EVALUATION THE AFFECTS OF MIMO BASED RAYLEIGH NETWORK CASCADED WITH UNSTABLE DEAD ZONE SATURATION SYSTEM

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Abstract

Nonlinear Control is the area of control engineering specifically involved with systems that are nonlinear, time-variant, or both. Many well-established analysis and design techniques exist for linear time-invariant (LTI) systems .However, one or both of the controller and the system under control in a general control system may not be an LTI system, and so these methods cannot necessarily be applied directly. Nonlinear control theory studies how to apply existing linear methods to these more general control systems. In this paper we generalized a MIMO based Rayleigh network where by diversity technique we improve the signal to noise ratio and see the effects if its passing through a dead zone saturation system. We also see the effects of Zero Order Hold technique to produce a smooth running system.

Keywords: Dead Zone Saturation System, MRC Diversity Technique, ABER, Zero Order Hold Technique

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1. INTRODUCTION

A system always follows some rules. Whenever we try to describe system for the timbering we generally establish a relation between output and input part of the system. For easy way, if, among the twos one of a system must follows linearity and homogeneity then by analysis on processing function of the system it would be try to find out the stability point of system. These happens for only linear system.

But naturally system are non linear in practice where the above stated principle are not applicable. For example, for a sinusoidal input, linear system demands output obviously follows input i.e sinusoidal in nature but in practice, the output signal may content so many harmonics and sub-harmonics of various unexpected amplitudes and phase difference. So this characteristics obviously unbounded and exist the properties of limit cycle. The output state of an unstable non; linear system can go to infinity time.

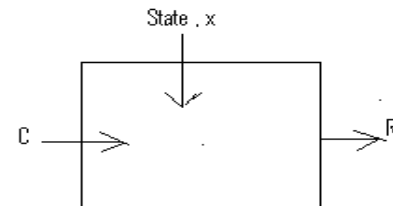So, the general model [1] of non linear system is as follows :

Fig 1: General Model of Non linear system

$x' = f(\tau, x, C, W)$: State Equation
$R = h(\tau, x, C, W)$: Response Equation

Where, $x \in D^i$; $C \in D^j$; $W \in D^k$

Here, 'x' is the state of the system and 'W' is the Creating Disturbance ; C is the input and R is the response (output) of the system ; D is the dynamical closed loop system ; i , j , k are the vector notation which trace the system in a well defined form.

In some cases [2] disturbance factor is not consider i.e $x' = f(\tau, x, C)$ and as well as even the system does not depend on input C, then its behave as an ***unforced system*** where the input not indicates any separate

Now, for an infinite dimensional system , the system is described with the help of ***distributed parameters*** i.e in that case, $f(\tau, x, C) = 0$

Where as a linear system can be modeled as:

$$R_L(\tau) = W_{0L}(\tau)C_L(\tau) + \int_0^{\tau} W_{1L}(\tau, \Delta\tau)C_L(\Delta\tau)d\Delta\tau \ . \ ...$$

(1)

Whereas, a Non Linear System can be modeled as :

$$R_{NL}(\tau) = W_{0NL}(\tau)C_{NL}(\tau) + \int_0^{\tau} W_{1NL}(\tau, \Delta\tau)C_{NL}(\Delta\tau)d\Delta\tau$$

$$+ \int_0^{\tau}\int_0^{\Delta\tau 1} W_{2NL}(\tau, \Delta\tau 1, \Delta\tau 2)C_{NL}(\Delta\tau 2)d\Delta\tau 1 d\Delta\tau 2 + ...$$

(2)

characteristics its itself as a function of τ { $C = f(\tau)$ }. But if x'= $f(x)$ only then it's a time invariantsystem which tends to force time to reach in origin.

2. CLASSIFICATION

In Broad sense [3] non satisfied superposition principle can be explained by two categories.

First one is very close to linear characteristics. Here, the problem of nonlinearity can be smoothly converted into linear problem by *feedback linearization method*.

On the other hand if reaching of stable point is very difficult, it indicates the another one. Here the nonlinearity problem can be produced in terms of *screendipitous, cautions, evolutionary, tactical*.

- *Screendipitious* the problem of constraints violets occasionally but no precautions are taken advance to avoid it.
- *Cautions* plays the roll in case of low gain controller system in which the objective is to produce closed loop asymptotic stability in bounded system. But at the cost of incurring performance is mitigated.
- *Evolutionary* is basically used in *antiwindup controller*. Here the broadly affections of constraints are suppressed, basically used in industry.
- *Tactial* is basically a model of predictive control where by chemical process the effect of constraints has to be reduced.

