TRANSITION FOR REGULAR TO MACH REFLECTION: HYSTERESIS PHENOMENA

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Abstract

The purpose of this research is to perform numerical calculations on supersonic flows in a convergent-divergent nozzle and more particularly to the study of the transition regular reflection to Mach reflection and vice versa – hysteresis loop phenomena. The latter method has been extensively studied in recent years. In this study the numerical simulations are performed for transient supersonic flow, detection of the transition from reflection to another. This was done by changing the upstream Mach number in the initial conditions over time. The viscosity was taken into account and all of the Navier-Stokes equations were solved. The results clearly show the existence of a hysteresis loop in the transition transient shock waves.

Keywords: *Shock wave, Interference of shock, Regular reflection, Mach reflection, Polar of shock.*

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1. INTRODUCTION

The reflection loop of shock waves has been reported for the first time in 1878 by E. Mach. He then distinguished two types of reflection: a so-called regular reflection involving an incident shock, reflected shock and reflection known posthumously Mach reflection involving in addition to the incident and reflected shocks, a strong shock near normal to the direction flow.

The reflection of shockwave on a flat surface affected by the presence of the boundary layer on the wall. The shock wave can cause separation of the boundary layer and the actual configuration of the remote configuration is predicted by the theory of ideal fluid. Similarly, the interaction of perfectly symmetric shocks is less likely. However the actual flows (air inlet, external flow) are often the seat of interaction of shock intensities and different families and the scope of their study is considerable.

The experimental works in this area are few or nonexistent. As in the case of interaction of two symmetrical shocks, there are two configurations of Mach interaction (MR) and the regular interaction (RR). An analytical and experimental study was conducted to establish firstly criteria of transition between these two types of interaction and secondly identify new phenomena inherent in this type of flow. The interaction (MR) and (RR) and corresponding geometric configurations ratings are presented in Figure 1. Regular interaction consists of two incidents shocks (i1) and (i2) and two reflected shock (r1) and (r2). The boundary conditions for the configuration (RR) are [1]:

$$\theta_1 - \theta_3 = \theta_2,\ \theta_4 = \delta \qquad (1)$$

$\delta = 0$ when, $\theta_1 = \theta_2$, i.e. when the interaction is symmetric.
The interaction of Mach includes more incidents reflected shocks and shocks, a strong near-normal shock connecting triple points (T1) and (T2). Two slip lines (s1) and (s2) complementary to the bumper system. The boundary conditions for a Mach interaction are as follows [1].

$$\theta_1 - \theta_3 = \delta_1 \text{ and, } \theta_2 - \theta_4 = \delta_2 \qquad (2)$$

$\delta_1 = \delta_2$ when $\theta_1 = \theta_2$.

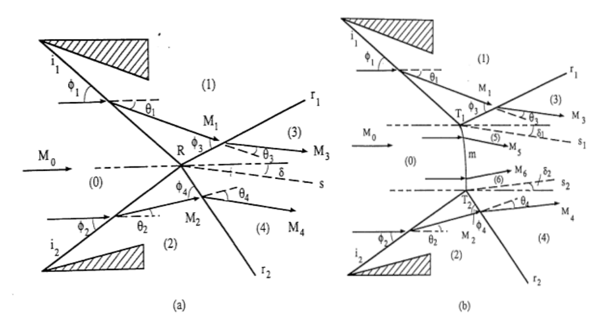


Fig -1: (a) Schematic of the regular interaction (b) Schematic of the Mach interaction [1].

