AVAILABILITY ANALYSIS OF PARALLEL TRANSIT FUEL SYSTEM IN PETROL ENGINE UNDER HEAD OF LINE REPAIR

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Abstract

In this paper, the author has considered a fuel system in petrol engine with one alternative fuel source (like LPG, CNG etc.) keeping in standby arrangement and followed online through a perfect switching device. The whole system can fail due to either its normal working or human error. All the failures follow exponential time distribution where as all repairs follow general time distribution.

Keywords: *Fuel system, standby, exponential time distribution, general time distribution.*

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1. INTRODUCTION

In this paper, the considered fuel system consists of three subsystems, namely *A*, *B* and *C*, connected in series. Subsystem *A* is fuel source having two units in standby redundancy. Subsystem *B* consists of two units (Transfer pump and carburetor) connected in series while subsystem *C* having N-cylinders [identical] in series and is of j-out-of-N:G nature, fig-1 shows the system configuration. Head-of-line policy is being adopted for repair purpose. The concept of human-error is also incorporated to make the model more compatible. Laplace transform of various state probabilities, availability and long run availability, some particular cases and profit function for the system are obtained. Fig-2 shows the state-transition diagram and states descriptions have mentioned in table-1.

By using supplementary variables technique and Laplace transform, various reliability parameters have been obtained. One numerical example together with graphical illustration has also been given at the end to highlight important results of the study.

Table-1: States Description

STATE	SUBSYSTEM			SYSTEM'S STATE	SUPPLEMENTARY VARIABLE USED
	A	B	C		
S_1	0	0	0	0	Nil
S_2	0	F	0	F	u
S_3	0	0	(N-j)0	0	Nil
S_4	0	0	F	F	v
S_5	1F	0	(N-j)0	0	Nil
S_6	1F	F	(N-j)0	F	y
S_7	0	F	(N-j)0	F	u
S_8	1F	0	0	0	Nil
S_9	1F	F	0	F	y
S_{10}	FH	FH	FH	FH	m
S_{11}	F	0	0	F	n
S_{12}	F	0	(N-j)0	F	r

O: All Operable; F : all Failed; FH : failure due to human-error; rF :r units have failed; rO : r units are operable.

2. ASSUMPTIONS

The following assumptions have been associated with this model:

1. Initially, all the units are new and the system is in good condition of full efficiency.
2. Repair has given only when the system is in failed condition and a single service channel is available.
3. After repair, system works like a new one and never damages anything.
4. 1-A unit is a petrol tank while 2-A unit is an alternative fuel source (like CNG, LPG etc.) and the perfect switching device has been used to online the standby unit.

3. NOTATIONS

The following notations have been used throughout the model:

$P_K(t)$: The probability that at time *t*, the system is operable and is in K^{th} state, where K=1, 3, 5, 8.

$P_K(s,t)\Delta$: The probability that at time *t*, the system is in *rth* failed state and elapsed repair time lies in the interval (s, s + Δ). K=2, 4, 6, 7, 9, 10, 11, 12 and s=u, v, y, u, y, m, n, r respectively.

b_1/b_2 : Constant failure rate of 1B/2B unit and note that $b_1+b_2=b$ (say).

a/c : Constant failure rate of any unit of subsystem A/C.

H : Human-error rate.

$\mu_K(s)\Delta$: The probability that the subsystem *K* will be repaired in the time interval (s,s+Δ) conditioned that it was not repaired up to time *s*, where K=A, B, C and s=u, v, y, n, r.

$\mu_h(m)\Delta$: The probability that the system will be repaired from state S_{10} in time interval (m, m+Δ) conditioned that it was not repaired up to time *m*.

$S_i(x)$: $\mu_i(x)\ \exp .\left\{\int_0^\infty \mu_i(x)dx\right\}$

M_k : Mean time to repair k^{th} unit $= -\bar{S}_k{}'(0)$, for all *k*.

P : Time independent probability.

$D_k(s)$: $\left[1-\bar{S}_K(s)\right]/s$, for all *k* =A,B and C.