Stability basically has been seen [4] is linear system. For example for a second order system, the damping ratio (ξ) indicated by radiating area shown in fig.2 below depicts the minimum covering area , $\xi = \cos\theta$ or $\sin\varphi$

The condition for stable system must be satisfied the following condition that , $\xi_{effective} > \xi_{min}$, and $\xi_{min} = \cos\theta = \sin\varphi = \xi'$, where $\theta + \varphi = \pi/2$
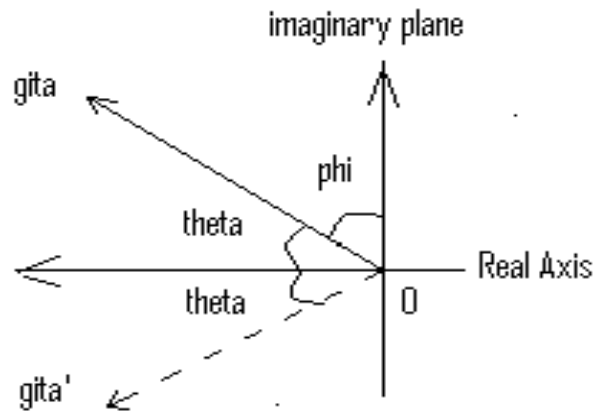

Fig 2 : Plot of radiating area of a Damping Ratio

Now, evaluation of roots for a processing function of a system on either side of numerator or denominator indicates if zeros and poles that plays a vital role in addition case of there.

Study Case 1 : $\dfrac{k}{s(s+a)} = \dfrac{add}{pole} > \dfrac{k}{s(s+a)(s+b)} =$

{Assuming, $|c| > |b| > |a|$ } $\dfrac{add}{pole} > \dfrac{k}{s(s+a)(s+b)(s+c)}$

Here the more roots added to increase the no of poles, the root locus goes through unstable position that means go through right handed side of origin shown below:

3. STABILITY ANALYSIS

3.1 'Root Locus of G(S) of Single Pole: In the Left Hand Side Hence the System is Stable for all Values of K'

ROOT LOCUS OF G(S) OF SINGLE POLE :IN THE LEFT HAND SIDE HENCE THE SYSTEM IS STABLE FOR ALL VALUES OF K
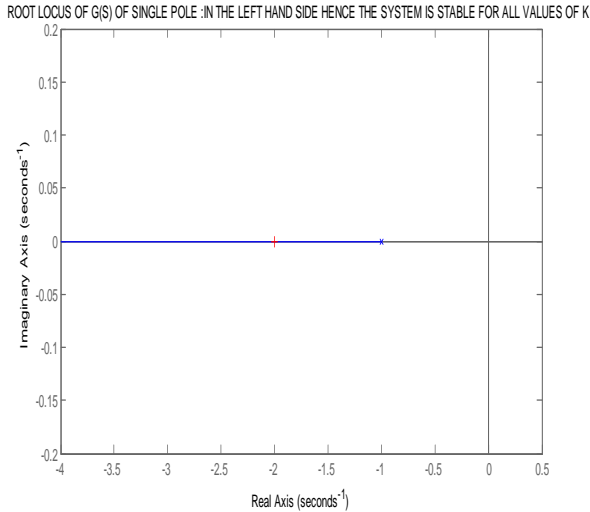
Select a point in the graphics window

selected_point = Empty matrix: 0-by-1 :   1

3.2 'Root Locus of G(S) with Two Pole on the LHS of S Plane: In the Left Hand Side hence the System is Stable for all Values of K'

ROOT LOCUS OF G(S) WITH TWO POLE ON THE LHS OF S PLANE : IN THE LEFT HAND SIDE HENCE THE SYSTEM IS STABLE FOR ALL VALUES OF K
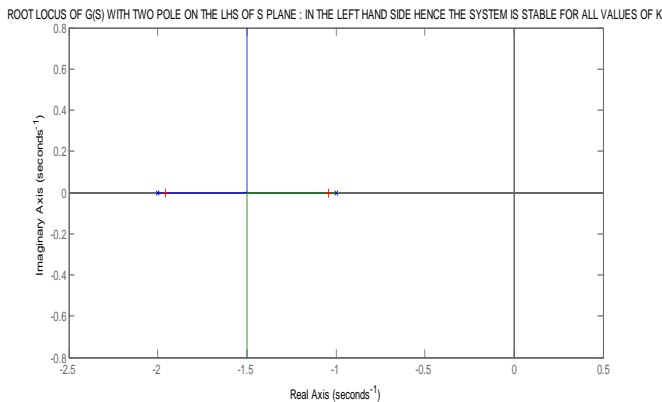
Select a point in the graphics window

selected_point = -1.0178 + 0.0373i :   0.0406

3.3 'Root Locus of G(S) with Increase of No of Poles Crosses the Imaginary Axix : Stability -> Unstability'

