RESTRAINED LICT DOMINATION IN GRAPHS

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Abstract

A set $D_r \subseteq V[n(G)]$ is a restrained dominating set of $n(G)$, where every vertex in $V[n(G)] - D_r$ is adjacent to a vertex in D_r as well as another vertex in $V[n(G)] - D_r$.

The restrained domination number of lict graph $n(G)$, denoted by $\gamma_{rn}(G)$, is the minimum cardinality of a restrained dominating set of $n(G)$. In this paper, we study its exact values for some standard graphs we obtained. Also its relation with other parameters is investigated.

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1. INTRODUCTION

In this paper, all the graphs considered here are simple, finite, non-trivial, undirected and connected. As usual p and q denote, the number of vertices and edges of a graph G. In this paper, for any undefined terms or notations can be found in Harary [4].

As usual, the maximum degree of a vertex in G is denoted by $\Delta(G)$.

The degree of an edge $e = uv$ of G is defined as $\deg e = \deg u + \deg v - 2$ and $\delta'(G)(\Delta'(G))$ is the minimum(maximum) degree among the edges of G.

For any real number x, $\lceil x \rceil$ denotes the smallest integer not less than x and $\lfloor x \rfloor$ denotes the greatest integer not greater than x. The complement $\overline{G}$ of a graph G has V as its vertex set, but two vertices are adjacent in $\overline{G}$ if they are not adjacent in G.

A vertex (edge) cover in a graph G is a set of vertices that covers all the edges (vertices) of G. The vertex (edge) covering number $\alpha_0(G)(\alpha_1(G))$ is a minimum cardinality of a vertex (edge) cover in G. The vertex (edge) independence number $\beta_0(G)(\beta_1(G))$ is the maximum cardinality of independent set of vertices (edges) in G.

The greatest distance between any two vertices of a connected graph G is called the diameter of G and is denoted by $diam(G)$.

We begin by recalling some standard definition from domination theory.

A set D of a graph $G = (V, E)$ is a dominating set if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set in G. The study of domination in graphs was begun by Ore [7] and Berge [1].

A set $D \subseteq V[L(G)]$ is said to be dominating set of $L(G)$ if every vertex not in D is adjacent to a vertex in D.The domination number of G is denoted by $\gamma[L(G)]$ is the minimum cardinality of dominating set in G.

A set F of edges in a graph G is called an edge dominating set of G if every edge in $E - F$ where E is the set of edges in G is adjacent to atleast one edge in F. The edge domination number $\gamma'(G)$ of a graph G is the minimum cardinality of an edge dominating set of G. The concept of edge domination number in graphs were studied by Gupta [3] and S.Mitchell and S.T.Hedetineim [6].

Analogously, we define restained domination number in lict graph as follows.
A dominating set D_r of lict graph is a restained dominating set, if every vertex not in D_r is adjacent in D_r and to a vertex in $V-D_r$.The restrained domination number of lict graph $n(G)$,denoted by $\gamma_{rn}(G)$ is the minimum cardinality of a restrained dominating set of $n(G)$.The concept of restrained domination in graphs was introduced by Domke et al [2].

In this paper, many bounds on $\gamma_{rn}(G)$ were obtained and expressed in terms of vertices, edges of G but not the elements of $n(G)$. Also we establish restained domination number of a lict graph $n(G)$ and express the results with other different domination parameters of G.

2. RESULTS

We need the following Theorems to establish our further results.

Theorem A [5]: For any connected (p,q) graph G, $\gamma'(G) \geq \left\lceil \frac{q}{\Delta'(G)+1} \right\rceil$.

Theorem B [5]: If G is a graph with no isolated vertex, then $\gamma'(G) \leq q - \Delta'(G)$.

Initially we begin with restrained domination number of lict graph of some standard graphs, which are straight forward in the following Theorem.

