APPROACHES TO THE NUMERICAL SOLVING OF FUZZY DIFFERENTIAL EQUATIONS

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Abstract

One of the main problems of the theory of numerical methods is the search for cost-effective computational algorithms that require minimal time machine to obtain an approximate solution with any given accuracy. In article considered fuzzy analog of the alternating direction, combining the best qualities of explicit and implicit scheme is unconditionally stable (as implicit scheme) and requiring for the transition from layer to layer a small number of actions (as explicit scheme).

Keywords: *fuzzy set, quasi-differential equation, the scheme of variable directions, finite-difference methods.*

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1. INTRODUCTION

The concept of fuzzy differential equations was introduced by O.Kaleva in 1987. In [1] he proved the theorem of existence and uniqueness of solutions of such equations. Later in [2-6] were been obtained properties of fuzzy differential equations and their solutions. To determine fuzzy derivative O.Kaleva used M.L.Pur and D.A.Rulesku's approach [7] to the differentiability of fuzzy mappings, which, in turn, is based on the M.Hukuhara's idea [8] about differentiability of multivalued mappings. In this regard, the O.Kaleva's approach adopted all the shortcomings typical differential equations with the Hukuhara's derivative.

In 1990 J.P.Aubin [9] and V.A.Baidosov [10,11] introduced into consideration fuzzy differential inclusion. Their approach to solving such equations is based on the latest information for ordinary differential inclusions. In the future, fuzzy inclusion were considered in works [12-15].

In [17], in the same way as was done in the theory of differential equations with multivalued right-hand side [16], introduced the concept of quasi-differential equation. This allows on the one hand to avoid the difficulties that arise in the solution of fuzzy differential equations and inclusions, and with other - to get some of their properties by available methods.

2. STATEMENT OF THE TASK

Let $Conv(R^n)$ - the space of nonempty convex compact subsets R^n with Hausdorf metric

$$h(F,G) = \max\left\{\sup_{f\in F}\inf_{g\in G}\|f-g\|, \sup_{g\in G}\inf_{f\in F}\|f-g\|\right\},$$

Where, under $\|\cdot\|$ refers Euclidean norm in space R^n.

Now we introduce the space E^n of the mapping $\mu : R^n \to [0,1]$, which satisfying the following conditions:

1. μ - semi-continuous from above, i.e. for any $x' \in R^n$ and for any $\varepsilon > 0$ there is $\delta(x',\varepsilon) > 0$ such that for all $\|x - x'\| < \delta$ runs condition $\mu(x) < \mu(x') + \varepsilon$;
2. μ - normally, i.e. there exists a vector $x_0 \in R^n$ such that $\mu(x_0) = 1$;
3. μ - fuzzy convex, i.e. for any $x', x'' \in R^n$ and any $\lambda \in (0,1]$ true inequality $\mu(\lambda x' + (1-\lambda)x'') \geq \min\{\mu(x'), \mu(x'')\}$;
4. The closure of the set $\{x \in R^n | \mu(x) > 0\}$ is compact.

Zero in the space E^n are elements

$$\theta(x) = \begin{cases} 1, & x = 0 \\ 0, & x \in R^n \setminus 0 \end{cases}$$

Definition 1 α - cutting $[\mu]^\alpha$ of the mapping $\mu \in E^n$ at $0 < \alpha \leq 1$ let's call a set $\{x \in R^n | \mu(x) \geq \alpha\}$. Zero cutting of the map $\mu \in E^n$ let's call the closure of the set $\{x \in R^n | \mu(x) > 0\}$.

Let's define in space E^n metric $D : E^n \times E^n \to [0,\infty)$, fancy $D(\mu,\nu) = \sup_{0\leq\alpha\leq1} h([\mu]^\alpha, [\nu]^\alpha)$.

Definition 2 Mapping $f : [0,T] \to E^n$ called weakly continuous at the point $t_0 \in (0,T)$, if for any fixed

$\alpha \in [0,1]$ and arbitrary $\varepsilon > 0$ there is $\delta(\varepsilon,\alpha) > 0$ such that $h(f_\alpha(t), f_\alpha(t_0)) < \varepsilon$ for all $t_0 \in [0,T]$ such, that $|t-t_0| < \delta(\varepsilon,\alpha)$.

Definition 3 Integral of the mapping $f: I \to E^n$ over the interval I is the element $g \in E^n$ such that $[g]^\alpha = \int_I f_\alpha(t)dt$ for all $\alpha \in (0,1]$.

Definition 4 Mapping $f: I \to E^n$ called differentiable at the point $t_0 \in I$, if fr all $\alpha \in [0,1]$ multivalued mapping $f_\alpha(t)$ differentiable by Hukuhara [198] in the point t_0, its derivative is equal to $D_H f_\alpha(t_0)$ and the $\{D_H f_\alpha(t_0) : \alpha[0,1]\}$ set family defines an element.

Definition 5 For $u, v \in E^n, w \in E^n$ called u and v, if $u = v + w$, and it is written as $w = u\underline{H}v$.