ROOT LOCUS OF G(S) WITH INCREASE OF NO OF POLES CROSSES THE IMAGINARY AXIX : STABILITY -> UNSTABILITY
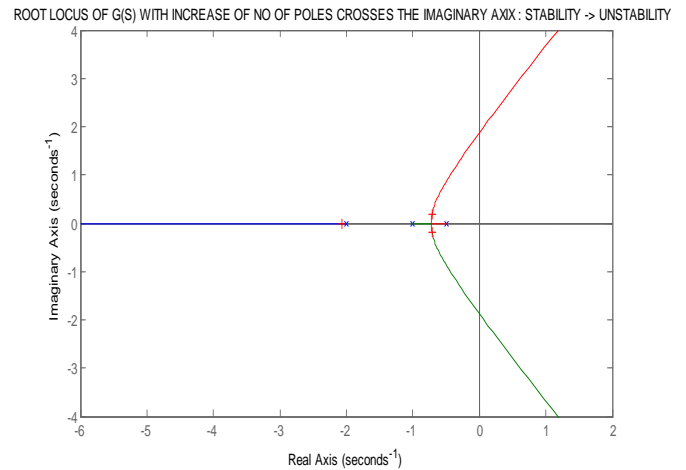
Select a point in the graphics window

selected_point = -1.2133 - 0.0373i :   0.1218

3.4 'Root Locus OG G(S) of Adding Complex Pole: System Tends to Restricted'

ROOT LOCUS OG G(S) OF ADDING COMPLEX POLE : SYSTEM TENDS TO RESTRICTED
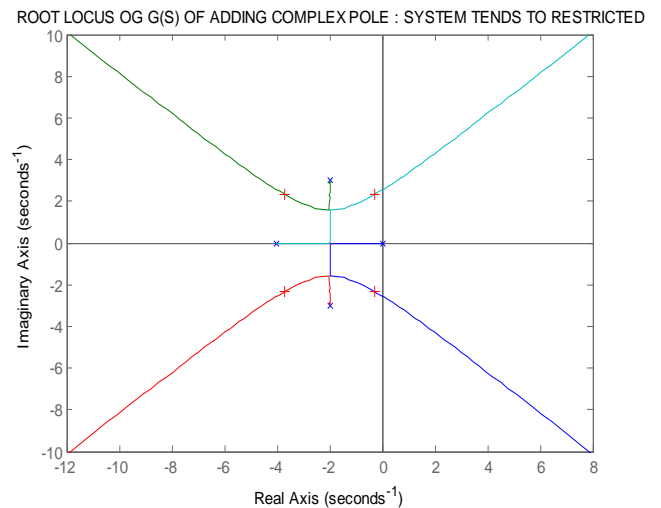
Select a point in the graphics window

selected_point =   0.6777 + 1.2112i :   107.7735

For considering complex axis, for the constant gain parameter $0<k<k_3<k_2<k_1$ the root contour goes more beyond of origin towards right hand side which enhances instability but for addition the complex root in denominator system tends to

became more restricted on stability and goes through L.H.S of s-plane.

Study Case2 : $\dfrac{k(s+a)}{(s+b)} = \dfrac{add}{zero} > \dfrac{k(s+a)(s+c)}{(s+b)} =$

{Assuming, $|c| > |b| > |a|$} $\dfrac{add}{zero} > \dfrac{k(s+a)(s+b)(s+c)}{(s+b)}$

Here the addition of *zeros* in the numerator indicates the left from origin that effects the improve of stability and for adding complex roots the stability is effected to reduce. Which is shown below :

3.5 'Root Locus of G(S) of Single Zero :In the Left Hand Side hence the System is Stable for all Values of K'

ROOT LOCUS OF G(S) OF SINGLE ZERO :IN THE LEFT HAND SIDE HENCE THE SYSTEM IS STABLE FOR ALL VALUES OF K
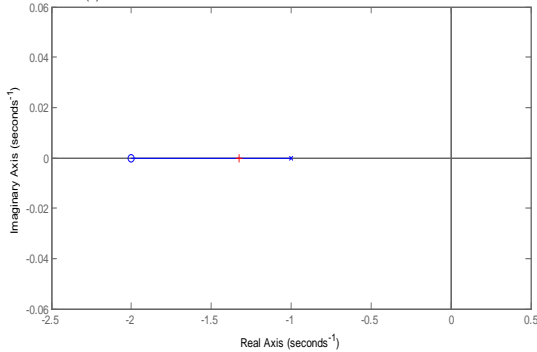
Select a point in the graphics window

selected_point = -0.0665 + 0.0013i :   0.4828

3.6 'Root Locus of G(S) with Two Zeros on the LHS of S Plane : In the Left Hand Side hence the System is More Stable for all Values of K'