2.1 Theorem 1:

(i) For any cycle C_p with $p \geq 3$ vertices,

$$\gamma_{rn}(C_p) = p - 2\left\lfloor \frac{p}{3} \right\rfloor.$$

(ii) For any path P_p with $p > 2$ vertices,

$$\gamma_{rn}(P_{2n-1}) = k$$

$$\gamma_{rn}(P_{2n}) = k \text{ , When } n = 2,3,4,5,\ldots\ldots\ldots$$

Then $k = 1,2,3,4,\ldots\ldots\ldots$

(iii) For any star $K_{1,p}$ with $p \geq 2$ vertices,

$$\gamma_{rn}(K_{1,p}) = 1.$$

(iv) For any wheel W_p with $p \geq 4$ vertices,

$$\gamma_{rn}(W_p) = 1 + \left\lceil \frac{p-3}{3} \right\rceil.$$

(v) For any complete bipartite graph K_{p_1,p_2} with $p_1, p_2 > 2$ vertices,

$$\gamma_{rn}(K_{p_1,p_2}) = \min\{p_1, p_2\}.$$

(vi) For any complete graph K_p with $p \geq 3$ vertices ,

$$\gamma_{rn}(K_p) = \left\lfloor \frac{p}{2} \right\rfloor.$$

In the following Theorem, we establish the upper bound for $\gamma_{rn}(T)$ in term of vertices of the G.

2.2 Theorem 2

For any tree T with $p > 2$ vertices and m end vertices $\gamma_{rn}(T) \leq p - m$. Equality holds if $T = K_{1,p}$ with $p \geq 2$ vertices.

Proof: If $diam(G) \leq 3$, then the result is obvious. Let $diam(T) > 3$ and $V_1 = \{v_1, v_2, v_3, \ldots\ldots\ldots, v_n\}$ be the set of all end vertices in T with $|V_1| = m$.Further $E = \{e_1, e_2, e_3, \ldots\ldots\ldots\ldots, e_q\}$ and $C = \{c_1, c_2, c_3, \ldots\ldots\ldots\ldots, c_i\}$ be the set of edges and cutvertices in G. In $n(G)$, $V[n(G)] = E(G) \cup C(G)$ and in G $\forall e_i$ incicident with $c_j, 1 \leq j \leq i$ forms a complete induced subgraph as a block in $n(G)$, such that the number of blocks in $n(G) = |C|$. Let $\{e_1, e_2, e_3, \ldots\ldots\ldots\ldots\ldots, e_j\} \in E(G)$ which are non end edges of G forms cutvertices

$C'(G) = \{c_1, c_2, c_3, ..., c_j\}$ in $n(G)$. Let $C_1' \le C'$ be a restrained dominating set in $n(G)$, such that $|C_1'| = \gamma_{rn}(G)$. For any non trivial tree $p > q$ and $|C_1'| \le p - m$ which gives $\gamma_{rn}(T) \le p - m$.

Further equality holds if $T = K_{1,p}$ then $n(K_{1,p}) = K_{p+1}$ and $\gamma_{rn}(K_{1,p}) = p - m.$

The following corollaries are immediate from the above Theorem.

Corollory 1: For any connected (p,q) graph G, $\gamma_{rn}(G) + \gamma(G) \le \alpha_0(G) + \beta_0(G)$. Equality holds if G is isomorphic to C_4 or C_5.

Corollory 2: For any connected (p,q) graph G, $\gamma_{rn}(G) + \gamma(G) \le \alpha_1(G) + \beta_1(G)$. Equality holds if G is isomorphic to C_4 or C_5.

2.3 Theorem 3

For any connected (p,q) graph G with $p > 2$ vertices, $\gamma_{rn}(G) \le \left\lceil \frac{p}{2} \right\rceil$. Equality holds if G is C_4 or C_5 or C_8 or K_p if p is even.

Proof: Let $E = \{e_1, e_2, e_3, ..., e_n\}$ be the edge set of G such that $V[n(G)] = E(G) \cup C(G)$, by definition of lict graph where $C(G)$ is the set of cutvertices in G. Let $D_r = \{v_1, v_2, v_3, ..., v_n\} \subseteq V[n(G)]$ be the restrained dominating set of $n(G)$. Suppose if $|V[n(G)] - D_r| \ge 2$, then $\{V[n(G)] - D_r\}$ contains atleast two vertices which gives $\gamma_{rn}(G) < \frac{p}{2} \le \left\lceil \frac{p}{2} \right\rceil$.

For the equality,

i) If G is isomorphic to C_4 or C_5 or C_8. For any cycle C_p with $p \ge 3$ vertices $n(C_p) \cong C_p$, which gives $|D_r| = \left\lceil \frac{p}{2} \right\rceil$. Therefore $\gamma_{rn}(C_p) = \left\lceil \frac{p}{2} \right\rceil$.

ii) If G is isomorphic to K_p, where is p even, , then by Theorem 1 ,

$$\gamma_{rn}(K_p) = \left\lceil \frac{p}{2} \right\rceil.$$

In the following Theorem, we obtain the relation between $\gamma_{rn}(G)$ and diameter of G.