The analytical study prepared by J. Von Neumann [5], highlighted two possible criteria for the transition reflection regular-Mach reflection. A test is in the absence of regular beyond the peel angle of the reflected shock reflection, the other states that the transition from one configuration to the other is seamless to the pressure angle Neumann between these two criteria are particularly distinct as the Mach number is high, there is a dual area where the two types of reflection is possible. One is led to ask the following question: What is the point of transition to a steady flow?. In the late 40 years and early 50 years authors such as Liepmann and Roshko (1957), Landau and Lifshitz (1957) and others had suggested the criterion of detachment as the correct test. Their conviction was based on experiments performed at moderately supersonic conditions (M <3). But for such flows, the dual zone is rather bit wide. A confusion of the two criteria into account experimental uncertainties is at higher Mach numbers (M> 5). Authors such as Henderson and Lozzi [6,7], and Kychakoff Hornung [8] revealed the criterion of von Neumann as the boundary crossing between the two configurations of reflection. In (1979) Hornung, Oertel and Sandemen [9] hypothesized the existence of a hysteresis phenomenon during the transition. According to this hypothesis the transition from regular reflection to Mach reflection should occur criterion detachment, while the reverse transition should occur criterion of von Neumann. However, past experiences and new experiences by Hornung and Robinson [10], have not confirmed this hypothesis and the standard von Neumann was chosen as the limit of transition between the two types of reflection.

2. MATHEMATICS MODELLING

It is recognized that the behavior of any flow testing the hypothesis of continuous media, whatever the nature of the fluid (compressible or not), and flow (laminar, turbulent) can be represented by the Euler equations or Navier Stokes which express the conservation of momentum, which are added the conservation equations of mass and total energy. The flow is described by the Navier-Stokes applied for supersonic flight. Equations are hyperbolic in nature and in an abbreviated form. [9]. the differential formulation of these equations is as follows, in Cartesian coordinates:

Continuity equation (or balance equation of mass)

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{v}) = 0 \qquad (3)$$

Balance equation of momentum

$$\frac{\partial (\rho \vec{v})}{\partial t} + div(\rho \vec{v} \otimes \vec{v}) = -\overrightarrow{grad}(p) + div(\overline{\overline{\tau}}) + \rho \vec{f} \qquad (4)$$

Equation of energy balance

$$\frac{\partial (\rho e)}{\partial t} + div[(\rho e + p)\vec{v} = div(\overline{\overline{\tau}}.\vec{v}) + \rho \vec{f}.\vec{v} - div(\vec{q}) + r \qquad (5)$$

With $\vec{q} = -\lambda \vec{\nabla} T$ lost heat flow by thermal conduction and the stress tensor

$$\overline{\overline{\tau}} = \mu[(\vec{\nabla} \otimes \vec{v}) + (\vec{\nabla} \otimes \vec{v})^t] + \eta(\vec{\nabla}.\vec{v})\overline{\overline{I}}.$$

The total energy can be decomposed into *u* internal energy and kinetic energy by $e = u + 1/2\vec{v}.\vec{v} \;=\; u + 1/2v^2$. The symbol μ, ρ, p, υ, h, $\vec{f}$, r, T, respectively, designate the molecular viscosity, specific gravity, the static pressure, velocity vector, enthalpy, the resultant of the mass forces in the fluid loss volumetric heat due to radiation, and static temperature.

3. NUMERICAL METHOD

3.1 Geometry and Flow Parameters

The creation of the geometry and the mesh are due to software Gambit 2.3.16. Several methods allow the creation of this geometry, or one based on predefined geometries, or simply enters the coordinates of the points (x, y) in 2D, create boundaries and finally create the surface. However, for our case, two main choices mesh arose. In this case, a mesh of quadrilaterals or based cells or triangular cells based. The use of a triangular mesh induce a surplus in the number of cells compared with cells quadrilaterals, hence the need for more resources and computing time. However, this is relatively simple geometry in which the flow follows substantially the form of the geometry. So using a quadrilateral mesh cells, we have an alignment of the flow with our mesh, then it will never be the case with triangular cells. Particular attention should be paid to the subsequent verification of mesh refinement near the walls to ensure that all phenomena are captured. A right symmetrical convergent nozzle (angle) diverged, was used to generate the shock waves, Figure (2).