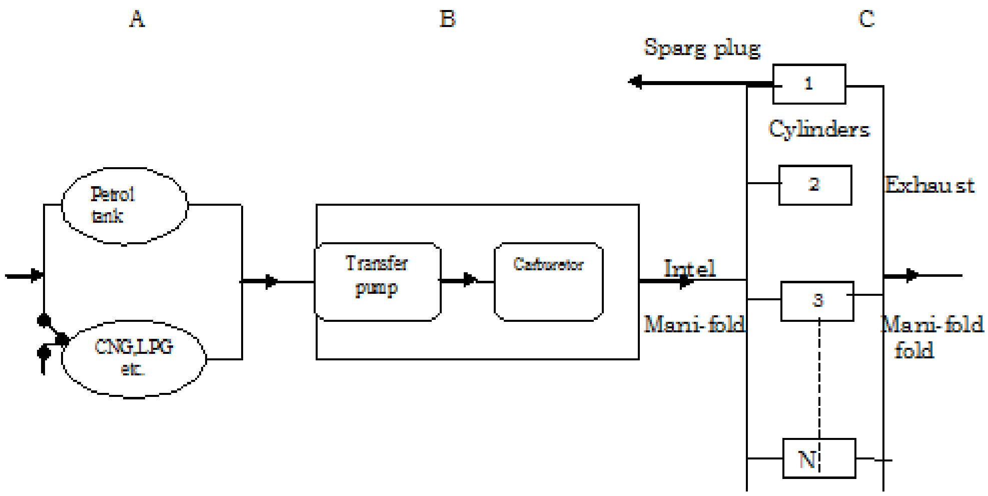

Fig-1: System configuration

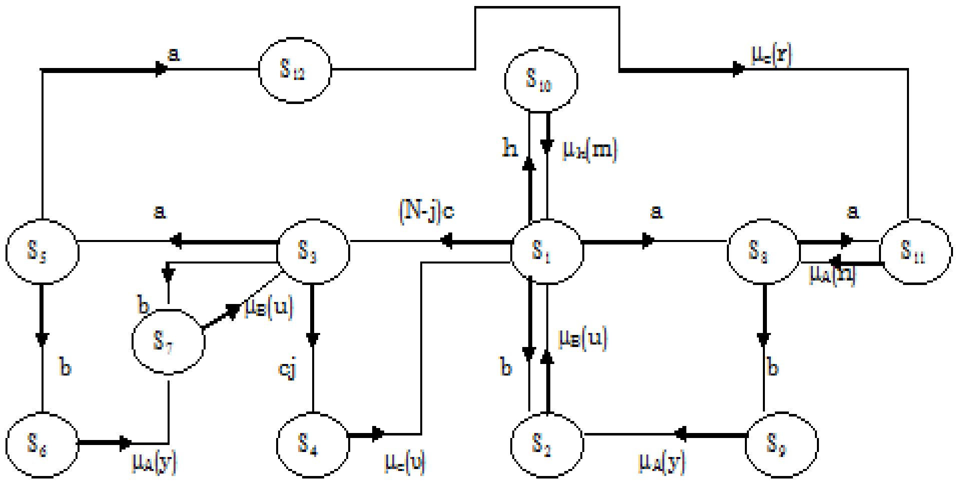

Fig-2: State-transition diagram

4. FORMULATION OF THE MATHEMATICAL MODEL

Using continuity argument, we obtain the following set of difference-differential equations governing the behavior of the system under consideration:

$$\left[\frac{d}{dt}+a+b+(N-j)c+h\right]P_1(t)=\int_0^\infty P_2(u,t)\mu_B(u)du + \int_0^\infty P_{10}(m,t)\mu_h(m)dm+\int_0^\infty P_4(\upsilon,t)\mu_c(\upsilon)d\upsilon \quad \ldots(1)$$

$$\left[\frac{\partial}{\partial u}+\frac{\partial}{\partial t}+\mu_B(u)\right]P_2(u,t)=0 \quad \ldots(2)$$

$$\left[\frac{d}{dt}+cj+b+a\right]P_3(t)=(N-j)cP_1(t)+\int_0^\infty P_7(u,t)\mu_B(u)du \quad \ldots(3)$$

$$\left[\frac{\partial}{\partial \upsilon}+\frac{\partial}{\partial t}+\mu_c(\upsilon)\right]P_4(\upsilon,t)=0 \quad \ldots(4)$$

$$\left[\frac{d}{dt}+a+b\right]P_5(t)=aP_3(t) \quad \ldots(5)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_A(y)\right]P_6(y,t)=0 \quad \ldots(6)$$