2.4 Theorem 4

For any connected (p,q) graph G, $\gamma_{rn}(G) \ge \left\lceil \frac{diam(G)+1}{3} \right\rceil$.

Proof: Let D_r be a restrained dominating set of $n(G)$ such that $|D_r| = \gamma_{rn}(G)$. Consider an arbitrary path of length which is a $diam(G)$. This diameteral path induces atmost three edges from the induced subgraph $\langle N(v) \rangle$ for each $v \in D_r$. Furthermore since D_r is γ_{rn}-set, the diametral path includes atmost $\gamma_{rn}(G) - 1$ edges joining the neighborhood of the vertices of D_r.

Hence $diam(G) \le 2\gamma_{rn}(G) + \gamma_{rn}(G) + 1$.
$diam(G) \le 3\gamma_{rn}(G) - 1.$

Hence the result follows.

The following theorem relates domination number of G and restrained domination number of $n(G)$.

2.5 Theorem 5

For any connected (p,q) graph G with $p \ge 3$ vertices,

$$\gamma_{rn}(G) \le P$$

$-\gamma(G)$. Equality holds if $G \cong C_4$ or C_5.

Proof: Let $D = \{u_1, u_2, u_3, ..., u_n\}$ be a minimal dominating set of $n(G)$ such that $|D| = \gamma(G)$. Further let $F_1 = \{e_1, e_2, e_3, ...e_n\}$ be the set of all edges which are incident to the vertices of D and $F_2 = E(G) - F_1$.

Let $C=\{c_1,c_2,c_3,..........,c_n\}$ be the cutvertex set of G. By definition of lict graph, $V[n(G)]=E(G)\cup C(G)$ and $F_1\subseteq V[n(G)]$.Let $I_1=\{e_1,e_2,e_3,.......e_k\}$; $1\le k\le i$,where $I_1\subseteq F_1$ and $I_2\subseteq F_2$.Since each induced subgraph which is complete in $n(G)$ may contain atleast one vertex of either F_1 or F_2. Then $(I_1\cup I_2)$ forms a minimal restrained dominating set in $n(G)$ such that $|I_1\cup I_2|=|D_r|=\gamma_{rn}(G)$.Clearly $|D|\cup|I_1\cup I_2|\le p$.Thus it follows that $\gamma(G)+\gamma_{rn}(G)\le p$.

For equality,
If $G\cong C_p$ for $p=4$ or 5, then by definition of Lict graph $n(C_p)\cong C_p$.Then in this case $|D|=|D_r|=\frac{p}{2}$. Clearly it follows that $\gamma_{rn}(G)+\gamma(G)=p$.

In [5] , they related $\gamma'(G)$ with line domination of G. In the following theorem we establish our result with edge dominationof G.

2.6 Theorem 6

For any non-trivial connected (p,q) graph G, $\gamma_{rn}(G)\ge\gamma'(G)$.

Proof: Let $E=\{e_1,e_2,e_3,........,e_n\}$ be the edge set of G and $C=\{c_1,c_2,c_3,......,c_n\}$ be the set of cutvertices in G, $V[n(G)]=E(G)\cup C(G)$. Let $F=\{e_1,e_2,e_3,..........,e_n\}$;

$\forall e_i$, where $1\le i\le n$ be the minimal edge dominating set of G, such that $|F|=\gamma'(G)$.

Since $E(G)\subseteq V[n(G)]$ every edge $e_i\in F$; $\forall e_i$; $1\le i\le n$ forms a dominating set in $n(G)$.Suppose $F_1=E(G)-F\subseteq V[n(G)]$, we consider $I_1=\{e_1,e_2,e_3,..............,e_n\}$;

$1\le k\le i$, where $I_1\subseteq F$ and $I_2\subseteq F_1$. Since each induced subgraph which is complete in $n(G)$ may contain atleast one vertex of either F or F_1. Then $|I_1\cup I_2|$ forms a minimal restrained dominating set in $n(G)$.Clearly it follows that $|F|\subseteq|I_1\cup I_2|$ in $n(G)$. Hence $\gamma'(G)\le\gamma_{rn}(G)$.