Definition 6 Function $F:[a,b] \to E^n$ differentiable in $t_0 \in (a,b)$, if there is a $F'(t_0) \in E^n$ such that there are limits $\lim\limits_{h\to 0+} \frac{F(t_0+h)\underline{H}F(t_0)}{h}$ and $\lim\limits_{h\to 0+} \frac{F(t_0)\underline{H}F(t_0-h)}{h}$ and they are equal to $F'(t_0)$.

If F differentiable in $t_0 \in (a,b)$, hen for all α slices $F_\alpha(t) = [F(t)]^\alpha$ there is a Hukuhara differential in t_0 and $[F'(t_0)]^\alpha = DF_\alpha(t)$, where DF_α is called Hukuhara differential F_α.

Definition 7 For function $f:[a,b] \to E^n$ Seikkala introduced the concept $SDf(t)$ in a such way $[SDf(t)]^\alpha = [f_1^{'}(t,\alpha), f_2^{'}(t,\alpha)],\quad 0 < \alpha \le 1$.

For $t \in [a,b]$, $[SDf(t)]^\alpha$ is fuzzy. If mapping $f: I \to E^n$ differentiable at the point $t_0 \in I$, then $f'(t_0)$ call fuzzy derivative $f(t)$ in the point t_0. Mapping $f: I \to E^n$ called differentiable on I, if it is differentiable at each point $t \in I$.

Definition 8 Mapping $f: I \to E^n$ is called uniformly continuous on $G \in E^n$, if for each $\varepsilon > 0$ there is $\delta(\varepsilon) > 0$ such that for all $x, y \in G$, satisfying the inequality $D(x,y) < \delta$ fair assessment $D(f(x), f(y)) < \varepsilon$.

Definition 9 Fuzzy number $\tilde{u}$ is called a fuzzy number of $L-R$ type, if

$$\mu_{\tilde{u}}(\alpha) = \begin{cases} \mu_L(u) = 1 - \dfrac{u - u_L(\alpha)}{u_L}, \\ \mu_R(u) = 1 - \dfrac{u_R(\alpha) - u}{u_R}, \end{cases}$$

where u - clear value of the number $\tilde{u}$, i.e. $u = u_L(1) = u_R(1)$; u_L and u_R respectively the left and right stretching of fuzzy number $\tilde{u}$; $u_L(\alpha)$ and $u_R(\alpha)$ - respectively the left and right values of the fuzzy number $\tilde{u}$ of definition α.

From the definition it follows that if

$\tilde{u}(\alpha) = \{u, u_L(\alpha), u_R(\alpha)\}$, then

$$u_L(\alpha) = u - (1-\alpha)u_L;\ u_R(\alpha) = u + (1-\alpha)u_R.$$

Consider the algebraic action on fuzzy $L-R$ type:
Addition:

$$\tilde{u} + \tilde{v} = \{u + v - (1-\alpha)(u_L + v_L); u + v + (1-\alpha)(u_R + v_R)\}.$$

Subtraction:

$$\tilde{u} - \tilde{v} = \{u - v - (1-\alpha)(u_L + v_L); u - v + (1-\alpha)(u_R + v_R)\}.$$

Multiplication:
1) for $u > 0; v > 0$

$$\tilde{u} \cdot \tilde{v} = \{u \cdot v; (1-\alpha)(uv_L + vu_L) - (1-\alpha)u_L v_L;\ (1-\alpha)(uv_R + vu_R) + (1-\alpha)u_R v_L\};$$

2) for $u > 0; v < 0$

$$\tilde{u}\cdot\tilde{v}=\{u\cdot v;(1-\alpha)(uv_R+vu_L)-(1-\alpha)u_Lv_L;\ (1-\alpha)(uv_L+vu_R)+(1-\alpha)u_Rv_L\};$$

3) for $u<0;\ v<0$

$$\tilde{u}\cdot\tilde{v}=\{u\cdot v;(1-\alpha)(uv_R+vu_L)-(1-\alpha)u_Rv_R;\ (1-\alpha)(uv_L+vu_L)-(1-\alpha)u_Lv_L\}.$$

Division:

$$\frac{\tilde{u}}{\tilde{v}}=\tilde{u}\cdot\frac{1}{\tilde{v}}.$$

Definition 10 We say that the mapping $\varphi:[0,\sigma]\times[0,\tau]\times[0,T]\times E^n\to E^n$ specifies the local quasi movement, if satisfied following conditions:

1) initial conditions axiom $\varphi(0,0,t,y)=y$;
2) quasiprimitive axiom:

$$D(\varphi(h_\gamma,\tau,0,y_0),\varphi(h_{N_\gamma},\tau_m,t_{m-1},y_{N_\gamma-1}))=0(h),$$

Where $h=\sum_{n_\gamma=1}^{N_\gamma}h_{n_\gamma}\quad \tau=\sum_{s=0}^{m}t_s$;

3) axiom of continuity: mapping $\varphi(h,\tau,y(x,t))$ is - weakly continuously.

