KINEMATIC PERFORMANCE ANALYSIS OF 4-LINK PLANAR SERIAL MANIPULATOR

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Abstract

This investigation is all about determining condition number for four link planner serial manipulator. Condition number which measures the accuracy of end effector velocity is obtained from the Jacobian matrix of the four link planner serial manipulator. Inverse of a non-square matrix in the calculation of the condition number. Pseudo inverse procedure is used for the calculation of inverse of jacobian matrix using condition number. Isotropic condition for manipulator is found. "MAT Lab" 6.1.0 version is used to determine the condition number which is recently developed for complex matrix manipulations. A computer program in "MAT Lab" is written for finding Jocobian matrix, inverse of Jacobian matrix, norm of jacobian and norm of Jacobian matrix and the condition number for various input joint angles and link lengths.

Keywords: Serial manipulator, Condition number, Jacobian matrix, MAT Lab,

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1. INTRODUCTION

Since the industrial Revolution there has been an ever-increasing demand to improve product quality and reducing cost. At the beginning of the 20th century, Ford motor company introduced the concept of mass production. In mass production each production machine is designed to perform a predetermined task, every time a new model is introduced, the entire production line has o be shut down and retooled. Retooling production line during each model change can be very expensive. To overcome the above problem robot manipulators have been introduced by manufacturing industries for performing certain production tasks, such as material handling, spot welding, spray painting and assembling. As robot manipulators controlled by computers or microprocessors, they can easily be reprogrammed for different tasks. Hence there is no need to replace these machines during a model change. We call this type of automation as flexible automation.

Robotics is the science of designing and building robots for real life applications in automated manufacturing and other non- manufacturing environments. Robots are the means of performing multi actives for man's welfare and integrate manner, maintain their arm flexibility to do any work effecting enhanced productivity, guarantying quality, assuring reliability and ensuring safety to the workers. There is only one definition of an industrial robot that is internationally accepted. It was developed by group of industrial scientists from the robotics industries association (R.I.A) formerly, Robotics Institute of American 1979. According to them, "An industrial robot is a reprogrammable, multi-functional manipulator, designed to move material, parts, tools or special devices through variable programmed motions for the performance of a variety of tasks'. An industrial robot is a general purpose programmable machine which possesses certain anthropomorphic or human like characteristics. The most typical human like characteristic of present day robots is their movable arms. The Word the Writer Kernel Capek introduced in "Robot" to the English language in 1921. In this work, the robots are machines that resemble people, but work tirelessly. Among science fiction writers, Isaac Asimov has contributed a number of stories staring in 1939 and introduced the term "Robotic".

The seventeenth and eighteenth centuries, there were number of mechanical devices that had some of the features of robots. During the late 1940's research programs were started the Oak Ride and Argon national laboratories to develop remotely controlled mechanical manipulator for the handling radioactive material. In the mid 1950's George C. Devol developed a device. Further development of this concept by Devol and Joseph F. Engelbreges led to the industrial Robot. In the 1960 the flexibility of these machines were enhanced significantly by the use of sensors feedback. In the 1970 the Stanford arm a small electrically powered robot arm, developed at Stanford University. Some of the most serious work in Robotics began as these arms were used as robot manipulators. During 1970's great deal of research work focused on use of external sensors to facilitate manipulative operations. In the 1980's several of the programming system were produced. Today we viewed robotics as much broader field of work than we did just a few years ago dealing with research development in a number of interdisciplinary areas including kinematics, planning systems, sensing, programming languages and machine intelligence.

Moving the robot joints. Four types of joints used on commercial robots. The first two types adapted from a SCARA type robot. The second set a revolute type robot. A sliding joint which moves along the single Cartesian axis. It is also known as a linear joint, a Translational joint, Cartesian joint or a prismatic joint. The remaining joints are rotational joints. Since they all result in angular movement. Their chief distinction is the plane of motion that they support. In the first type, the two links are in different planes and angular motion is in the plane of the second link (θ_1), the second type provides rotation in a plane 90^0 from the line of either arm link (θ_2), in the case the two links always lie long a straight line. The third joint is a hinge type (θ_3) that restricts angular movement to the of both arm links. In this case the links always stay in the same plane. The combination of joints determines the type of robots. Most robot have three or four major joints that define their work envelope (i.e the area that their end effector can reach) actions that control wrist motion usually provide only tool orientation and do not affect work envelope.