$$\left[\frac{\partial}{\partial u}+\frac{\partial}{\partial t}+\mu_B(y)\right]P_7(u,t)=0 \quad \ldots(7)$$

$$\left[\frac{d}{dt}+a+b\right]P_8(t)=aP_1(t)+\int_0^\infty P_{11}(n,t)\mu_A(n)dn \quad \ldots(8)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_A(y)\right]P_9(y,t)=0 \quad \ldots(9)$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+\mu_h(m)\right]P_{10}(m,t)=0 \quad \ldots(10)$$

$$\left[\frac{\partial}{\partial n}+\frac{\partial}{\partial t}+\mu_A(n)\right]P_{11}(n,t)=0 \quad \ldots(11)$$

$$\left[\frac{\partial}{\partial r}+\frac{\partial}{\partial t}+\mu_c(r)\right]P_{12}(r,t)=0 \quad \ldots(12)$$

Boundary conditions are:

$$P_2(0,t) = bP_1(t) + \int_0^\infty P_9(y,t)\mu_A(y)dy \quad \ldots(13)$$

$$P_4(0,t) = cjP_3(t) \quad \ldots(14)$$

$$P_6(0,t) = bP_5(t) \quad \ldots(15)$$

$$P_7(0,t) = bP_3(t) \quad \ldots(16)$$

$$P_9(0,t) = bP_8(t) \quad \ldots(17)$$

$$P_{10}(0,t) = hP_1(t) \quad \ldots(18)$$

$$P_{11}(0,t) = aP_8(t) + \int_0^\infty P_{12}(r,t)\mu_c(r)dr \quad \ldots(19)$$

$$P_{12}(0,t) = aP_5(t) \quad \ldots(20)$$

Initial conditions are:

$$P_1(0) = 1, \text{ otherwise zero.} \quad \ldots(21)$$

5. SOLUTION OF THE MODEL

Taking Laplace transforms of equations (1) through (20) by making use of (21) and then on solving them, we have following Laplace transforms of different state probabilities:

$$\overline{P}_1(s) = \frac{1}{A(s)} \quad \ldots(22)$$

$$\overline{P}_2(s) = \frac{b}{A(s)}\left[1 + a\overline{S}_A(s)\left\{\frac{1}{B(s)} + \frac{a}{s+a+b}\frac{C(s)}{B(s)}\overline{S}_c(s)\overline{S}_A(s)\right\}\right]D_B(s) \quad \ldots(23)$$

$$\overline{P}_3(s) = \frac{C(s)}{A(s)} \quad \ldots(24)$$

$$\overline{P}_4(s) = cj\frac{C(s)}{A(s)}D_c(s) \quad \ldots(25)$$

$$\overline{P}_5(s) = \frac{a}{s+a+b}\frac{C(s)}{A(s)} \quad \ldots(26)$$

$$\overline{P}_6(s) = \frac{ab}{s+a+b}\frac{C(s)}{A(s)}D_A(s) \quad \ldots(27)$$

…(28)

$$\overline{P}_7(s) = b\frac{C(s)}{A(s)}D_B(s)$$

$$\overline{P}_8(s) = \frac{a}{A(s)}\left[\frac{1}{B(s)} + \frac{a}{s+a+b}\frac{C(s)}{A(s)}\overline{S}_c(s)\overline{S}_A(s)\right] \quad \ldots(29)$$

$$\overline{P}_9(s) = b\overline{P}_8(s)D_A(s) \quad \ldots(30)$$

$$\overline{P}_{10}(s) = \frac{h}{A(s)}D_h(s) \quad \ldots(31)$$

$$\overline{P}_{11}(s) = a\left[\overline{P}_8(s) + \frac{a}{s+a+b}\frac{C(s)}{A(s)}\overline{S}_c(s)\right]D_A(s) \quad \ldots(32)$$

$$\ldots(33)$$

$$\overline{P}_{12}(s) = \frac{a^2}{s+a+b}\frac{C(s)}{A(s)}D_c(s)$$

where, $B(s) = s + b + a\left[1 - \overline{S}_A(s)\right]$ …(34)

$$C(s) = \frac{(N-j)c}{s+a+cj+b\left[1-\overline{S}_B(s)\right]} \quad \ldots(35)$$

and $A(s) = s + a + b + (N-j)c + h\left[1 - \overline{S}_h(s)\right] - cj\overline{S}_c(s)C(s) - b\overline{S}_B(s)$

$$- ab\overline{S}_A(s)\overline{S}_B(s)\left\{\frac{1}{B(s)} + \frac{a}{s+a+b}\frac{C(s)}{B(s)}\overline{S}_c(s)\overline{S}_A(s)\right\} \quad \ldots(36)$$

