there are total 12 reactors [1 to 12].
Where j is for product and there are total 12 products [a to j]
Six parameters considered for this program
a(i) = Total time available in hours for each reactor
b(j) = Total time available in hours for each Product
c(j) = Batch size in kg for each product
d(i,j)= Fresh water required for washing each reactor to each
product
e(i,j)= Recycle water collected after washing each reactor to
each product
g(i,j)= Waste water generated after washing each reactor to each product
f(j) = Profit in Rs. per batch for each product

5. CONSTRAINTS

Available time of each reactor, Batch Size of Each product, No of batches of each product, Minimum production of each product and waste generation through successive washing of each reactor has been considered to solve the given model
Constraints Equations

– Waste as a lim $\Sigma((i,j),d(i,j)*x(i,j))/90<=19000$
– Reactors Time $\Sigma(j,c(j)*x(i,j)) <= a(i)$
– Batch length $\Sigma(i,c(j)* x(i,j)) <= b(j)$

6. RESULTS & DISCUSSION

The results of this case study is obtained using GAMS 24.1.1 on a 2.4 GHz,. The computational results for this problem are compared with actual data of the company on quarterly basis.

Table- 6: Objective value

Optimized No of Batches produced	165
Waste water Generated	1701.7 kilo liters
Optimum Amount of water Recycled	7.737 kilo liters
Maximized Profit	Rs. 58.02/-Lakhs

Table- 7: GAMS result of no of batches produced for product and reactor wise

Reactors	a	b	c	d	E	f	g	h	i	j	k	l	Reactor
1	1				5					5	2		13
2		15											15
3	1	15											16
4	7												7
5	5		1	4		2	5	5					22
6	7			6		6							19
7	1	11											12
8		1			5							4	10
9		13	2										15
10	4	5											9
11	9												9
12	6		1			6			5				18
Total no Batches	41	60	4	10	10	14	5	5	5	5	2	4	165

Table- 8: Actual result of no of batches produced for product and reactor wise

Reactors	a	b	c	d	E	f	g	h	i	j	k	l	Reactor
1			4					3					7
2		13											13
3	5	7				1		1					14
4	3	3			5	1							12
5	5	3											8
6	4	2			4	3	2		2				17
7	6	2					1			3			12
8		13		5									18
9		11		4		4							19
10	4					2			3		2		11
11	6									1		2	9
12	5											2	7
Total no Batches	38	54	4	9	9	11	3	4	5	4	2	4	147

The results shown in Table- 7 are the optimized no of batches that can be produced for the given data and actual constraint. These results are compared with the actual production of the plant which is available in Table- 8. Hence, GAMS data suggests that for the same set of data 18 additional batches can be produced.

Table- 9: Comparison of GAMS Data v/s Actual Data Product wise

Type of Product	Actual Data (number of batches)	GAMS Data (number of batches)
A	38	41
B	54	60
C	4	4