2. ROBOT KINEMATICS

Robot kinematics is a study of the geometry of manipulator arm motions. Since the performance of specific tasks is achieved through the movement of manipulator arm linkages, kinematics is a fundament tool in manipulator design and control. A manipulator consists of a series of rigid bodies (links) connected by mean of kinematic pairs or joints. Joints can be essentially of two types; revolute and prismatic. The whole structure forms a kinematic chain. One end of the other end allowing manipulation of objects in space.

A kinematic chain is termed open when there is only sequence of links connecting the two ends of the chain. Alternatively, a manipulator contains a closed kinematic chain when a sequence of link forms a loop. For a robot to perform a specific task, the location of the end effector relative to the base should be established first. This is called position analysis problem. There are two types of position analysis problems; direct position or direct kinematic and inverse position or inverse kinematics problems. For direct kinematic, the joint variables are given and the problem is to find the location of the end effector. The key to the solution of the direct kinematics problem was the Denavit-Hartenberg representation. The relationship between the joint velocities and the corresponding effector linear and angular velocities and derived by differential kinematic. This relation described by matrix, termed geometric Jacobian, which depends on the manipulator configuration.

3.   THE   DENAVIT-HARTENBERG REPRESENTATION

For describing the kinematic relationship between a pair of adjacent links involved in a kinematic chain. The D-H notation is a systematic method of describing the kinematic relationship. The method is based on 4x4 matrix representation of a rigid body position and orientation. It uses minimum number of parameters to completely describe the kinematic relationship.
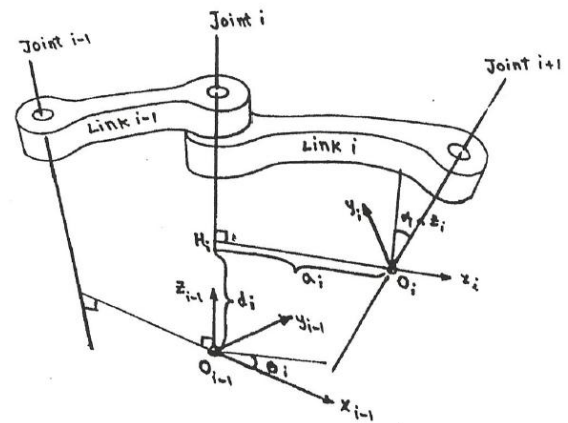

Fig. Shows a pair of adjacent links; link i-1 and link I and their associated joint i-1 and i+1; the origin of the ith frame is located at the inter section of joint axes i+1 and common normal between the axes I and i+1. The x_i axis is directed along the extension link of the common normal. While Z_i axis is along the joint axis i+1. Finally the y_i axis chosen such that relative resultant frame o_i-$x_i y_i z_i$ forms right hand coordinate systems.

θ_i is the angle between x_{i-1} and the common normal $H_i O_i$ measured about Z_{i-1} axis in right hand hand sense. Alpha is the angle between joint axis i and Z_i axis. The parameter a_i and α_i are constant and they are determined by the geometry of the link. θi and d_i are varies as the joint moves. Fig shows two link coordinates frames o_i-$x_i y_i z_i$ and o_{i-1}-$x_{i-1} y_{i-1} z_{i-1}$ and intermediate frame H_i-$x'_i y'_i z'_1$ attached at point H_i.

4. FORMULATION OF JACOBIAN MATRIX

The relationship between the joint velocities and corresponding end-effector linear and angular velocities are mapping by a matrix which is known as "Jacobian Matrix". Consider n- degree of mobility of manipulator. The direct kinematic equation can be written in the form

$$A(q) = \begin{pmatrix} R(q) & P(q) \\ O^T & 1 \end{pmatrix}$$

Where q=$[q_1 \ldots \ldots \ldots q_n]^T$ is the vector of joint variables. If 'q' varies end effector position and orientation varies.