ROOT LOCUS OF G(S) WITH TWO ZEROS ON THE LHS OF S PLANE : IN THE LEFT HAND SIDE HENCE THE SYSTEM IS MORE STABLE FOR ALL VALUES OF K
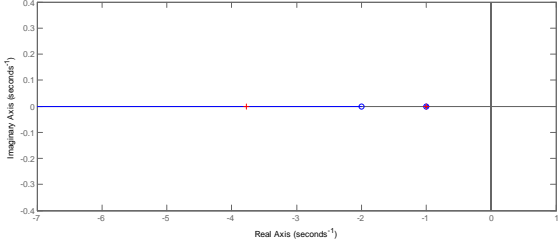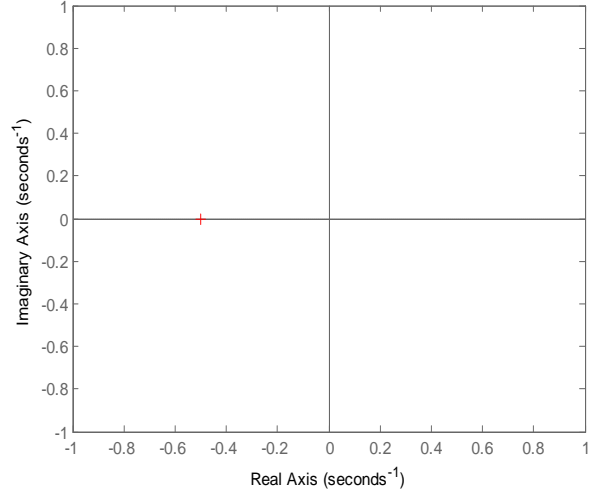
Select a point in the graphics window

selected_point = -0.2227 + 0.0261i :   0.5626

3.7 'Root Locus of G(S) with Increase of No of Zeors: System is Under Stable Totally'

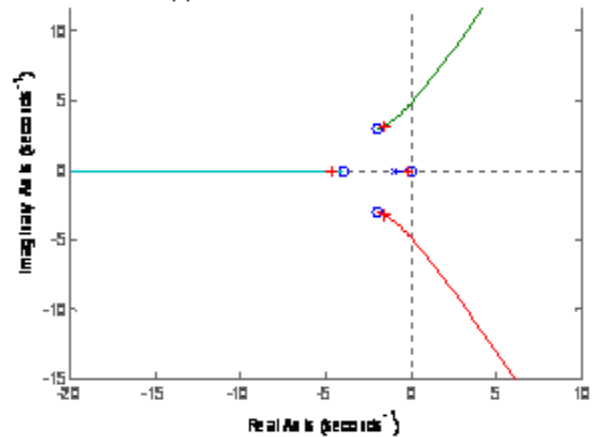OOT LOCUS OF G(S) WITH INCREASE OF NO OF ZEORS : SYSTEM IS UNDER STABLE TOT

Select a point in the graphics window

selected_point = 0.3246 + 0.0466i :   1

3.8 'Root Locus OG G(S) of Adding Complex Pole: System Stability Effected'

ROOT LOCUS OG G(S) -ADDING COMPLEX ZEROS : STABILITY EFFECTED

Select a point in the graphics window

selected_point = -2.6185 + 2.9348i :   0.0708

Now these all are bounding for linear system basically. For Non linear the system demands boundation for every harmonics. Practically this is very difficult.

4. NON LINEAR CONTROL WITH DESCRIBING FUNCTION IN MIMO NETWORK:

Due to affects of harmonics present in this system, its better to represent the processing function by the complex ratio of the fundamentals harmonics present in this system to the input component.