Approximation equation

$$D(y(x+h,t+\tau),\varphi(h,\tau,t,y(x,t)))=0(h,\tau)$$

,

$$y(x,0)=u_0(x),\quad x\in\overline{D},$$

$$y(x,t)=\psi(x,t),\quad x\in\Gamma,\quad t\in[0,T]$$

we will call fuzzy quasi-differential equation.

Definition 11 Continuous map $y:Q_T\to E^n$, satisfying the approximation equation will be called a solution of fuzzy quasi-differential equation.

In this paper discusses some of the issues of building fuzzy analogs of finite difference methods.

Let's consider the initial-boundary task

$$\frac{\partial u}{\partial t}-Lu=f(x,t),\ \ (x,t)\in Q_T, \tag{1}$$

$$u(x,0)=u_0(x),\quad x\in\overline{D}, \tag{2}$$

$$u(x,t)=\psi(x,t),\quad x\in\Gamma,\quad t\in[0,T], \tag{3}$$

$$Lu=\Delta u=L_1u+L_2u,\ \ L_1u=\frac{\partial^2u}{\partial x_1^2},\ \ L_2u=\frac{\partial^2u}{\partial x_2^2},$$

$$\overline{D}=\{0\le x_\gamma\le l_\gamma;\ \gamma=1,2\},$$

$$Q_T=D\times(0,T],\ \ x=(x_1,x_2).$$

Suppose that $u_0(x),\psi(x,t),f(x,t)$ - are fuzzy functions.

$$u_0(x)\in\bigcup_{\alpha\in[0,1]}\alpha[U_0(x)]^\alpha,$$

$$\psi(x,t)\in\bigcup_{\alpha\in[0,1]}\alpha[\Psi(x,t)]^\alpha, \tag{4}$$

$$f(x,t)\in\bigcup_{\alpha\in[0,1]}\alpha[F(x,t)]^\alpha,\ \alpha\in[0,1] \tag{5}$$

We introduce two-dimensional spatial grid and temporary one-dimensional grid:

$$\overline{w}_h\equiv\overline{w}_{h_1,h_2}=w_h+\gamma_h\equiv\{(x_{n_1},x_{n_2})\in\overline{D};\ 0\le n_\gamma\le N_\gamma;\ \gamma=1,2\},$$

$$t=t_{s+\frac{1}{2}}=t_1+0{,}5\tau;\ \ \overline{y}=y^{s+\frac{1}{2}},\ \Phi^s=[f(x_n,t^s)]^\alpha$$

In this case, the construction of the discrete solution of the task (1)-(5) is reduced to the definition in the grid of fuzzy numbers $[y^s]^\alpha$, such, that for for exact solutions $u(x,t)$ of the task (1)-(3) with any initial condition $u_0(x)\in\bigcup_{\alpha\in[0,1]}\alpha[U_0(x)]^\alpha$ and

$\psi(x,t) \in \bigcup_{\alpha\in[0,1]} \alpha[\Psi(x,t)]^{\alpha}$, fair inclusions $u(x_s,t) \in \bigcup_{\alpha\in[0,1]} [y^s]^{\alpha}$.

Let the function $f(x,t) \in \bigcup_{\alpha\in[0,1]} \alpha[F(x,t)]^{\alpha}$, $\alpha \in [0,1]$ defined for points $(x,t) \in Q_T$ and $F(x,t)$ satisfies the following conditions:

1) $[F(x,t)]^{\alpha}$ - defined and continuous for all $(x,t) \in Q_T$.

2) $[F(x,t)]^{\alpha}$ - monotonously to inclusion of h_0, i.e. from $X_1 \subset X_2,\ T_1 \subseteq T_2$ ensue $[F(X_1,T_1)]^{\alpha} \subseteq [F(X_2,T_2)]^{\alpha}$.

3) There is a real constant $l > 0$, such that when $(x,t) \in Q_T$ true inequality $\varpi([F(x,t)]^{\alpha}) \le l(\varpi([x]^{\alpha}) + \varpi([t]^{\alpha}))$.

Describe the fuzzy variant of the finite-difference methods for tasks (1)-(5).

3. FUZZY VARIANT OF THE FINITE-DIFFERENCE METHODS.

Replace differential operators with finite-difference:

$$Lu \to \Lambda y = \Lambda_1 y + \Lambda_2 y, \Lambda_\gamma y = y_{\bar{x}_\gamma, x_\gamma},\ \gamma = 1,2.$$

One of the main problems of the theory of numerical methods is the search for cost-effective computational algorithms that require minimal time machine to obtain an approximate solution with any given accuracy $\varepsilon > 0$.

As is known, the explicit scheme requires a lot of activity, but its stability is at a sufficiently small time step, the implicit scheme is unconditionally stable, but it requires a large number of arithmetic operations.