The differential kinematics gives the relationship between joint velocities and end effector linear velocity and angular velocities. It is desired to express the end effector linear velocities p^o, angular ω as a function of joint velocities q^o by following relations.

$$P^0 = J_p(q)q_0 - (1)$$

$$W = J_0(q)_q - (2)$$

J_p is (3xn) Matrix relative to the contribution of joint velocities q^o to the end effector linear velocities P.

J_o is (3xn) Matrix relative to the contribution of joint velocities q^o to the end effector angular velocities ω.

The above 1 and 2 equations can be written as

$$V = \begin{pmatrix} P_o \\ \omega \end{pmatrix} = J(q)(q_0) \ldots\ldots\ldots\ldots(3)$$

Equation (3) represents the manipulator differential equation. J is (6xn) matrix is the manipulator geometric Jacobian matrix.

$$J = \begin{pmatrix} J_p \\ J_0 \end{pmatrix}$$

5. CONDITION NUMBER

Condition number of matrix A is defined as the product of the norm of A and the norm of A inverse.

Norm of A is given by $\|A\| = \sqrt{[\text{trace of } (AWA^T)]}$

Trace is the sum of the matrix.

Condition number of a matrix is always greater or equal to one. If it is close to one the matrix is well conditioned which means its inverse can be computed with good accuracy. If the condition number is large, then the matrix is ill-conditioned and the consumption of its inverse, or solution of linear system of equations is prone to large numerical errors. A matri that is not invariable has condition number equal to infinity.

Condition number shows how much a small in the input data can be magnified in the output.

Let Ax = b, then relative solution error is measured by

$$\|\$\|/\|X\| = \|A\| \|A^{-1}\| (\|\$b\|/\|A\|)$$

Where $\|A\| \|A^{-1}\| = Z$, condition number.
$\$b$ = small perturbations of b.
$\$$ = deviation of the solution of perturbed system from original solution.

When condition number criteria is applied to Jacobian matrix of the velocity transformation equation, $=J\theta$. Condition number of Jacobian is given by

$$Z = \|J\| \|J^{-1}\|$$

Therefore condition number of a Jacobian matrix gives how much error in joint velocities magnified into Cartesian velocities.

As the end effector moves from one location to another location the condition number varies. The points in the workspace of a manipulator are called isotropic points, when the condition number of matrix is one. Condition number is the best tool for the control of the velocity of end effector. Condition number of the accuracy of the end effector's velocity

Jocobian matrix for 4 link planner serial manipulator with respect to base frame:

$$^0J_4 = \begin{pmatrix} Z_0 X(p-p_0) & Z_1 X(p-p_1) & Z_2 X(p-p_2) & Z_3 X(p-p_3) \\ Z_0 & Z_1 & Z_2 & Z_3 \end{pmatrix}$$

Z_o, Z_1, Z_2, Z_3, are the unit vectors along the joint axis.
P, P_o, P_1, P_2, P_3 are the vectors defined from origin of link frame.

6. RESULTS AND DISCUSSIONS

Simplified notations are used for easy understanding and standardized notations are used for finding condition number and matrices too. Graphs were constructed varying each and every parameter and graphs were drawn for each and every table.