Processing Transfer Function :

$$\frac{\left|\text{Amplitudes of the systems Harmonics}\right|}{\left|\text{Amplitudes of the inputs component}\right|} \; , \; \angle\theta$$

θ is the phase shift of harmonics w.r.t input components

Consider a system which is combined for a MIMO based system cascaded with *dead zone* saturation system with a negative unity feedback.
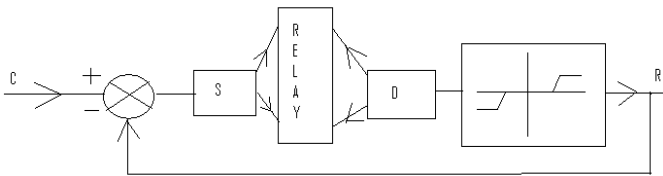

Fig 3: Model of MIMO based network with Non Linear Saturation System

Figure.3 shows a wireless communication system where terminal S is communicating with terminal D through the relay terminal R. The relay R is equipped with (M=2) receiving antennas for reception of signal transmitted from terminal S and (N=2) transmitting antenna to convey the signal to terminal D after suitable amplification or decoding (as the case may be) at the Relay. The Relay is cascaded with *Dead Zone Saturation Non Linear system*. We *as*sume that *Maximum Ratio Combining (MRC)* is used for receiving the signal at R and *Maximum Ratio Transmission (MRT)* is used for conveying it to the destination D. In MRC, [5] the branch with the instantaneous SNR is selected as the output of the combiner is given by :

$$\gamma_{mrc} = \sum_{i=1}^{2} \gamma i$$

Where the instantaneous signal-to-noise ratio (SNR) of the ith branch is, : $\gamma_i = E_s/N_0[\alpha_i^2]$, i=1, 2...With α_i being the Rayleigh fading amplitude of the channel

Between terminals S and the antennas (i = 1, 2) at relay R, E_S is the energy of the transmitted signals and N_0 is the one-sided noise power spectral density per branch.

We assume the channel is frequency non-selective and slowly varying such that it is constant over the transmitted symbols interval. Noting that α_i is Rayleigh distributed, α_i^2 is the exponentially distributed random variables. If we further assume that $(SNR)_i$ is the same for all diversity branches (i.e.,$\overline{\gamma_1} = \overline{\gamma_2}$, the output SNR of the MRC combiner(or, the SNR between S and R,$\gamma_{P,Q}$) has the probability density function (pdf) is given by

$$\mathrm{p}\Gamma_{S,R}(\gamma_{S,R}) = \frac{\gamma_{S,R}^{2-1}}{\bar{\gamma}_S^2 \, \Gamma(2)} e^{-\frac{\gamma_{S,R}}{\gamma_S}} \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$\mathrm{p}\Gamma_{S,R}(\gamma_{S,R}) = \frac{\gamma_{S,R}^{1}}{\bar{\gamma}_S^2 \, \Gamma(2)} e^{-\frac{\gamma_{S,R}}{\gamma_S}} \quad \ldots\ldots\ldots\ldots \quad (4)$$

Assuming α_j , j=1 and 2 the fading amplitude of channel between two antennas at R and D as Rayleigh type, the $(SNR)_j$ is the same for all diversity branches, namely $\overline{\gamma_R}$ similar way the pdf of $\Gamma_{R,D}$ can be written by :

$$\mathrm{p}\Gamma_{R,D}(\Gamma_{R,D}) = \frac{\gamma_{R,D}^{2-1}}{\bar{\gamma}_D^2 \, \Gamma(2)} e^{-\frac{\gamma_{R,D}}{\bar{\gamma}_R}} \quad \ldots\ldots \quad (5)$$

$$\mathrm{p}\Gamma_{R,D}(\Gamma_{R,D}) = \frac{\gamma_{R,D}^{1}}{\bar{\gamma}_D^2 \, \Gamma(2)} e^{-\frac{\gamma_{R,D}}{\bar{\gamma}_R}} \quad \ldots\ldots\ldots\ldots \quad (6)$$

Where $\overline{\gamma_R}$ is the average SNR of the R to D link.

Choosing the appropriate gain at the relay terminal before re-transmission, the overall SNR at the receiving terminal R can be very closely upper bounded as,

$$\gamma_{eq} = \frac{\Gamma_{S,R}\Gamma_{R,D}}{\Gamma_{S,R} + \Gamma_{R,D}} \quad \ldots\ldots\ldots\ldots \quad (7)$$

Where $\Gamma_{R,D}$ is the output SNR of the MRC, and $\Gamma_{R,D}$ is the instantaneous SNR between R and D with MRT

We also obtained [6],

$$\int_0^\infty S^{k-1} {}_2\sqrt{\beta} s k_1(\sqrt{\beta} s) ds = \frac{\Gamma(k+1)\Gamma(k)}{\beta^k} \; \ldots \; (8)$$

And hence reduce to:

$$M_x(S) = \sum_{K=0}^{\infty} \frac{(-S)^K}{\Gamma(K+1)} \left[\frac{\Gamma(K+1)\Gamma(K)}{\beta^K \Gamma(K)} = \frac{\beta}{S+\beta} \right. \quad \text{........ (9)}$$