6. ERGODIC BEHAVIOUR OF THE SYSTEM

By making use of Abel's Lemma viz., $\lim_{s\to 0} s\overline{F}(s) = \lim_{t\to\infty} F(t) = F(say)$, provided the limit on right exists, we have the following time independent state probabilities from equations (22) through (33):

$$P_1 = \frac{1}{A'(0)} \quad \ldots(37)$$

$$P_2 = \frac{1}{A'(0)}\left[b + a\left\{1 + \frac{aH}{a+b}\right\}\right]M_B \quad \ldots(38)$$

$$P_3 = \frac{H}{A'(0)} \quad \ldots(39)$$

$$P_4 = \frac{cjH}{A'(0)}M_c \quad \ldots(40)$$

$$P_5 = \frac{aH}{a+b}\cdot\frac{1}{A'(0)} \quad \ldots(41)$$

$$P_6 = \frac{abH}{(a+b)A'(0)} M_A \quad \ldots(42)$$

$$P_7 = \frac{bH}{A'(0)} M_B \quad \ldots(43)$$

$$P_8 = \frac{a}{bA'(0)}\left[1 + \frac{aH}{a+b}\right] \quad \ldots(44)$$

$$P_9 = \frac{a}{A'(0)}\left[1 + \frac{aH}{a+b}\right] M_A \quad \ldots(45)$$

$$P_{10} = \frac{h}{A'(0)} M_h \quad \ldots(46)$$

$$P_{11} = \frac{a^2}{bA'(0)}[1+H] M_A \quad \ldots(47)$$

$$P_{12} = \frac{a^2 H}{(a+b)A'(0)} M_c \quad \ldots(48)$$

where, $H = \frac{(N-j)c}{a+cj}$; and $A'(0) = \left[\frac{d}{ds}A(s)\right]_{s=0}$ …(49)

6. SOME PARTICULAR CASES

6.1 When Repairs Follow Exponential Time Distribution:

Setting $\overline{S}_A(s) = \mu_A/(s+\mu_A)$ etc. in equations (22) through (33), we obtained:

$$\overline{P}_1(s) = \frac{1}{E(s)} \quad \ldots(50)$$

$$\overline{P}_2(s) = \frac{b}{E(s)}\left[1 + \frac{a\mu_A}{s+\mu_A}\left\{\frac{1}{F(s)} + \frac{a}{s+a+b}\frac{G(s)}{F(s)}\frac{\mu_c}{s+\mu_c}\cdot\frac{\mu_A}{s+\mu_A}\right\}\right]\frac{1}{s+\mu_B} \quad \ldots(51)$$

$$\overline{P}_3(s) = \frac{G(s)}{E(s)} \quad \ldots(52)$$

$$\overline{P}_4(s) = cj\frac{G(s)}{E(s)}\frac{1}{s+\mu_c} \quad \ldots(53)$$

$$\overline{P}_5(s) = \frac{a}{s+a+b}\frac{G(s)}{E(s)} \quad \ldots(54)$$

$$\overline{P}_6(s) = \frac{ab}{s+a+b}\frac{G(s)}{E(s)}\frac{1}{s+\mu_A} \quad \ldots(55)$$

$$\overline{P}_7(s) = b\frac{G(s)}{E(s)}\frac{1}{s+\mu_B} \qquad \ldots(56)$$

$$\overline{P}_8(s) = \frac{a}{E(s)}\left[\frac{1}{F(s)} + \frac{a}{s+a+b}\frac{G(s)}{F(s)}\frac{\mu_c}{s+\mu_c}\frac{\mu_A}{s+\mu_A}\right] \qquad \ldots(57)$$

$$\overline{P}_9(s) = b\overline{P}_8(s)\frac{1}{s+\mu_A} \qquad \ldots(58)$$

$$\overline{P}_{10}(s) = \frac{h}{E(s)}\frac{1}{s+\mu_h} \qquad \ldots(59)$$

$$\overline{P}_{11}(s) = a\left[\overline{P}_8(s) + \frac{a}{s+a+b}\frac{G(s)}{E(s)}\frac{\mu_c}{s+\mu_c}\right]\frac{1}{s+\mu_A} \qquad \ldots(60)$$