We can build a system that combines the best of explicit and implicit scheme is unconditionally stable (as implicit scheme) and requiring for the transition from layer to layer a small number of actions (as explicit scheme).

One of the first cost-effective schemes is the scheme of variable directions, built in 1955 by Pismen and Recordon.

Pismen and Reckford scheme makes the transition from layer S on a layer $S+1$ in two steps, using intermediate (fractional) layer.

$$\frac{[y^{s+\frac{1}{2}}]^{\alpha} - [y^s]^{\alpha}}{0,5\tau} = \Lambda_1[y^{s+\frac{1}{2}}]^{\alpha} + \Lambda_2[y^s]^{\alpha} + [\Phi^s]^{\alpha}, \tag{6}$$

$$\frac{[y^{s+1}]^{\alpha} - [y^{s+\frac{1}{2}}]^{\alpha}}{0,5\tau} = \Lambda_1[y^{s+\frac{1}{2}}]^{\alpha} + \Lambda_2[y^s]^{\alpha} + [\Phi^s]^{\alpha}, \tag{7}$$

$$[y(x,0)]^{\alpha} = [U_0(x)]^{\alpha}, \quad x \in \bar{w}_h, \tag{8}$$

$$[y^{s+1}]^{\alpha} = [\Psi]^{\alpha}, \quad n_2 = 0,\ n_2 = N, \tag{9}$$

$$[y^{s+\frac{1}{2}}]^{\alpha} = [\Psi]^{\alpha},\ n_1 = 0,\ n_2 = N\ . \tag{10}$$

Equation (6) is implicit in the first direction and clear for the second, and equation (7) is explicit in the first direction, and implicit in the second.

From (6) and (7) we get

$$\frac{2}{\tau}[\bar{y}]^{\alpha} - \Lambda_1[\bar{y}]^{\alpha} = [F]^{\alpha};\ \ [F]^{\alpha} = \frac{2}{\tau}[y]^{\alpha} + \lambda[y]^{\alpha} + [\Phi]^{\alpha},$$

$$\frac{2}{\tau}[\hat{y}]^{\alpha} - \Lambda_2[\hat{y}]^{\alpha} = [\bar{F}]^{\alpha};\ \ [\bar{F}]^{\alpha} = \frac{2}{\tau}[\bar{y}]^{\alpha} + \Lambda_1[\bar{y}]^{\alpha} + [\bar{\Phi}]^{\alpha},$$

$$\frac{1}{h^2}[\bar{y}_{n_1-1}]^{\alpha} - 2\left(\frac{1}{h_1^2} + \frac{1}{\tau}\right)[\bar{y}_{n_1}]^{\alpha} + \frac{1}{h_1^2}[\bar{y}_{n_1+1}]^{\alpha} = -[F_{n_1}]^{\alpha},\ n_1 = 1,2,...,N_1 - 1,$$

$$[\bar{y}_{n_1}]^{\alpha} = [\Psi_{n_1}]^{\alpha};\ n_1 = 0,\ n_1 = N_1.$$

$$\frac{1}{h^2}[\hat{y}_{n_2-1}]^{\alpha} - 2\left(\frac{1}{h_2^2} + \frac{1}{\tau}\right)[\hat{y}_{n_2}]^{\alpha} + \frac{1}{h_2^2}[\hat{y}_{n_2+1}]^{\alpha} = -[\bar{F}_{n_2}]^{\alpha},\ n_2 = 1,2,...,N_2 - 1,$$

$$[\bar{y}_{n_2}]^{\alpha} = [\Psi_{n_2}]^{\alpha};\ n_2 = 0,\ n_2 = N_2,$$

$$x_n = (n_1h_1, n_2h_2), [F]^{\alpha} = [F_{n_1n_2}]^{\alpha},$$

$$[y]^{\alpha} = [y_{n_1n_2}]^{\alpha}.$$

Thus, the construction of the discrete solution of problem (1)-(5) is reduced to the definition in the grid of fuzzy numbers $[y^s]^{\alpha}$, such that for exact solutions $u(x,t)$ of the task (1)-(3) with any initial condition $u_0(x) \in \bigcup_{\alpha\in[0,1]} \alpha[U_0(x)]^{\alpha}$ and $\psi(x,t) \in \bigcup_{\alpha\in[0,1]} \alpha[\Psi(x,t)]^{\alpha}$, fair inclusions $u(x_s,t) \in \bigcup_{\alpha\in[0,1]} \alpha[y^s]^{\alpha}$.