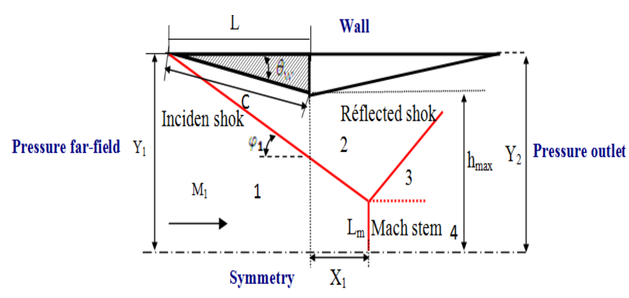


Fig -2: Geometry of the 2D nozzle

3.2 Grid Refinement and Time Independency

To see the influence of the mesh on the numerical solution, we performed calculations with four meshes of different sizes. Figure (3). Shows the distribution of pressure along the axis of symmetry for different meshes. We examined the influence of different grids on the regular reflection, including the impact of the incident shock on the symmetry axis of the nozzle. The reflection of the particular Mach position Mach disk. Than 90,511 cells almost the same distribution of the finest mesh. He was chosen for reasons of economy of computing time (CPU time). The same for the second step is the independence of the computation time [4] investigation. To this end, we examined three different time scales (0.01, 0.02, and 0.005 sec) for the mesh that has been selected. Again the pressure distributions are shown in the plane of symmetry for the three different time positions, Figure (4). The figure shows that the choice of not 0.005 sec time is acceptable.

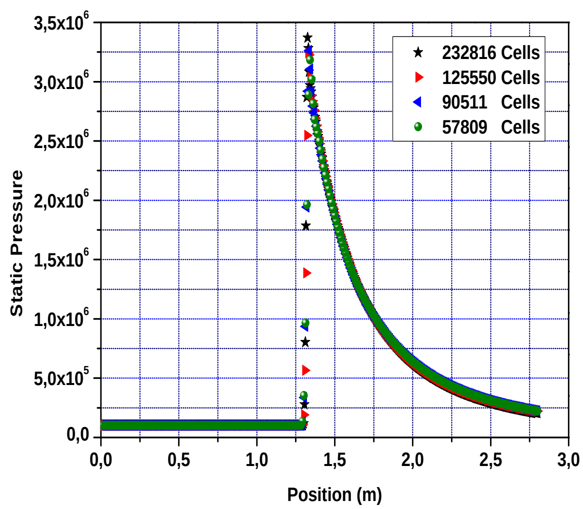
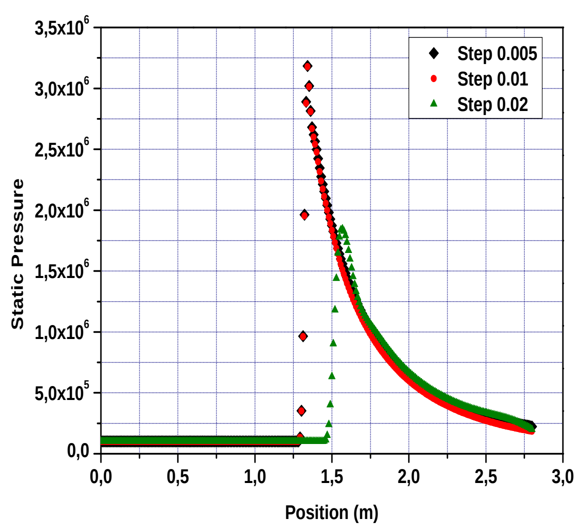
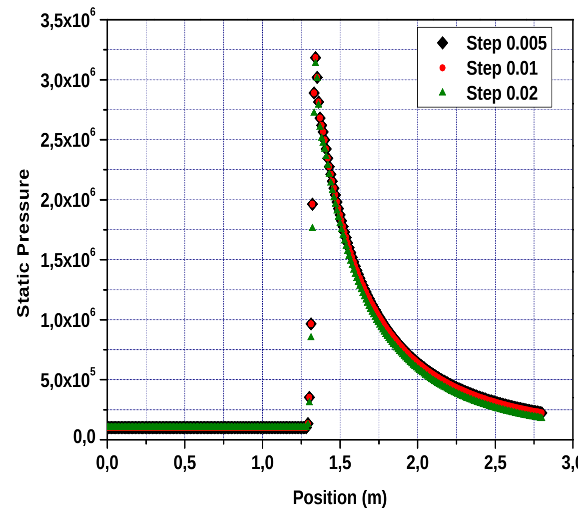