$$\overline{P}_{12}(s) = \frac{a^2}{s+a+b}\frac{G(s)}{E(s)}\frac{1}{s+\mu_c} \qquad \ldots(61)$$

where,

$$F(s) = s+b+\frac{as}{s+\mu_A} \quad ; \quad G(s) = \frac{(N-j)c}{s+a+cj+\frac{bs}{s+\mu_B}} \qquad \ldots(62)$$

and $E(s) = s+a+b+(N-j)c+\frac{hs}{s+\mu_h} - cj\frac{\mu_c}{s+\mu_c}G(s) - \frac{b\mu_B}{s+\mu_B}$

$$-\frac{ab\mu_A\mu_B}{(s+\mu_A)(s+\mu_B)}\left\{\frac{1}{F(s)} + \frac{a}{s+a+b}\frac{G(s)}{F(s)}\frac{\mu_c\mu_A}{(s+\mu_c)(s+\mu_A)}\right\} \qquad \ldots(63)$$

6.2 Availability and Cost Function for the System :

Now, Laplace transform of availability of petrol engine is given by:

$$\overline{P}_{up}(s) = \frac{1}{s+a+b+(N-j)c+h}\left[1+\frac{(N-j)c}{s+a+cj+b}+\frac{a}{s+a+b}\right] \qquad \ldots(64)$$

on inverting this, one can obtain the availability at any time 't' as under

$$P_{up}(t) = Qe^{-[a+b+(N-j)c+h]t} + \mathrm{Re}^{-(a+cj+b)t} + Se^{-(a+b)t} \qquad \ldots(65)$$

where $Q = 1 - \frac{(N-j)c}{(N-2j)c+h} - \frac{a}{(N-j)c+h}$; …(66)

$$R = \frac{(N-j)c}{(N-2j)c+h} \quad \text{and} \quad S = \frac{a}{(N-j)c+h} \qquad \ldots(67)$$

Note that $P_{up}(0) = 1$

Also, Profit function for the system, working up to the time t, is given by

$$K(t) = C_1 \int_0^t P_{up}(t)dt - C_2 t - C_3 \quad \ldots(68)$$

where,

$$\int_0^t P_{up}(t) = Q\frac{1-e^{-(a+b+(N-j)c+h)t}}{a+b+(N-j)c+h} + R\frac{1-e^{-(a+cj+b)t}}{a+cj+b} + S\frac{1-e^{-(a+b)t}}{a+b} \quad \ldots(69)$$

where, C_1 ,C_2 and C_3 are the revenue, repair and establishment cost per unit time respectively also Q ,R and S are given in the equations (66)and (67).

6.3 Steady- State Availability

The steady state availability of the system is given by

$$A_{VSS} = \frac{(a+Nc)(a+b)}{b(a+cj)[a+b+(N-j)c+h]} \quad (70)$$

Where, $H = (N-j)c / a + cj$ (71)

6.4 Numerical Computation

For a numerical computation, let us consider the values: N=5, j=2, a=0.01, b=0.02, c=0.03, h=0.001, C_1=Rs.5.00, C_2=Re1.00 and C_3 =Rs2.00.

Also t= 0, 1, 2,-----------10 and h=0.001,0.0015, 0.002----------- 0.01.

Table-2

Time	Availability
0	1
1	0.976302
2	0.94806
3	0.916399
4	0.882266
5	0.846451
6	0.809611
7	0.772289
8	0.734929
9	0.697892
10	0.661467
11	0.625883
12	0.591317
13	0.557903
14	0.525739
15	0.494891