4. NUMERICAL EXPERIMENT

Let's consider the idea of the alternating direction method of class cheap schemes, applied to the solution of the first initial-boundary value task for two-dimensional heat equation.

$$[q(z)]^{\alpha}\left[\frac{\partial u}{\partial t}\right]^{\alpha}=\left[\frac{\partial}{\partial z}(k_z\frac{\partial u}{\partial z})\right]^{\alpha}-[w]^{\alpha}\left[\frac{\partial u}{\partial z}\right]^{\alpha}+\left[\frac{\partial}{\partial y}(k_y\frac{\partial u}{\partial y})\right]^{\alpha}-[au]^{\alpha},$$

$(z,y,t)\in(0,L)\times(-M,M)\times(0,T)$,

$[u(z,y,0)]^{\alpha}=[\psi(z,y)]^{\alpha}, \quad (z,y)\in[0,L]\times(-M,M)$ - initial conditions,

$[u(z,y,t)]^{\alpha}=[\varphi(z,y,t)]^{\alpha}, \quad (z,y)\in\Gamma,\ 0<t\le T$ - boundary conditions

где $\Omega=(0,L)\times(-M,M),\quad \Gamma$ is -Ω domain border;

$[\psi(z,y)]^{\alpha},[\varphi(z,y,t)]^{\alpha}$ - α - sections of the set of fuzzy functions. L,M,T - given numbers.

Initial equation is approximated by the combination of two difference schemes, each of which corresponds to only one spatial direction. Each element of the sum approximated by explicit and implicit structures.

To do this, along with a layer $t=t^n$, alculation of which is carried out at this stage, the inclusion of an additional layer $t=t^{n+\frac{1}{2}}$. Then for the transition from layer $t=t^n$ to the layer $t=t^{n+1}$ by using the layer $t=t^{n+\frac{1}{2}}$ the original differential equation is approximated by two differential equations, one of which connects layers $t=t^n$ and $t=t^{n+\frac{1}{2}}$, and the second layers $t=t^{n+\frac{1}{2}}$ and $t=t^{n+1}$.

First phase. Transition to more smart $n+\frac{1}{2}$ - th layer with step $\frac{\tau}{2}$. Time derivative is approximated by the formula

$$\dot{u}_t(z_i,y_j,t^n)=\frac{\hat{u}_{ij}^{n+\frac{1}{2}}\underline{H}\hat{u}_{ij}^{n}}{\frac{\tau}{2}},$$

Derivative by z approximated on $n+\frac{1}{2}$ - th layer, derivative by y on n - th layer.

Included in the right part of the initial equation fuzzy function $[f(z,y,t,q)]^{\alpha}$ is replaced by its grid view. Corresponding difference scheme has the form:

$$[q_i]^{\alpha}\cdot\frac{\left[\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}-\left[\hat{u}_{ij}^{n}\right]^{\alpha}}{\left[\frac{\tau}{2}\right]^{\alpha}}=\frac{\left[k_{z_{i-1}}\hat{u}_{i+1,j}^{n+\frac{1}{2}}\right]^{\alpha}-\left[(k_{z_{i-1}}-k_{z_i})\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}+\left[k_{z_i}\hat{u}_{i-1,j}^{n+\frac{1}{2}}\right]^{\alpha}}{\left[h_z^2\right]^{\alpha}}-[w]^{\alpha}\frac{\left[\hat{u}_{i+1,j}^{n+\frac{1}{2}}\right]^{\alpha}-\left[\hat{u}_{i-1,j}^{n+\frac{1}{2}}\right]^{\alpha}}{\left[2h_z\right]^{\alpha}}+$$

$$\frac{\left[k_{y_{j-1}}\hat{u}_{i,j+1}^{n}\right]^{\alpha}-\left[(k_{y_{j-1}}+k_{y_j})\hat{u}_{ij}^{n}\right]^{\alpha}+\left[k_{y_j}\hat{u}_{i,j-1}^{n}\right]^{\alpha}}{\left[h_y^2\right]^{\alpha}}-\left[\frac{a}{2}\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}, \qquad 1\le i\le I-1,\ 1\le j\le J-1,$$

Where $\left[\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}=\left[\hat{u}(z_i,y_i,t^n+\frac{1}{2}\tau)\right]^{\alpha}$ or in traditional recording

$$\left[\frac{\tau}{2h_z^2}k_{z_i}+\frac{\tau}{4h_z}w\right]^{\alpha}\cdot\left[\hat{u}_{i-1,j}^{n+\frac{1}{2}}\right]^{\alpha}-\left[\frac{\tau}{2h_z^2}k_{z_i}+\frac{\tau}{2h_z^2}k_{z_{i-1}}+q_i+\frac{a\tau}{4}\right]^{\alpha}\cdot\left[\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}+$$
$$\left[\frac{\tau}{2h_z^2}k_{z_{i-1}}-\frac{\tau}{4h_z}w\right]^{\alpha}\cdot\left[\hat{u}_{i+1,j}^{n+\frac{1}{2}}\right]^{\alpha}=\left[F_{ij}^{n}\right]^{\alpha}, \qquad 1\le i\le I-1, 1\le j\le J-1, \tag{11}$$

where $\left[F_{ij}^{n}\right]^{\alpha}=\left[q_i\hat{u}_{ij}^{n}+\frac{\tau}{2}\frac{k_{y_{j-1}}\hat{u}_{i,j+1}^{n}-(k_{y_{j-1}}+k_{y_j})\hat{u}_{ij}^{n}+k_{y_j}\hat{u}_{i,j-1}^{n}}{h_y^2}\right]^{\alpha}$.