For the program it is clear that for link length ratios 1:1:1:1. The suitable condition number varying joint angle '2' is "1.0032" at 50^0. The suitable condition number varying joint angle '3' is "1.0473" at 40^0. The suitable condition number varying joint angle '4' is "1.0861" at 20^0.
   1.   The condition number when varying link length 1 "a_1" keeping all the terms constant joint '1' is 15^0, joint angle '2' is 50^0, joint angle '3' is 13^0, and joint angle '4' is 61^0, and with a_2:a_3:a_4 =1:1:1 is at a_1 =1 and the condition number is "1.0032".
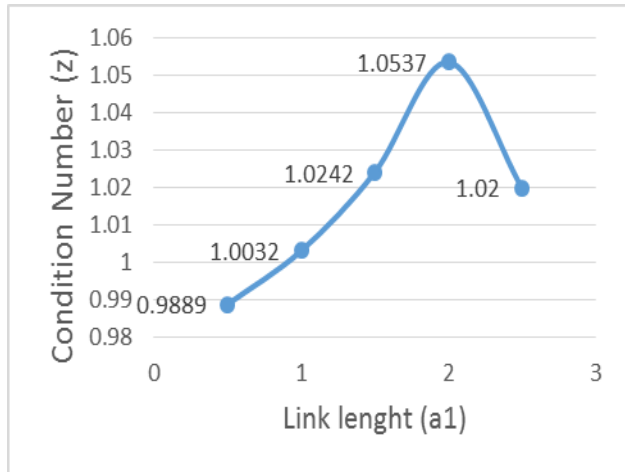   2.   The condition number when varying link length 2 "a_2" keeping all the terms constant joint '1' is 15^0, joint angle '2' is 50^0, joint angle '3' is 13^0, and joint angle '4' is 61^0, and with a_2:a_3:a_4 =1:1:1 is at a_2 =1 and the condition number is "1.0032".
   3.   The condition number when varying link length 3 "a_3" keeping all the terms constant joint '1' is 15^0, joint angle '2' is 50^0, joint angle '3' is 13^0, and joint angle '4' is 61^0, and with a_2:a_3:a_4 =1:1:1 is at a_3 =1 and the condition number is "1.0032".
   4.   The condition number when varying link length 1 "a_1" keeping all the terms constant joint '4' is 15^0, joint angle '2' is 50^0, joint angle '4' is 13^0, and joint angle '4'

is 61^0, and with $a_2{:}a_3{:}a_4 = 1{:}1{:}1$ is at $a_4 = 1$ and the condition number is "1.0032".

1. Condition number values for joint angles

$$\theta_1 = 15^0; \theta_2 = 50^0; \theta_3 = 13^0; \theta_4 = 61^0,$$

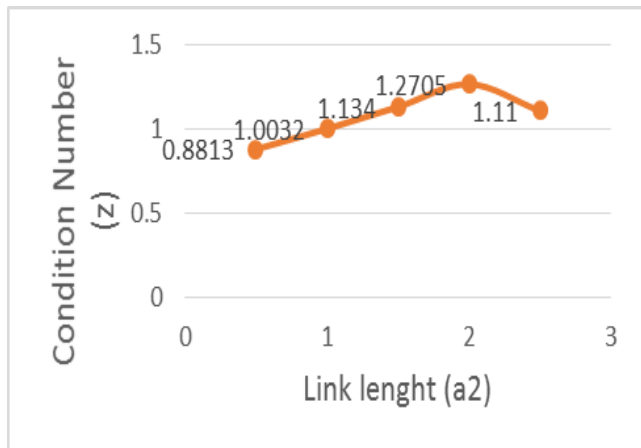Link lengths $a_1{:}\ a_2{:}\ a_{3=1{:}1{:}1}$ varying a_1.

2. Condition number values for joint angles

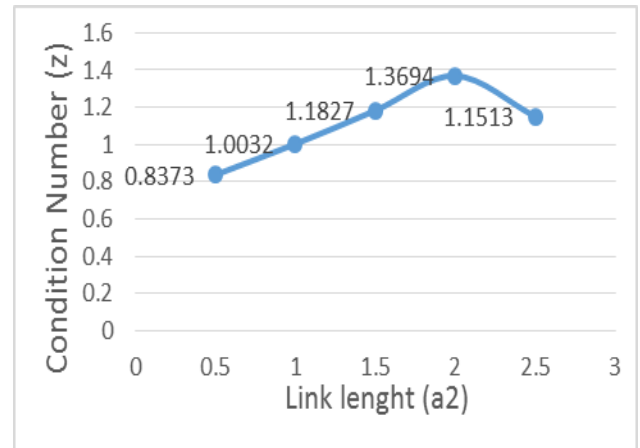$$\theta_1 = 15^0; \theta_2 = 50^0; \theta_3 = 13^0; \theta_4 = 61^0,$$

