THE LONGITUDINAL PERTURBATED FLUID VELOCITY OF THE DUSTY FLUID IN THE INCOMPRESSIBLE FLOW IN CYLINDRICAL POLAR COORDINATE

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Abstract

The effect of finite volume fraction of suspended particulate matter on axially symmetrical jet mixing of incompressible dusty fluid has been considered. However, this assumption is not justified when the fluid density is high or particle mass fraction is large Here we are assuming the velocity and temperature in the jet to differ only slightly from that of surrounding stream, a perturbation method has been employed to linearized the equation those have been solved by using Laplace Transformation technique. Numerical computations have been made to discuss the longitudinal perturbated fluid velocity.

Keywords: *Particulate suspension, Boundary layer characteristics, Volume fraction, Incompressible flow.*

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1. INTRODUCTION

Many researchers have been studied, in the incompressible laminar jet mixing of a dusty fluid issuing from a circular jet with negligible volume fraction of SPM. However, this assumption is not justified when the fluid density is high or particle mass fraction is large. In the present paper, we find the magnitude of Longitudinal perturbated velocity of fluid phase. Assuming the velocity and temperature in the jet to differ only slightly from that of the surrounding stream, a perturbation method has been employed to lineralize the governing differential equations. The resulting lineralize equations have been solved by using Laplace transformation technique. Numerical computations have been made to find the Longitudinal velocity profiles of fluid phase.

2. MATHEMATICAL FORMULATION

The equation governing the study two-phase boundary layer flow in axi-symmetric case can be written in cylindrical polar coordinates as

Equation of Countinuity in fluid phase

$$\frac{\partial}{\partial z}(ru) + \frac{\partial}{\partial r}(rv) = 0 \tag{1}$$

Equation of motion in fluid phase

$$(1-\phi)\rho\left(u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}\right) = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\rho_p}{\tau_m}(u_p-u) \tag{2}$$

Equation of heat in fluid phase

$$(1-\phi)\rho C_p\left(u\frac{\partial T}{\partial z}+v\frac{\partial T}{\partial r}\right) = K\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+\rho_p C_s\frac{(T_p-T)}{\tau_t} \tag{3}$$

To study the boundary layer flow, we introduce the dimensionless variables are

$$\bar{z}=\frac{z}{\lambda},\ \bar{r}=\frac{r}{(\tau_m\upsilon)^{\frac{1}{2}}},\bar{u}=\frac{u}{U},\bar{v}=v\left(\frac{\tau_m}{\upsilon}\right)^{\frac{1}{2}},\ \bar{u}_p=\frac{u_p}{U},\bar{v}_p=v_p\left(\frac{\tau_m}{\upsilon}\right)^{\frac{1}{2}},\alpha=\frac{\rho_{p_0}}{\rho}=\text{const}$$

$$\bar{\rho}_p=\frac{\rho_p}{\rho_{p_0}},\ \bar{T}=\frac{T}{T_0},\ \bar{T}_p=\frac{T_p}{T_0},\ \lambda=\tau_m U,\ \tau_m=\frac{2}{3}\frac{C_p}{C_s}\frac{1}{p_r}\tau_T, p_r=\frac{\mu C_p}{K}.$$

Now considering the flow from the orifice under full expansion we can assume that the pressure in the mixing region to be approximately constant. Hence, the pressure at the exit is equal to that of the surrounding stream. Therefore, both the velocity and the temperature in the jet is only slightly different from that of the surrounding stream. The coefficient of viscosity μ and thermal conductivity K are assumed to be constant. Then it is possible to write $u=u_0+u_1,\ v=v_1, u_p=u_{p_0}+u_{p_1},\ v_p=v_{p_1}, T=T_0+T_1,\ T_p=T_{p_0}+T_{p_1},\ \rho_p=\rho_{p_1}$ where the subscripts 1 denotes the perturbed values which are much smaller than the basic values with subscripts '0' of the surrounding stream, i.e. $u_0 >> u_1, u_{p_0} >> u_{p_1}, T_0 >> T_1, T_{p_0} >> T_{p_1}$.

Using the dimensionless variable and the perturbation method to the non linear above equations (1) to (3) becomes

$$\frac{\partial}{\partial z}(ru_1)+\frac{\partial}{\partial r}(rv_1)=0 \tag{4}$$

$$(1-\phi)u_0\ \frac{\partial u_1}{\partial z}=\frac{1}{r}\ \frac{\partial u_1}{\partial r}+\frac{\partial^2 u_1}{\partial r^2}+\alpha\,\rho_{p_1}\left(u_{p_0}-u_0\right) \tag{5}$$

$$(1-\phi)\ u_0\frac{\partial T_1}{\partial z}=\frac{1}{p_r}\left(\frac{\partial^2 T_1}{\partial r^2}+\frac{1}{r}\ \frac{\partial T_1}{\partial r}\right)+\frac{2\alpha}{3p_r}\rho_{p_1}\left(T_{p_0}-T_0\right) \tag{6}$$

The boundary conditions for u_1, v_1, u_{p_1} and v_{p_1} are

$$u_1(0,r)=\begin{cases} u_{10}, & r\le 1\\ 0, & r>1\end{cases} \tag{A}$$

$$\frac{\partial u_1}{\partial r}(z,0)=0,\ u_1(z,\infty)=0 \tag{B}$$

$$v_1(0,r)=0 \tag{C}$$

3. METHOD OF SOLUTION

The governing differential equation (4) have been solved by using Laplace transform technique and using the relevant conditions from (A) to (C) we get,