D	9	10
E	9	10
F	11	14
G	3	5
H	4	5
I	5	5
J	4	5
K	2	2
L	4	4
Total	147	165

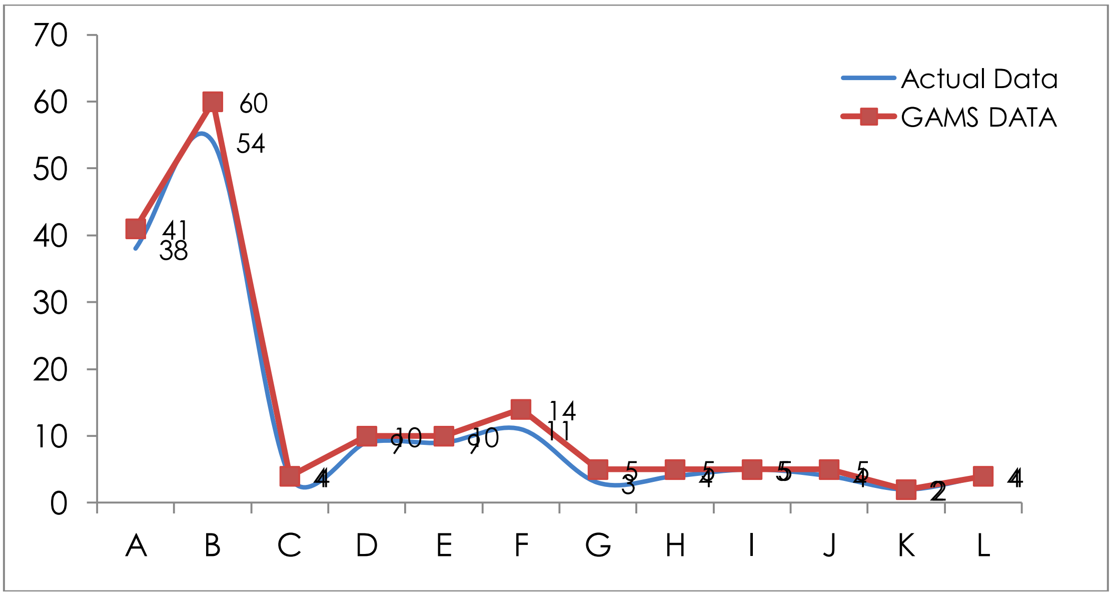

Chart 1 Comparison of GAMS Data v/s Actual Data Product wise

The GAMS data is compared with actual data on the basis of batches produced of each product the result is shown in Table- 9 and the same has been presented in Chart1. This suggests that the batches of most of the product can be optimized compared to actual production.

Table- 10: Comparison of GAMS Data v/s Actual Data Reactor wise

Reactor No	Actual Data (number of batches)	GAMS Data (number of batches)
1	9	13
2	12	14
3	13	16
4	12	11
5	20	24
6	23	26
7	10	9
8	9	10
9	10	11
10	8	7
11	9	9
12	12	15
Total	147	165

The GAMS data is compared with actual data on the basis no of batches produced in each reactor the result is shown in Table-10 and has been presented in Chart 2. This suggests that the batches of most of the product can be optimized compare to actual production which is possible because it takes the optimum occupancy of most of reactors.

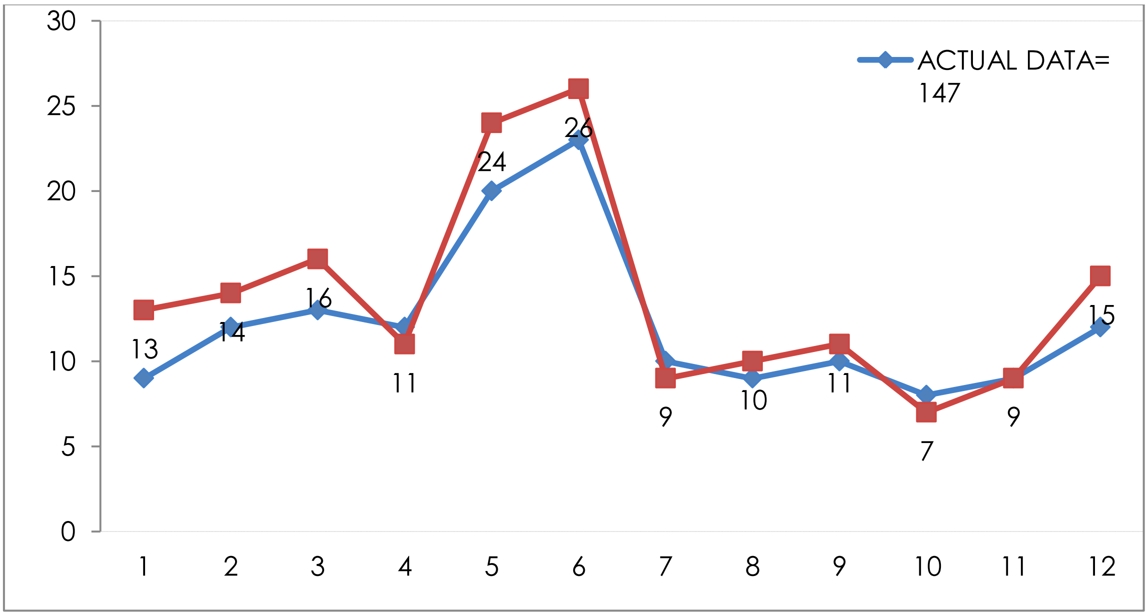

Chart 2 Comparison of GAMS Data v/s Actual Data Reactor wise

Table- 11: Flexibility of GAMS Program for future planning

Sr.No	Total waste water (per day) as a limit in liters	No Of Batches	Recycle water in liters	Profit in lac of Rs.
1	19000	165	7737	58
2	17000	160	7292	55.6
3	15000	126	6605	40.9
4	12000	92	5412	31
5	10000	64	4626	21.4
6	8000	60	3778	19.7
7	5000	47	2492	17.02

The GAMS program offers so much flexibility for future planning. The no of batches of each product produced is optimized by considering the total waste on daily basis as a major constraint. Total waste water generated and recycle water for further use is calculated. The profitability is also calculated by considering the fresh water cost.