For each $j=1,...,J-1$ need to solve three-diagonal system of fuzzy linear algebraic equations, as each equation contains three unknown values $\left[\hat{u}_{i-1,j}^{n+\frac{1}{2}}\right]^{\alpha}, \left[\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}, \left[\hat{u}_{i+1,j}^{n+\frac{1}{2}}\right]^{\alpha}$, rest of the values are taken from n - th layer. In other words, under this approach, the scheme is implicit in the direction z and explicit in direction y. The desired value in the intermediate layer are calculated by the fuzzy sweep method by direction z , i.e. by the longitudinal direction.

Second phase. The transition to $(n+1)$ - th temporary layer from intermediate $n+\frac{1}{2}$ - th with step $\frac{\tau}{2}$. Time derivative is approximated by the formula

$$\dot{u}_t(z_i,y_j,t^n)=\frac{\hat{u}_{ij}^{n+1}\underline{H}\hat{u}_{ij}^{n+\frac{1}{2}}}{\frac{\tau}{2}},$$

derivative by z approximated on the $n+\frac{1}{2}$ - th layer, derivative by y on the $(n+1)$ - th layer.

Corresponding difference scheme has the form:

$$[q_i]^{\alpha}\cdot\frac{\left[\hat{u}_{ij}^{n+1}\right]^{\alpha}-\left[\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}}{\left[\frac{\tau}{2}\right]^{\alpha}}=\frac{\left[k_{z_{i-1}}\hat{u}_{i+1,j}^{n+\frac{1}{2}}\right]^{\alpha}-\left[(k_{z_{i-1}}-k_{z_i})\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}+\left[k_{z_i}\hat{u}_{i-1,j}^{n+\frac{1}{2}}\right]^{\alpha}}{\left[h_z^2\right]^{\alpha}}-[w]^{\alpha}\frac{\left[\hat{u}_{i+1,j}^{n+\frac{1}{2}}\right]^{\alpha}-\left[\hat{u}_{i-1,j}^{n+\frac{1}{2}}\right]^{\alpha}}{\left[2h_z\right]^{\alpha}}+$$
$$\frac{\left[k_{y_{j-1}}\hat{u}_{i,j+1}^{n+1}\right]^{\alpha}-\left[(k_{y_{j-1}}+k_{y_j})\hat{u}_{ij}^{n+1}\right]^{\alpha}+\left[k_{y_j}\hat{u}_{i,j-1}^{n+1}\right]^{\alpha}}{\left[h_y^2\right]^{\alpha}}-\left[\frac{a}{2}\hat{u}_{ij}^{n+1}\right]^{\alpha}, \qquad 1\le i\le I-1,\ 1\le j\le J-1,$$

or in the traditional record:

$$\left[\frac{\tau}{2h_y^2}k_{y_j}\right]^{\alpha}\cdot\left[\hat{u}_{i,j-1}^{n+1}\right]^{\alpha}-\left[\frac{\tau}{2h_y^2}k_{y_j}+\frac{\tau}{2h_y^2}k_{y_{j-1}}+q_i+\frac{a\tau}{4}\right]^{\alpha}\cdot\left[\hat{u}_{ij}^{n+1}\right]^{\alpha}+$$
$$\left[\frac{\tau}{2h_y^2}k_{y_{j-1}}\right]^{\alpha}\cdot\left[\hat{u}_{i,j+1}^{n+1}\right]^{\alpha}=\left[Q_{ij}^{n+\frac{1}{2}}\right]^{\alpha}, \qquad 1\le i\le I-1, 1\le j\le J-1, \tag{12}$$

Where

$$\left[Q_{ij}^{n+\frac{1}{2}}\right]^{\alpha}=\left[q_i\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}+\left[\frac{\tau}{2}\frac{k_{z_{i-1}}\hat{u}_{i+1,j}^{n+\frac{1}{2}}-(k_{z_{i-1}}+k_{z_i})\hat{u}_{ij}^{n+\frac{1}{2}}+k_{z_i}\hat{u}_{i-1,j}^{n+\frac{1}{2}}}{h_y^2}\right]^{\alpha}-\left[\frac{w\tau}{2}\frac{\hat{u}_{i+1,j}^{n+\frac{1}{2}}-\hat{u}_{i-1,j}^{n+\frac{1}{2}}}{2h_z}\right].$$

For each $i=1,...,I-1$ need to solve three-diagonal system of fuzzy linear algebraic equations, as each equation contains three unknown values $\left[\hat{u}_{i,j-1}^{n+1}\right]^{\alpha}, \left[\hat{u}_{ij}^{n+1}\right]^{\alpha}, \left[\hat{u}_{i,j+1}^{n+1}\right]^{\alpha}$, the rest of the values are taken from $(n+\frac{1}{2})$ - th layer. In other words, under this approach, the scheme is implicit in the direction z and a clear by direction y. The desired value in the intermediate layer are calculated by the sweep method in the direction y, i.e. the transverse direction. Diagonal elements prevail.