Link lengths $a_1{:}\ a_2{:}\ a_{3=1{:}1{:}1}$ varying a_2

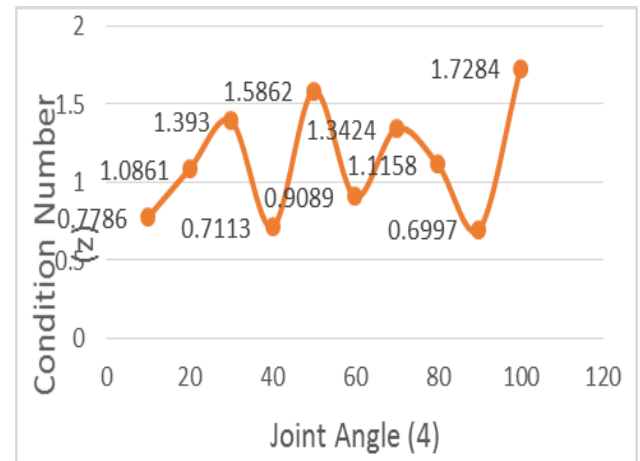3. Condition number values for joint angles

$$\theta_1 = 15^0; \theta_2 = 50^0; \theta_3 = 13^0; \theta_4 = 61^0,$$
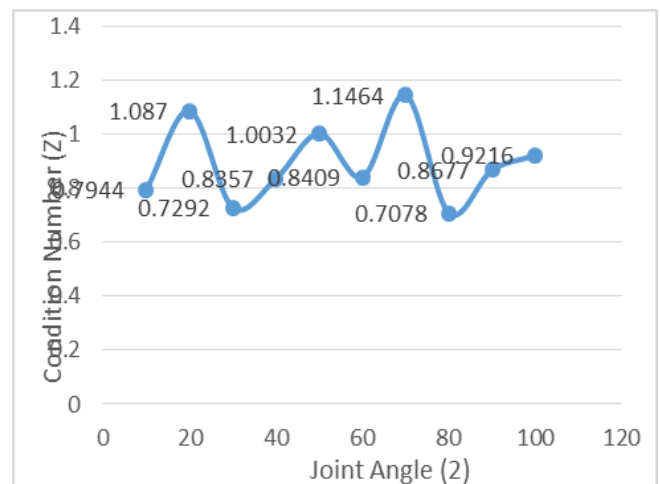
Link lengths $a_1{:}\ a_2{:}\ a_{3=1{:}1{:}1}$ varying a_3.
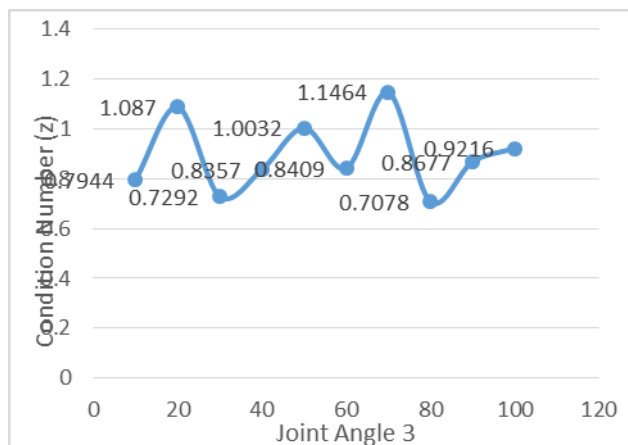

4. Condition number values for link length ratio 1:1:1:1 varying joint angle "4"

5. Condition number values for link length ratio 1:1:1:1 varying joint angle "2."

6. Condition number values for link length ratio 1:1:1:1 varying joint angle "3".

7. CONCLUSIONS

Condition number for four – link planar serial manipulator was determined. As the end effector moves from one location to another, the condition number will vary. It is noticed that condition number for four-linked planar serial manipulator is dependent for first joint angle. Condition number is best tool for the velocity of the end effector.

A program in "MAT LAB" (A high level programming language for technical computing) was written for finding condition number and inverse of Jacobian matrix. Condition number is given by the product of norm of jacobian matrix and form of inverse of jacobian matrix. It must be strictly nearer of unity. Best performance of end effector will be obtained when the condition number reaches unity.

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