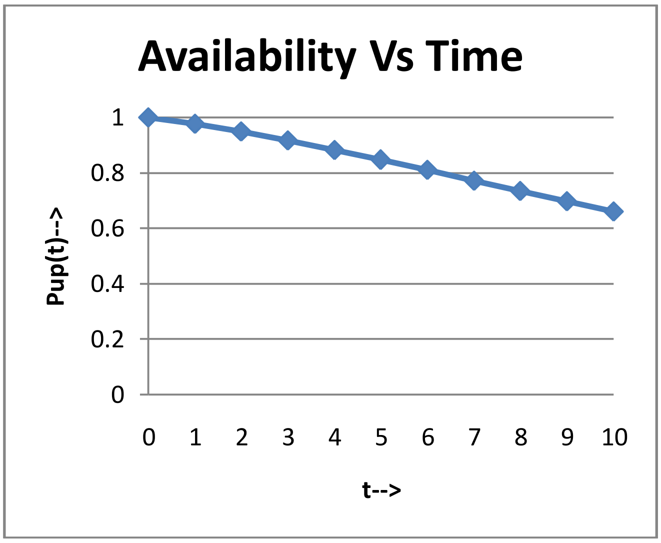

Fig-3

Table-3

Time	Profit Function
0	-2
1	-0.03425
2	1.853275
3	3.650692
4	5.349202
5	6.942611
6	8.426888
7	9.799798
8	11.06059
9	12.2097
10	13.24855
11	14.17934
12	15.00487
13	15.7284
14	16.35354
15	16.88414
16	17.32424
17	17.67793
18	17.94936
19	18.14267
20	18.26195

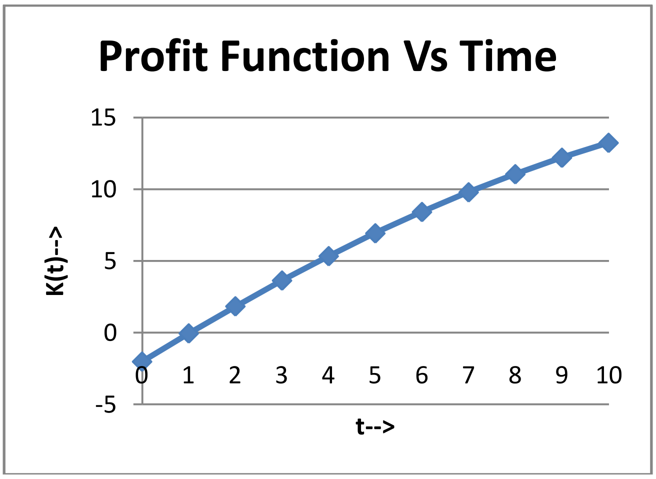

Fig-4

Table-4

h	A VSS
0.001	0.283353
0.0015	0.282187
0.002	0.28103
0.0025	0.279883
0.003	0.278746
0.0035	0.277617
0.004	0.276498
0.0045	0.275387
0.005	0.274286
0.0055	0.273193
0.006	0.272109
0.0065	0.271033
0.007	0.269966
0.0075	0.268908
0.008	0.267857
0.0085	0.266815
0.009	0.265781
0.0095	0.264755
0.01	0.263736

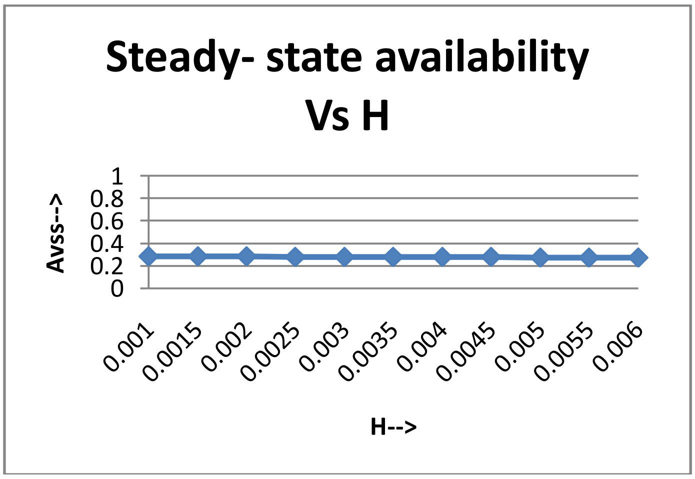

Fig-5

7. CONCLUSIONS

The plots of equations (65), (68) and (70) have shown in figs-(3), (4) and (5), respectively.

Fig-3 shows that availability of the system decreases slowly with time initially but there after it decreases approximately in a constant manner.

Fig-4 shows that profit with the system under consideration. For t=0 and 1, there is no profit but after this profit increases in constant way.

Fig-5 shows long run availability of the system. Its value is 0.283353. Now as we increase the value of h, the long run availability decreases correspondingly.

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