Laplace transform of $(1-\phi)u_0\ \frac{\partial u_1}{\partial z}=\frac{1}{r}\ \frac{\partial u_1}{\partial r}+\frac{\partial^2 u_1}{\partial r^2}+\alpha\,\rho_{p_1}\left(u_{p_0}-u_0\right)$ is

i.e

$$L\left\{(1-\phi)u_0\frac{\partial u_1}{\partial z}\right\}=L\left\{\frac{1}{r}\frac{\partial u_1}{\partial r}+\frac{\partial^2 u_1}{\partial r^2}+\alpha\rho_{p1}*(u_{p0}-u_0)\right\}$$

i.e.

$$L\left\{(1-\phi)u_0\frac{\partial u_1}{\partial z}\right\}=L\left\{\frac{1}{r}\frac{\partial u_1}{\partial r}+\frac{\partial^2 u_1}{\partial r^2}\right\}+L\left\{\alpha\rho_{p1}(up_0-u_0)\right\} \tag{7}$$

We have

$$L\left\{\frac{1}{r}\frac{\partial u_1}{\partial r}\right\}= p2\ U1\ (z,\ s)$$

And

$$L\left\{\frac{\partial^2 u_1}{\partial r^2}\right\}= s2\ U1\ (z,s) - s\ u\ (z,0) - u / (z,0) = s2\ U1\ (z,s)$$

$$L\{\alpha\rho_{p1}\} = \int_0^\infty \alpha\rho_{p1}(z,r)\ e^{-sr}\, dr = \alpha\rho_{p1}*(z,s)$$

Putting all the above values in equation (7) , it becomes

$$(1-\phi)\ u_0\ \frac{\partial u_1^*}{\partial z} = p^2 u_1^* + s2\,u_1^* + \alpha\rho_{p1}*(up_0-u_0)$$

Or

$$\frac{\partial u_1*}{\partial z}-\frac{(p^2+s^2)}{(1-\phi)\,u_0}u_1*=\alpha\frac{(up_0-u_0)}{(1-\phi)\,u_0}\rho_{p1}*$$

Or

$$\frac{\partial u_1*}{\partial z}+A\ k2\,u_1*=\alpha\ E\,\rho_{p1}*$$

Which is Linear Partial differential equation.

where A = $-\frac{p^2+s^2}{(1-\phi)\,u_0}$, B= $\frac{(up_0-u_0)}{(1-\phi)\,u_0}$ and p2 +s2 = k2

The Integrating Factor is I.F = $e^{-AK^2 z}$

The solution of the above differential equation using Laplace Transformation is

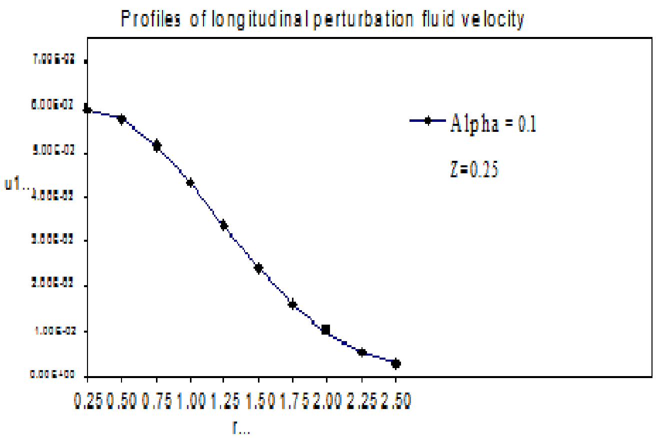
$$u_1*=\left(u_{10}-\frac{\alpha E\rho_{p1}*}{Ak^2}\right)\left(\frac{1-e^{-s}}{s}\right)e^{-AK^2z}+\frac{\alpha E\rho_{p_{10}}}{Ak^2} \tag{8}$$

The Laplace inverse Transformation of (8) gives

$$u_1(z,r)=L^{-1}\left\{(u_{10}-\frac{\alpha E\rho_{p1}*}{Ak^2})\frac{(1-e^{-s})}{s}e^{-AK^2z}+\frac{\alpha E\rho_{p_{10}}}{Ak^2}\right\} \tag{9}$$

4. CONCLUSIONS

Numerical computation have been made by taking $Pr = 0.72$, $u_{10} = u_{p10} = T_{10} = Tp_{10} = \rho_{p10} = 0.1$, $\varphi = 0.01$. The velocity and temperature
at the exit are taken nearly equal to unity.The given figures show the profiles of longitudinal perturbation fluid velocity u_1 for $\alpha = 0.1$, and the values of $Z = 0.25$. It is observed that the flow of the perturbated fluid velocity decreases with increase of the radius r.



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NOMENCLATURE

(x, y, z)	Space coordinates
(u, v, w)	Velocity components of fluid phase
(u_p, v_p, w_p)	Velocity components of particle phase
$(\bar{u}, \bar{v}, \bar{w})$	Dimensionless velocity components of fluid phase
$(\bar{u}_p, \bar{v}_p, \bar{w}_p)$	Dimensionless velocity components of particle phase
K	Thermal conductivity
R_e	Fluid phase Reynolds number
R_{e_p}	Particle phase Reynolds number
E_c	Eckret number
T	Temperature of fluid phase
T_p	Temperature of particle phase

C_{f_0}, C_{f_1} Skin friction coefficients at the lower and upper plates respectively

C_p, C_s Specific heats of fluid and SPM respectively