Under implementation is the lack of dividing the interval containing zero on each α cutting, and by sustainability - limited impact of an error in the calculation at some stage of the final result.

The advantages of the method, achieved through the introduction of intermediate layer, should include the splitting of the initial task into two simpler, solved with the help of sweep algorithm.

The initial and boundary conditions is presented in the form:initial layer

$$\left[\hat{u}_{ij}^{0}\right]^{\alpha}=\left[\psi_{ij}\right]^{\alpha},\quad 0\le i\le I;\quad 0\le j\le J,$$

on planes perpendicular to the axis OZ

$$\left[\hat{u}_{0,j}^{n}\right]^{\alpha}=\left[\varphi_{0,j}^{n}\right]^{\alpha};\left[\hat{u}_{I,j}^{n}\right]^{\alpha}=\left[\varphi_{I,j}^{n}\right]^{\alpha},\quad 0\le j\le J;\quad 0\le n\le N,$$

on planes perpendicular to the axis OY

$$\left[\hat{u}_{i,0}^{n}\right]^{\alpha}=\left[\varphi_{i,o}^{n}\right]^{\alpha};\left[\hat{u}_{i,J}^{n}\right]^{\alpha}=\left[\varphi_{i,J}^{n}\right]^{\alpha},\quad 0\le j\le J;\quad 0\le n\le N,$$

Where

$$\left[\varphi_{i,j}^{n}\right]^{\alpha}=\left[\varphi(x_i,y_j,t^n)\right]^{\alpha},\ \left[\psi_{ij}\right]^{\alpha}=\left[\psi(x_i,y_j)\right]^{\alpha}.$$

For the intermediate layer is required values $\left[\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}$ on the sides of the settlement of the region defined by the equations $x=0$ and $x=L$. Subtracting (12) from (11), we obtain

$$\left[\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}=\left[\frac{\hat{u}_{ij}^{n}+\hat{u}_{ij}^{n+1}}{2}\right]^{\alpha}+\left[\frac{\tau}{4q_i}\right]^{\alpha}\frac{\left[k_{y_{j-1}}\hat{u}_{i,j+1}^{n}\right]^{\alpha}-\left[(k_{y_{j-1}}-k_{y_j})\hat{u}_{ij}^{n}\right]^{\alpha}+\left[k_{y_j}\hat{u}_{i,j-1}^{n}\right]^{\alpha}}{\left[h_y^2\right]^{\alpha}}-$$
$$\left[\frac{\tau}{4q_i}\right]^{\alpha}\frac{\left[k_{y_{j-1}}\hat{u}_{i,j+1}^{n+1}\right]^{\alpha}-\left[(k_{y_{j-1}}+k_{y_j})\hat{u}_{ij}^{n+1}\right]^{\alpha}+\left[k_{y_j}\hat{u}_{i,j-1}^{n+1}\right]^{\alpha}}{\left[h_y^2\right]^{\alpha}}+\left[\frac{a\tau}{8q_i}\right]^{\alpha}\cdot\left[\hat{u}_{ij}^{n+1}-\hat{u}_{ij}^{n+\frac{1}{2}}\right]^{\alpha}.$$

Hence when $i=0\,(x=0)$ we have

$$\left[\hat{u}_{0j}^{n+\frac{1}{2}}\right]^{\alpha}=\left[\frac{\hat{u}_{0j}^{n}+\hat{u}_{0j}^{n+1}}{2}\right]^{\alpha}+\left[\frac{\tau}{4q_0}\right]^{\alpha}\frac{\left[k_{y_{j-1}}\hat{u}_{0,j+1}^{n}\right]^{\alpha}-\left[(k_{y_{j-1}}-k_{y_j})\hat{u}_{0j}^{n}\right]^{\alpha}+\left[k_{y_j}\hat{u}_{0,j-1}^{n}\right]^{\alpha}}{\left[h_y^2\right]^{\alpha}}-$$
$$\left[\frac{\tau}{4q_0}\right]^{\alpha}\frac{\left[k_{y_{j-1}}\hat{u}_{0,j+1}^{n+1}\right]^{\alpha}-\left[(k_{y_{j-1}}+k_{y_j})\hat{u}_{0j}^{n+1}\right]^{\alpha}+\left[k_{y_j}\hat{u}_{0,j-1}^{n+1}\right]^{\alpha}}{\left[h_y^2\right]^{\alpha}}+\left[\frac{a\tau}{8q_0}\right]^{\alpha}\cdot\left[\hat{u}_{0j}^{n+1}-\hat{u}_{0j}^{n+\frac{1}{2}}\right]^{\alpha}.$$