Table- 12: Summary of optimized results

	GAMS Result	Actual Data	Benefit
No of Batches	165	147	12.24 % Increase
Waste water Generated [liters]	10,02000	14,94,500	32.95 % Decrease
Fresh Water Reduction [liters]	1013670	1698300	40.31% Decrease
Profitability [Rs.]	58,02,000/-	51,18,000/-	13.36 % Increase
Profitability considering fresh water cost [Rs.]	64,98,330/-	53,21,800/-	22.10 % Increase

The overall comparative study has been carried out for GAMS results with actual plant data and same is summarized in Table-12.. This indicates that the GAMS results have shown significant benefit in all the aspects like production, waste water generation, fresh water requirement and in terms of profitability.

7. AREA & SPACE REQUIREMENT OF HOLDING TANKS

The major part of waste is generated through washing of reactors carried out after the changeover of batch products. This waste is further categorized as high COD waste which is directed to ETP plant and low COD waste water which can be recycled and re used for the next washing cycle. The two cases have been considered either to have individual holding tank along with each reactor or to have two bigger centralized holding tank which can take care of high and low COD water of entire 12 reactors.

Table- 13: Area & Space Requirement for Re-Cycle (Low COD) Water Tank

Reactor	Re-cycle water in liters	Net capacity in liters	Overall height in (mm)	Overall diameter in (mm)	Area of storage tank base Area= πr^2(m^2)	Cost of frp tank in Rs.
R-01	1688	2000	1630	1370	1.47	45000
R-02	2700	3000	1860	1530	1.84	65000
R-03	4500	5000	2050	1900	2.83	100000
R-04	5625	6000	2250	1950	2.98	120000
R-05	6750	7000	2400	2050	3.30	140000
R-06	6750	7000	2400	2050	3.30	140000
R-07	4500	5000	2050	1900	2.83	100000
R-08	1418	1500	1350	1235	1.20	35000
R-09	2250	2500	1650	1500	1.77	55000
R-10	4500	5000	2050	1900	2.83	100000
R-11	5625	6000	2250	1950	2.98	115000
R-12	6750	7000	2400	2050	3.30	135000
				(30.64) m^2		1150000
TOTAL	53055	60000	4875	4250	14.18 m^2	800000

Table- 14: Area & Space Requirement for Waste (High COD) Water Tank

Rector	High COD	Net capacity [liters]	Overall ht. In (mm)	Overall dia. in (mm)	Base area= $A = \pi r^2$(m^2)	Cost of FRP tank in Rs.
R-01	2250	2500	1650	1500	1.77	55000
R-02	3600	4000	1870	1720	2.32	85000
R-03	6000	7000	2400	2050	3.30	135000
R-04	7500	8000	2500	2150	3.62	150000
R-05	9000	10000	2700	2300	4.15	190000
R-06	9000	10000	2700	2300	4.15	190000
R-07	6000	7000	2400	2050	3.30	135000
R-08	1890	2000	1630	1370	1.47	45000
R-09	3000	3500	1800	1650	2.14	75000
R-10	6000	7000	2400	2050	3.30	135000
R-11	7500	8000	2500	2150	3.62	150000
R-12	9000	10000	2700	2300	4.15	190000
					(37.29) m^2	1535000
TOTAL	70740	800000	5300	4700	17.34 m^2	1100000

The comparative study has been considered whether to have individual tanks of low COD and high COD nearer to each reactor or to have two centralized tanks one for each low and high COD, which should take care of all twelve reactors waste water. Area & Space Requirement for low and high COD tanks for both the cases is calculated and same is presented in Table- 13 and 14. If we combine the results presented in Table- 13 and 14, the total area required for the centralized tanks (High COD tank 17.35 +Low COD tank 14.18) is 31.5 m^2 whereas the total area required for individual tanks(High COD tank37.29 +Low COD tank 30.64) is 68 m^2 which is almost 53% less. The total cost required for centralized tank(High COD tank Rs.11 lakh+ +Low COD tank Rs.8 lakh) is Rs.19 lakh which is almost 30% less than the cost required for individual tanks (High COD tank Rs.15.35 lakh+ +Low COD tank Rs.11 lakh) Both comparisons clearly indicate that centralized tank is much better than the individual tank in terms of area and cost

8. CONCLUSION

In this case study the product optimization of multiproduct pharmaceutical plant having 12 different products and 12 reactors of different size is done successfully by using MINLP formulation and commercial solver GAMS 24.1.1 .This study shows that the no. of batches within the same source of available resources can be increased by 12.24% and the profitability can be increased to 13.36%. On the other hand waste generation is reduced to almost 33% because of recycle and re-use opportunities. The fresh water intake can be reduced to 40% which would also help to increase profitability by 22%. The comparative study of holding tanks has shown that the centralized (bigger) tanks of each low as well high COD occupies 50% of area compared to individual holding tanks and even cost wise also the centralize tanks are 29% cheaper than individual holding tanks.