Using the above results for mgf of destination SNR one can derive the average bit error rate (ABER) of several digital modulation schemes over the proposed relayed link by adopting the mgf-based approach. For demonstration, we consider here the ABER of binary differential phase-shift keying (DPSK). For amplify-and – forward relaying the ABER is given by:

$$P_b(e) = \frac{1}{2} M_r(1) \quad \text{........................ (10)}$$

For decode –and-forward relaying the ABER can be shown to be given by

$$P_b(e) = P_b(e_1) + P_b(e_2) - 2P_b(e_1)P_b(e_2)$$

Where $P_b(e_1)$= the ABER for S to R link and is given by:

$$P_b(e_1) = \int_0^{\infty} \frac{1}{2} e^{-\gamma_{s,r}} p_{\Gamma_{s,r}}(\gamma_{s,r}) d\gamma_{S,r} \tag{11}$$

$$P_b(e_1) = \frac{1}{2} \left(\frac{\beta_1}{1+\beta_1}\right)^2 \text{............... (12)}$$

Similarly the ABER for R to D link and is obtained as,

$$P_b(e_2) = \frac{1}{2} \left(\frac{\beta_2}{1+\beta_2}\right)^2 \text{............(13)}$$

The ABER for *decode –and forward relaying* can be written as:

$$P_b(e) = \frac{1}{2} \left(\frac{\beta_1}{1+\beta_1}\right)^2 + \frac{1}{2} \left(\frac{\beta_2}{1+\beta_2}\right)^2 -$$
$$2 \times \frac{1}{2} \left(\frac{\beta_1}{1+\beta_1}\right)^2 \times \frac{1}{2} \left(\frac{\beta_2}{1+\beta_2}\right)^2 \quad \text{........ (14)}$$

The ABER curves for decode-and-forward (DF) relaying over the proposed relay over Rayleigh fading channel with respect to the normalized SNR. The horizontal axis represents the instantaneous SNR in dB and the vertical axis represents the average bit error rate (ABER). Observations are noted for the cases of decode-and-forward relaying systems from the plots of Fig.4, where improvements are even less. This nature of the graph is highly expected because, with diversity, merely S →□ R link SNR is improved whereas overall performance is determined by both S →□ R and R→ D links The (*S/N*) ratio is improved for M=2 and N=2.
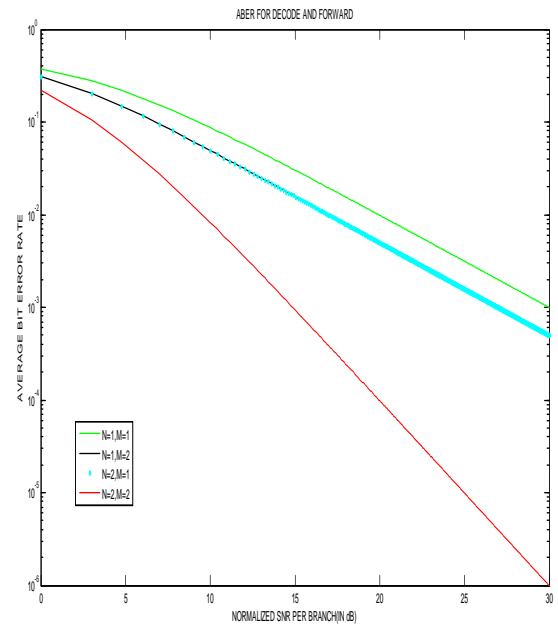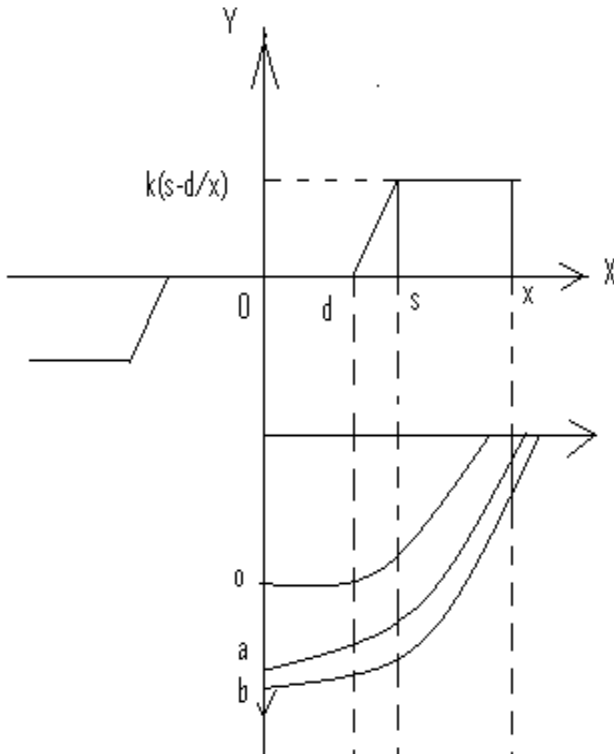

Fig 4 : Plot of ABER verses SNR (dB) for decode and forward relaying with transmitting antenna M=1,N=1;M=1,N=2;M=2,N=1;M=2,N=2.