Similarly, we get the relation at $i=I\,(x=L)$

$$\left[\hat{u}_{Ij}^{n+\frac{1}{2}}\right]^{\alpha}=\left[\frac{\hat{u}_{Ij}^{n}+\hat{u}_{Ij}^{n+1}}{2}\right]^{\alpha}+\left[\frac{\tau}{4q_I}\right]^{\alpha}\frac{\left[k_{y_{j-1}}\hat{u}_{I,j+1}^{n}\right]^{\alpha}-\left[(k_{y_{j-1}}-k_{y_j})\hat{u}_{Ij}^{n}\right]^{\alpha}+\left[k_{y_j}\hat{u}_{I,j-1}^{n}\right]^{\alpha}}{\left[h_y^2\right]^{\alpha}}-$$
$$\left[\frac{\tau}{4q_I}\right]^{\alpha}\frac{\left[k_{y_{j-1}}\hat{u}_{I,j+1}^{n+1}\right]^{\alpha}-\left[(k_{y_{j-1}}+k_{y_j})\hat{u}_{Ij}^{n+1}\right]^{\alpha}+\left[k_{y_j}\hat{u}_{I,j-1}^{n+1}\right]^{\alpha}}{\left[h_y^2\right]^{\alpha}}+\left[\frac{a\tau}{8q_I}\right]^{\alpha}\cdot\left[\hat{u}_{Ij}^{n+1}-\hat{u}_{Ij}^{n+\frac{1}{2}}\right]^{\alpha}.$$

Theorem 1 Suppose that for the system

$$\left[b_0\right]^{\alpha}\left[\hat{u}_0\right]^{\alpha}+\left[c_0\right]^{\alpha}\left[\hat{u}_1\right]^{\alpha}=\left[f_0\right]^{\alpha},$$
$$\left[a_i\right]^{\alpha}\left[\hat{u}_{i-1}\right]^{\alpha}+\left[b_0\right]^{\alpha}\left[\hat{u}_i\right]^{\alpha}+\left[c_i\right]^{\alpha}\left[\hat{u}_{i+1}\right]^{\alpha}=\left[f_i\right]^{\alpha},$$
$$\left[a_n\right]^{\alpha}\left[\hat{u}_{n-1}\right]^{\alpha}+\left[b_n\right]^{\alpha}\left[\hat{u}_n\right]^{\alpha}=\left[f_n\right]^{\alpha},$$

by the formulas

$$\left[x_0\right]^{\alpha}=-\left[c_0\right]^{\alpha}/\left[b_0\right]^{\alpha},$$
$$\left[x_i\right]^{\alpha}=-\left[c_i\right]^{\alpha}/\left[b_i+a_ix_{i-1}\right]^{\alpha},\quad i=1,...,I-1,$$
$$\left[y_0\right]^{\alpha}=\left[f_0\right]^{\alpha}/\left[b_0\right]^{\alpha},$$
$$\left[y_i\right]^{\alpha}=\left[f_i-a_iy_{y-1}\right]^{\alpha}/\left[b_i+a_ix_{i-1}\right]^{\alpha},\quad i=1,...,I-1,$$
$$\left[u_n\right]^{\alpha}=\left[y_n\right]^{\alpha},$$
$$\left[u_i\right]^{\alpha}=\left[x_iu_{i+1}\right]^{\alpha}+\left[y_i\right]^{\alpha},\quad i=I-1,I-2,...,0$$

defined $[u_i]^\alpha$

Then we have

$$\bigcup_{\alpha} \alpha \hat{u}_i^\alpha \subseteq \bigcup_{\alpha} \alpha u_i^\alpha$$

Where

$$[a_i]^\alpha = \left[\frac{\tau}{2h_y^2} k_{y_j}\right]^\alpha,$$

$$[b_i]^\alpha = \left[\frac{\tau}{2h_y^2} k_{y_j} + \frac{\tau}{2h_y^2} k_{y_{j-1}} + q_i + \frac{a\tau}{4}\right]^\alpha,$$

$$[c_i]^\alpha = \left[\frac{\tau}{2h_y^2} k_{y_{j-1}}\right]^\alpha, \qquad [f_i]^\alpha = \left[Q_{ij}^{n+\frac{1}{2}}\right]^\alpha \qquad \text{в (2) и}$$

$$[a_i]^\alpha = \left[\frac{\tau}{2h_z^2} k_{z_i} + \frac{\tau}{4h_z} w\right]^\alpha,$$

$$[b_i]^\alpha = \left[\frac{\tau}{2h_z^2} k_{z_i} + \frac{\tau}{2h_z^2} k_{z_{i-1}} + q_i + \frac{a\tau}{4}\right]^\alpha,$$

$$[c_i]^\alpha = \left[\frac{\tau}{2h_z^2} k_{z_{i-1}} - \frac{\tau}{4k_z} w\right]^\alpha, \quad [f_i]^\alpha = \left[F_{ij}^n\right]^\alpha .$$

5. CONCLUSIONS

Introduced analogues of numerical solutions of fuzzy differential equations by the method of variable directions generalize earlier reviewed fuzzy differential equations and inclusions and provide an opportunity to examine their properties.

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