In the next Theorem, we obtain the relation between domination number of G and restrained domination number of $n(G)$ in terms of vertices and diameter of G.

2.7 Theorem 7

For any connected (p,q) graph G with $p>2$ vertices,

$$\gamma_{rn}(G)\le p+\gamma(G)-diam(G).$$

Proof: Let $V=\{v_1,v_2,v_3,.......,v_n\}$ be the set of vertices in G. Suppose there exists two vertices $u,v\in V(G)$ such that $\text{dist}(u,v)=diam(G)$.Let $D=\{v_1,v_2,v_3,........,v_p\}$; $1\le p\le n$ be a minimal dominating set in $n(G)$. Now we consider $F=\{e_1,e_2,e_3,.......,e_n\}$; $F\subseteq E(G)$ and $\forall e_i\in V[n(G)],1\le i\le n$ in $n(G)$.The $V[n(G)]=E(G)\cup C(G)$,where $C(G)$ is the set of cutvertices in G.Suppose F_1,C_1 are the subsets of F and C.Then $\exists$ a set $\{J\}\in V[n(G)]-\{F_1\cup C_1\}$ such that $\langle J\rangle$ has no isolates .Clearly $|F_1\cup C_1|=\gamma_{rn}(G)$.Let $u,v\in V(G)$, $d(u,v)=diam(G)$, then $\{F_1\cup C_1\}\cup diam(G)<p\cup|D|$.Hence $\gamma_{rn}(G)+diam(G)\le p+\gamma(G)$ which implies $\gamma_{rn}(G)\le p+\gamma(G)-diam(G)$.

The following Theorem, relates restrained domination number of $n(G)$ in terms of vertices and $\Delta(G)$.

2.8 Theorem 8

For any connected (p,q) graph G, $\gamma_{rn}(G)\le p-\Delta(G)$.Equality holds if G is P_3 or C_p

$(3 \le p \le 5)$ or $K_{1,p}\,(p \ge 2)$ or K_{p_1,p_2} with $p_1, p_2 > 2$.

Proof: We consider the following cases.

Case 1: Suppose G is tree. Then $E'(G) = \{e_1, e_2, e_3, ..., e_n\}$ be the set of all end edges and $E(G) = \{e_1, e_2, e_3, ..., e_i\}$; $\forall e_i \in E(G)$ be the set of all non-end edges. Let $D_r = \{u_1, u_2, u_3, ..., u_n\}$ be the vertex set of $n(G)$ which corresponds to the set F and is a minimal edge dominating set of $n(G)$ where $F \subseteq V[n(G)] = E(G) \cup C(G), C(G)$ a set of cutvertices . Further let $I_1 \subseteq V[n(G)] = F$ and $I_2 \subseteq F_1$, where $F_1 = E(G) - F \subset V[n(G)]$. Then $(I_1 \cup I_2)$ forms a minimal restrained dominating set of $n(G)$, where $I_1, I_2 \in E(G) \subseteq V[n(G)]$. Since it is clear that $\gamma_{rn}(G) < p$ and $p = q+1$.Let there exists a vertex $v_i \in \Delta(G)$ and also by Theorem 3, $\gamma_{rn} < \dfrac{p}{2}$, which gives the result of minimal restrained dominating set of $n(G)$ such that $|D_r| \le p - \Delta(G)$ and is $\gamma_{rn}(G) \le P - \Delta(G)$.

Case 2: Suppose G is not a tree , again we consider the following subcases of case 2.

Subcase 2.1: Assume G is a cycle $C_p\,(p \ge 3)$,Since for any cycle C_p with $p \ge 3$ vertices $\Delta(G) = 2$ and by Theorem 1, $\gamma_{rn}(C_p) = p - 2\left\lceil \dfrac{p}{3} \right\rceil = p - \Delta(G)$.