Now the output of this MIMO network is passing through the *dead zone saturation* system and the corresponding diagram is shown below :

For symmetric case, with $\beta_1 = \beta_2 = \beta = \frac{1}{\gamma}$ may be reduced to

$$P_b(e) = \frac{1}{2} \left[2 \left\{ \frac{1}{(1+\bar{\gamma})} \right\}^2 - \left\{ \frac{1}{(1+\bar{\gamma})} \right\}^4 \right] \quad \text{........ (15)}$$

Output:

Here in between the region *'d' -> 'x'* , the slope is *k* , so the portion of the output of MIMO network over *ABER vs S/N* ratio shows linear characteristics, before the '*d*' i.e *0<x<d.* the output is dead zone and after s<x<∞ the output = *k(s-d/x)* ,

So the improvement of Signal to Noise Ratio is somehow hampering for some time period span, that's affects the system is unstable condition. Now if we connect a *Zero Order Hold* circuit the stability is improvement that we see via simulink design.
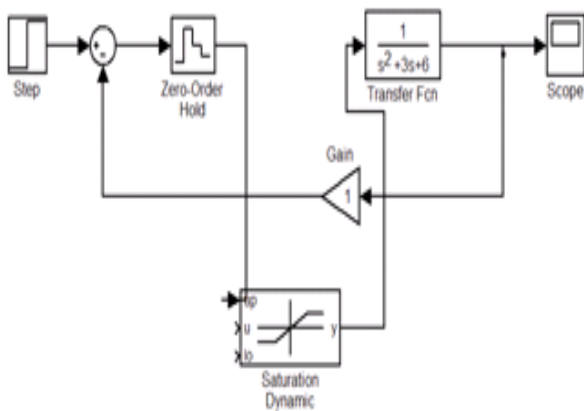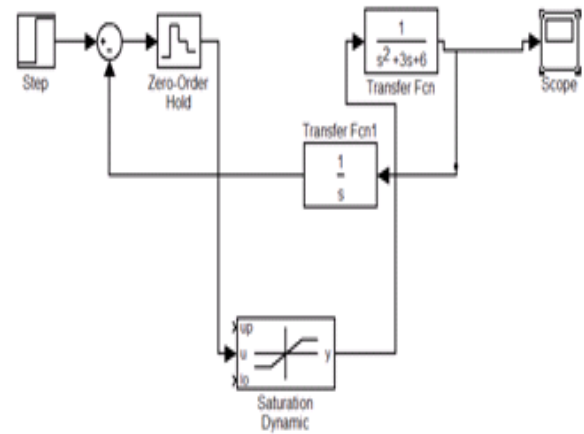
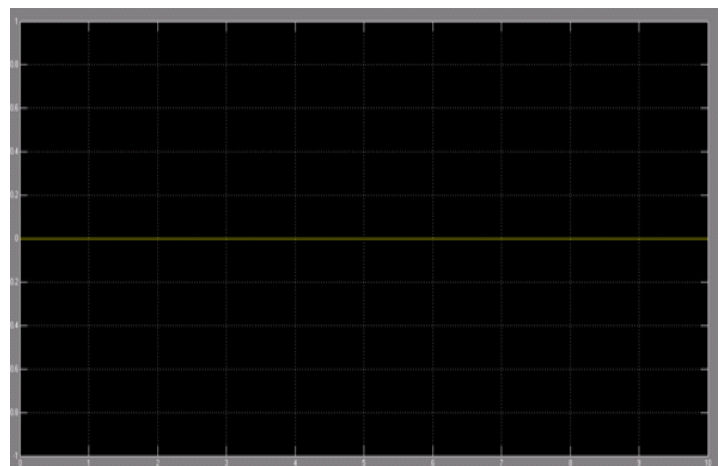**Fig 5(b):** ZOH connected in saturation system with step input and Step feedback processing function

Output:

Fig 5(a): ZOH connected in saturation system

Fig 5(c): ZOH connected in saturation system with pulse input and Step feedback processing function
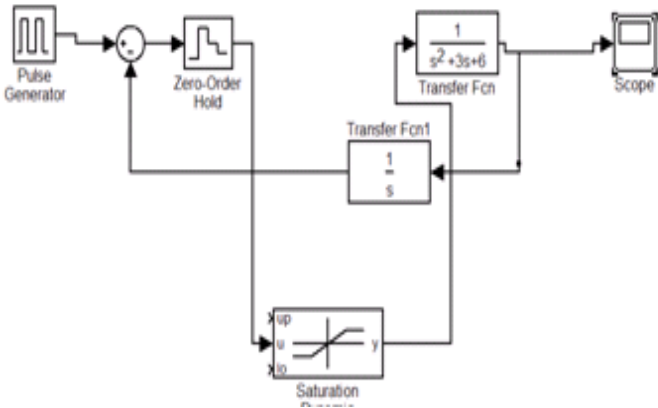
Output:

Output:

Now if the *summer* output connect up portion of *saturation* level then the stability is unchanged though the external trigger is connected.

Figure 5(e): ZOH connected in saturation system with sinusoidal input with external trigger input and Step feedback processing function : Summer output in up portion of saturation

Now if the system is disturbed by some external trigger the system's stability is hampering. For example consider the following diagram for a sinusoidal input with an external trigger input. Here the *summer* output connect below portion of *saturation* level
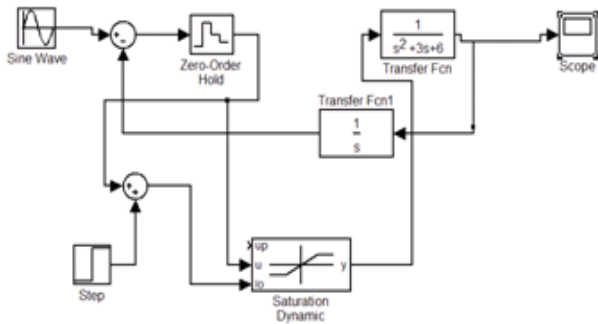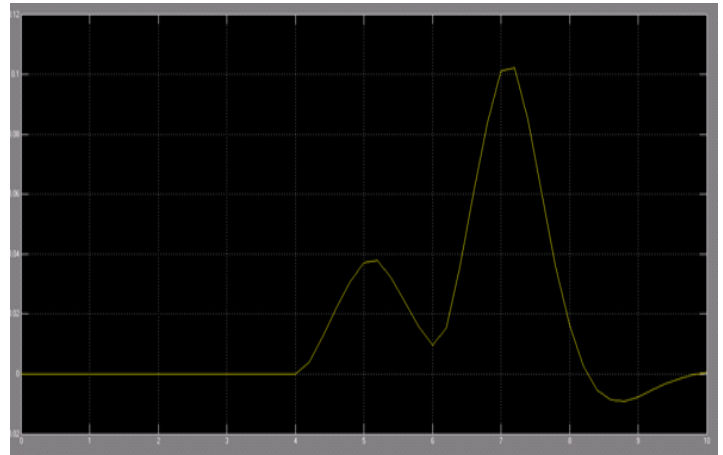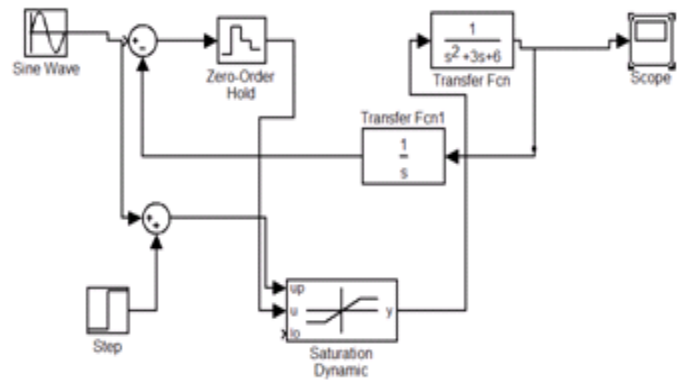

Fig 5(d): ZOH connected in saturation system with sinusoidal input with external trigger input and Step feedback processing function

Output:

5. CONCLUSIONS

From the above diagrams we see the effects of *Zero Order Hold* technique for smoothly running a system. Basically Nonlinearities affects a system beyond of boundary. For that the Region of Convergence of that system shows uncertainty condition. This paper has given a brief overview of some of the simpler strategies for nonlinear control. Some of these methods, e.g., feedback linearization can be seen to be very close to linear methods and, as such, are easy to use in practice.

ACKNOWLEDGMENTS

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BIOGRAPHIES

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