REFERENCES:

[1] Seid, R., Majozi, Optimization of energy and water use in multipurpose batch plants using an improved mathematical formulation. Chemical Engineering Science, 2014 – Elsevier

[2] Esmael R. Seid † and Thokozani Majozi Design and Synthesis of Multipurpose Batch Plants Using a Robust Scheduling Platform. *Ind. Eng. Chem. Res.*, 2013, 52 (46), pp 16301–16313

[3] R. Parand, H. M. Yao, M. O. Tadé, and V. Pareek. Composite Table- Algorithm - A Powerful Hybrid Pinch Targeting Method for Various Problems in Water Integration. International Journal of Chemical Engineering and Applications, Vol. 4, No. 4, August 2013

[4] Bhanu Prasad Behera, Dr. Rati Ranjan Dash, Dr.Arun kumar Panda3.Evaluation of Minimum Makespan using Modified Evolutionary Algorithm Journal of Engineering Innovation and Research Volume: IV, Issue: 1

[5] Adekola, Stamp, J. D., Majozi, T. Simultaneous Optimization of Energy and Water in Multipurpose Batch Plants, computers in operations and information processing.

[6] Marianthi Ierapetritou. Modeling and Managing Uncertainty in Process Planning and Scheduling. Springer Optimization and Its Applications 30, DOI 10.1007/978-0-387-88617-63

[7] Jaime Cerdá Cerda Dynamic scheduling in multiproduct batch plants Computers & Chemical Engineering. Volume 27, Issues 8–9, 15 September 2003, Pages 1247–1259

[8] P. Oliver' ,R. Rodríguez, S. Udaquiola Water use optimization in batch process industries. Design of the water network Journal of Cleaner Production Dec.2008 1275-1286

[9] Majozi, T., Gouws, J.F., A mathematical optimization approach for wastewater minimization in multipurpose batch plants: Multiple contaminants. *Comp. Chem. Eng*. 33, 1826-1840. 2009.

[10] Zukui Li, Marianthi Ierapetritou Process scheduling under uncertainty: Review and challenges. Computers & Chemical Engineering , vol. 32, no. 4-5, pp. 715-727, 2008

[11] Méndez, C.A., Cerdá, J., Grossmann, I.E., Harjunkoski, I., Fahl, M., 2006. State of- the- art review of optimization methods for short-term scheduling of batch processes. *Comput. Chem. Eng.* 30, 913-946.

[12] Majozi, T., Gouws, J.F., 2009. A mathematical optimization approach for wastewater minimization in multipurpose batch plants: Multiple contaminants. *Comp. Chem. Eng*. 33, 1826-1840.

[13] Seid, R., Majozi, T., 2012. A robust mathematical formulation for multipurpose batch plants**.** *Chem. Eng. Sci*. 68, 36–53.