Subcase 2.2: Assume G is a cyclic graph .Then there exists a cycle or block in G which contains cycles. Let $F = \{e_1, e_2, e_3, ..., e_n\}$ be the edge dominating set in G, such that atleast one of $e_i \in \langle G' \rangle$; $1 \le i \le n$, where G' is a block or a cycle in G. In $n(G)$ the set F gives a minimal dominating set and let $I_1 = \{v_1, v_2, v_3, ..., v_n\}$ be the set of vertices in $n(G)$ and $I_1 \in E(G) \subseteq V[n(G)] - F$ and let $I_2 \subseteq F_1$ where $F_1 = E(G) - F$ .Then $(I_1 \cup I_2)$ is a minimal restrained dominating set of $n(G)$, which gives $|I_1 \cup I_2| = |D_r| = \gamma_{rn}(G)$. Suppose $v \in \Delta(G)$ and atleast two edges which are incident to v are the element of F which gives $p - \Delta(G)$.So one can easily verify that $p - \Delta(G) \ge \gamma_{rn}(G)$.

For equality,

i) If $G \cong P_3$, then $n(P_3) \cong C_3$ and $|D_r| = 1$,which gives $\gamma_{rn}(G) = p - \Delta(G)$.

ii) If $G \cong C_p$ for $3 \le p \le 5$,since for any cycle $n(C_p) \cong C_p$ and $\Delta = 2$. Then $|D_r| = p - \Delta(G)$, which gives $\gamma_{rn}(C_p) = p - \Delta(G)$.

iii) If $G \cong K_{1,p}$ for $p \ge 2$ vertices , then $n(K_{1,p}) \cong K_p$. For any star $\Delta = p - 1$ and $p = q+1$ and also $|D_r| = 1$,which gives $\gamma_{rn}(K_{1,p}) = p - \Delta(G)$.

iv) If $G \cong K_{p_1,p_2}$ with $p_1, p_2 > 2$ vertices , then in this case $\Delta = \max\{p_1, p_2\}$ and also $p_1 \cup p_2 = V(G)$. Since by Theorem 1, $|D_r| = \min\{p_1, p_2\} = V(G) - \max\{p_1, p_2\}$, which gives $\gamma_{rn}(K_{p_1,p_2}) = \min\{p_1, p_2\} = p - \Delta(G)$.

The following Theorem gives the relation between the restrained domination number of $n(G)$ and vertex covering number of G .

2.9 Theorem 9

For any connected (p,q) graph G with $p > 2$ vertices, $\gamma_{rn}(G) \le \alpha_0(G)$

Proof: Let $B = \{v_1, v_2, v_3, ..., v_m\} \subset V(G)$ be the minimum number of vertices which covers all the edges , such that $|B| = \alpha_0(G)$ and $E_1 = \{e_1, e_2, e_3, ..., e_k\} \subset E(G)$, such that $\forall v_i \in B; 1 \le i \le n$ is incident with e_i, for $1 \le i \le k$. We consider the following cases.

Case 1: Suppose for any two vertices $v_1, v_2 \in B$ and $v_1 \in N(v_2)$.Then an edge e incident with v_1 and v_2 covers all edges incident with v_1 and v_2. Hence e belongs to γ_{rn}- set of G. Further for any vertex $v_i \in B$ covering the edge $e_i \in E_1$ incident with a cutvertex v_c of G, e_i belongs to the $\gamma_{rn}-$ set of G. Thus $\gamma_{rn}(G) \le |B| = \alpha_0(G)$.

Case 2: Suppose for any two vertices $v_1, v_2 \in B$ and $v \notin N(v_2)$.Then $e_1, e_2 \in E_1$ covers all the edges incident with v_1 and v_2. Since B consist of the vertices which covers the edges that are incident all the cutvertices of G, the corresponding edges in E_1 covers the cutvertices of G. Thus $\gamma_{rn}(G) \le |B| = \alpha_0(G)$.

Next we obtain a bound of restrained lict domination number in terms of number of edges and maximum edge degree of G.

2.10 Theorem 10

For any connected (p,q) graph G with $p \ge 3$, $\gamma_{rn}(G) \le q - \Delta'(G)$. **Proof:** We consider the following cases.

Case 1 : Suppose G is non-seperable. Using Theorem 6 and Theorem B , the result follows.

Case 2 : Suppose G is seperable. Let e be an edge with degree Δ' and let M be a set of edges adjacent to e in G.

Then $E(G) - M$ covers all the edges and all the cutvertices of G.But some of the $e_i's \in E(G) - M$, for $1 \le i \le n$ forms a minimal restrained dominating set in $n(G)$.

Then $\gamma_{rn}(G) \le |E(G) - M|$, which gives $\gamma_{rn}(G) \le q - \Delta'(G)$.