ANNEXURE

GAMS CODE:

```
set i /1,2,3,4,5,6,7,8,9,10,11,12/
j /a,b,c,d,e,f,g,h,i,j,k,l/;
Parameters
a(i) /1 980, 2 1800, 3 1980, 4 1300, 5 1690, 6 1600, 7 1550, 8 1850, 9 1740, 10 1320, 11 1620,12 1620/
b(j) / a 7380, b 7200, c 280, d 300, e 1000, f 280, g 250, h 300, i 350, j 100, k 150, l 1000/
c(j) / a 180, b 120, c 70, d 30, e 100, f 20, g 50, h 60, i 70, j 20, k 75, l 250/
f(j) / a 23000, b 30000, c 45000, d 35000, e 40000, f 45000, g 60000, h 60000, i 70000, j 45000, k 42000, l 60000/;
Table d(i,j) :   Fresh  water  required (liters) for washing each reactor to each product
a      b      c              d      e          f        g      h          i      j      k      l
1   5250   3750   2250   2250   2250   2250   3750   3000   3000   2250  3750   2250
2   8400   6000  3600   3600   3600    3600   6000   4800    4800  3600   6000  3600
3   14000 10000  6000   6000   6000    6000  10000  8000    8000  6000  10000 6000
4   17500 12500  7500   7500   7500    7500  12500 10000 10000  7500   12500 7500
5   21000 15000  9000   9000   9000   9000    15000 12000 12000  9000   15000 9000
6   21000 15000  9000   9000   9000   9000    15000 12000 12000  9000   15000 9000
7   14000 10000  6000   6000   6000   6000    10000  8000   8000   6000   10000 6000
8   4410    3150   1890   1890   1890   1890    3150   2520   2520  1890    3150  1890
9   7000    5000   3000   3000   3000   3000    5000  4000   4000   3000    5000  3000
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```
10  14000 10000  6000  6000   6000   6000    10000 8000   8000   6000     10000 6000
11  17500 12500  7500  7500   7500   7500    12500 10000 10000 7500     12500 7500
12  21000 15000  9000   9000   9000   9000    15000 12000 12000 9000    15000  9000

Table E(i,j): Recycle water (liters)  collected after  washing each reactor to each product
a              b         c           d            e           f             g           h            i            j            k           l
1   2250  1500      750        750        750        750        1500      1500      3000      3000      6000      750
2   3600  2400   1200   1200   1200     1200    2400   2400       2400     1200      2400   1200
3   6000   4000   2000   2000   2000   2000    4000  4000        4000     2000      4000   2000
4   7500   5000   2500   2500   2500   2500    5000  5000         5000     2500     5000   2500
5   9000   6000   3000   3000   3000   3000    6000  6000         6000    3000      6000   3000
6   9000   6000   3000   3000   3000   3000    6000  6000         6000    3000      6000   3000
7   6000   4000   2000   2000   2000   2000    4000  4000        4000     2000      4000   2000
8   1890   1260    630     630     630    630      1260  1260       1260       630      1260   630
9   3000   2000   1000   1000   1000   1000    2000  2000         2000     1000      2000   1000
10  3600  2400   1200   1200   1200     1200    2400   2400       2400     1200      2400   1200
11  7500   5000   2500   2500   2500   2500    5000  5000         5000     2500     5000   2500
12  9000   6000   3000   3000   3000   3000    6000  6000         6000    3000      6000   3000

Table G(i,j): Waste water (liters)  generated after washing each reactor to each product
a       b         c        d          e        f        g       h        i        j      k        l
1   3000  2250   1500   1500   1500   1500  2250  1500  1500  1500 2250  1500
2   4800  3600   2400   2400   2400   2400  3600  2400  2400  2400 3600  2400
3   8000  6000   4000   4000   4000   4000  6000  4000  4000  4000 6000  4000
4   10000 7500   5000   5000   5000   5000  7500  5000  5000  5000 7500  5000
5   12000 9000   6000   6000   6000   6000  9000  6000  6000  6000 9000  6000
6   12000 9000   6000   6000   6000   6000  9000  6000  6000  6000 9000  6000
7   8000  6000   4000   4000   4000   4000  6000  4000  4000  4000 6000  4000
8   2520  1890   1260   1260   1260   1260  1890  1260  1260  1260 1890  1260
9   4000  3000   2000   2000   2000   2000  3000  2000  2000  2000 3000  2000
10  8000  6000   4000   4000   4000   4000  6000  4000  4000  4000 6000  4000
11  10000 7500   5000   5000   5000   5000  7500  5000  5000  5000 7500  5000
12  12000 9000   6000   6000   6000   6000  9000  6000  6000  6000 9000  6000 ;
variable z,z1,z2,z3;
integer variable x(i,j);
equations Production,Recycle,Waste,profit,a1,a2,a3,a4,a5,a6,a7,a8,a9,a10;
Production..        z=e=sum((i,j),x(i,j));
Recycle..           z1=e=sum((i,j),E(i,j)*x(i,j))/90;
Profit..             z2=e=sum((i,j),f(j)*x(i,j));
Waste..             z3=e=sum((i,j),G(i,j)*x(i,j))/90;
a1..                sum((i,j),d(i,j)*x(i,j))/90=e=19500;
a2(i)..             sum(j,c(j)*x(i,j))=L=a(i);
a4..                c("c")*x("1","c")=l=b("c");
a3(j)..             sum(i,c(j)* x(i,j))=L=b(j);
a5(i,"a")..         x(i,"a")=l=9;
a6(i,"b")..         x(i,"b")=l=15;
a7(i,"d")..         x(i,"d")=l=6;
a8(i,"f")..         x(i,"f")=l=6;
a9(i,"g")..         x(i,"g")=l=8;
a10(i,"e")..        x(i,"e")=l=5;
Model apd /all/;
Solve apd using MINLP maximizing Z1;
```