The following Theorem relates the domination number of $L(G)$ and and restrained domination number of $n(G)$.

2.11 Theorem 11

For any connected graph G with $p > 2$ vertices, $\gamma_{rn}(G) \le q - \gamma[L(G)]$

Proof: Let $E = \{e_1, e_2, e_3, ..., e_n\}$ be the edge set of G and $C = \{c_1, c_2, c_3, ..., c_n\}$ be the cutvertex set of G, then $V[n(G)] = E(G) \cup C(G)$ and $V[L(G)] = E(G)$,by definition. Suppose $H = \{u_1, u_2, u_3, ..., u_n\} \subseteq V[L(G)]$ be the set of vertices of degree $\deg(u_i) \ge 2, 1 \le i \le n$.Then $D' \subseteq H$ forms a minimal dominating set of $L(G)$ such that $|D'| = \gamma[L(G)]$. Further let $H' = \{u'_1, u'_2, u'_3, ..., u'_i\}; 1 \le i \le n$, where $H' \subseteq H$ .Then $H' \cup D'$ forms a minimal restrained dominating set in $n(G)$. Since $V[L(G)] = E(G) = q$ and also $V[L(G)] \subseteq V[n(G)]$.Clearly it follows that $|D' \cup H'| \cup |D'| \le q$. Thus $\gamma_{rn}(G) + \gamma[L(G)] \le q$.

To prove our further result, we give the following two observations.

Observation 1: For connected (p,q) graph G, $\gamma_{rn}(G) \le q - 2$.

Proof: Suppose D_r is a restrained dominating set of $n(G)$.Then by definition of restrained domination $|V[n(G)]| \ge 2$. Further by definition of $n(G)$, $q - \gamma_{rn}(G) \ge 2$. Clearly it follows that $\gamma_{rn}(G) \le q - 2$.

Observation 2: Suppose D_r be any restrained dominating set of $n(G)$, such that $|D_r| = \gamma_{rn}(G)$.Then $|V[n(G) + D_r]| \le \sum_{v_i \in D_r} \deg v_i$.

Proof: Since every vertex in $V[n(G)] + D_r$ is adjacent to atleast one vertex in $V[n(G)] + D_r$ contributes atleast one to the sum of the degrees of the vertices of D_r. Hence the proof.

2.12Theorem 12

For any connected (p,q) graph G, $\frac{q}{\Delta'(G)+1} \le \gamma_{rn}(G) \le q - \delta'(G)$.

Proof: Let $e \in E(G)$, now without loss of generality , by definition of lict graph , $e = u \in V[n(G)]$ and let D_r be the restrained dominating set of $n(G)$ such that $|D_r| = \gamma_{rn}(G)$. If $\delta(G) \leq 2$,then by observation 1, $\gamma_{rn}(G) \leq q-2 \leq q-\delta'(G)$. If $\delta'(G) > 2$, then for any edge $f \in N(e)$ and by definition of $n(G)$, $f = w \in N(u)$, $D_r \subseteq \{V[n(G)] - N(u)\} \cup \{w\}$. Then $\gamma_{rn}(G) \leq [q-(\delta'(G)+1)+1] = q-\delta'(G)$.

Now for the lower bound we have by observation 2 and the fact that any edge $e \in E(G)$ and $\deg e \leq \Delta'(G)$, we have

$$q - \gamma_{rn}(G) \leq |V[n(G)] + n(G)| \leq \sum_{v \in D_r} \deg v \leq \gamma_{rn}(G).\Delta'(G)$$

.

Therefore $\frac{q}{\Delta'(G)+1} \leq \gamma_{rn}(G)$.

2.13Theorem 13

For any non-trivial connected (p,q) graph G,

$$\gamma_{rn}(G) \geq \left\lceil \frac{q}{\Delta'(G)+1} \right\rceil.$$

Proof: Using Theorem 6 and Theorem A, the result follows.

Finally we obtain the Nordhous – Gaddum type result.

2.14 Theorem 14

Let G be a connected (p,q) graph such that both G and $\overline{G}$ are connected, then

i) $\gamma_{rn}(G) + \gamma_{rn}(\overline{G}) \geq \left\lceil \frac{p}{2} \right\rceil$.

ii) $\gamma_{rn}(G).\gamma_{rn}(\overline{G}) \geq \left\lceil \frac{3p}{2} \right\rceil$.

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