Fig -3: Pressure distribution along the axis of symmetry for the different meshes





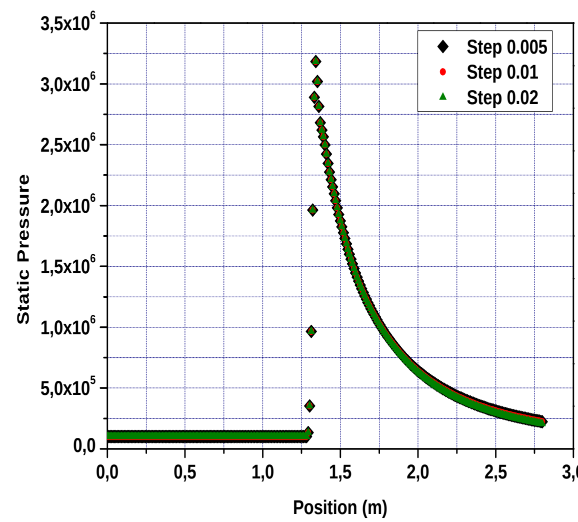


Fig-4: Pressure distribution along the axis of symmetry for three different time: t = 0.6 s, 0.9 s, 1.3 s and from top to bottom, respectively

4. MODELLING AND NUMERICAL SOLUTION

The above differential equations are solved by the finite volume technique, it's assumed that the flow is two-dimensional compressible laminar. Two commercial codes were used for the numerical solution of the supersonic flow inside the nozzle, FLUENT 2D version 6.3.26 (solver) and GAMBIT 2.3.16. (Mesh generator), both supplied by Fluent.

The computer code used, solved the Navier-Stokes equations, in Upwind scheme uses the centred second order for the central convective terms, and the terms of distribution, based on the flow of a Roe-FDS CFL = 0.5. The temporal discretization is second order. The formulation of this plan was fully implicit, and the system of equations for each time step was solved by an iterative method developed classical Gauss-Seidel iterative. For the sake of accelerating the convergence, a step of pseudo-relaxation time was used in each time step with a suitable expansion factor.

The fluid used is air, considered as ideal gas. Admission requirements (initial condition of the flow) are shown in Table.1. The density is calculated using the ideal gas law (isentropic flow). Sutherland's law was chosen to calculate the molecular viscosity μ describes the variations of the viscosity with respect to temperature, because in a supersonic flow, there are large temperature gradients.

$$\mu(T) = \mu_0 (\frac{T}{T_0})^{1.5} \frac{T_0 + 110.56}{T + 110.56}$$

Table -1: Physical Parameters of the inflow

Definition	Value and unit
Number of local current imposes Cfl	0.5
Mach number in the upstream infinity (ideal gas)	Vari
Value of the energy upstream infinity	1683 (Pa)
Value of temperature upstream infinity	76.51 (K)

5. RESULTS AND DISCUSSION

To reproduce the hysteresis sequences, a dihedral angle of 25° has been established, and the incident angle of impact was decreased from a regular configuration to obtain a Mach configuration, by varying the upstream Mach number in the initial conditions over time. This operation was repeated in the opposite direction (decrease or increase), that is to say the Mach configuration of up to regular configuration. The sequences were reproduced hysteresis for six values of the Mach number equal to 3.4, 3.6, 3.8, 4, 4.2 and 4.4. Transition points are shown on Figure 5. Arrows represent the hysteresis loop and the points correspond reflections and transitions MR / RR. The figure shows the theoretical transition criteria, the various realms of existence interactions RR and MR and the dual area. This figure shows not only the hysteresis phenomenon predicted by Hornung but more it shows a very good agreement between the theoretical levels of transition and numerical values.

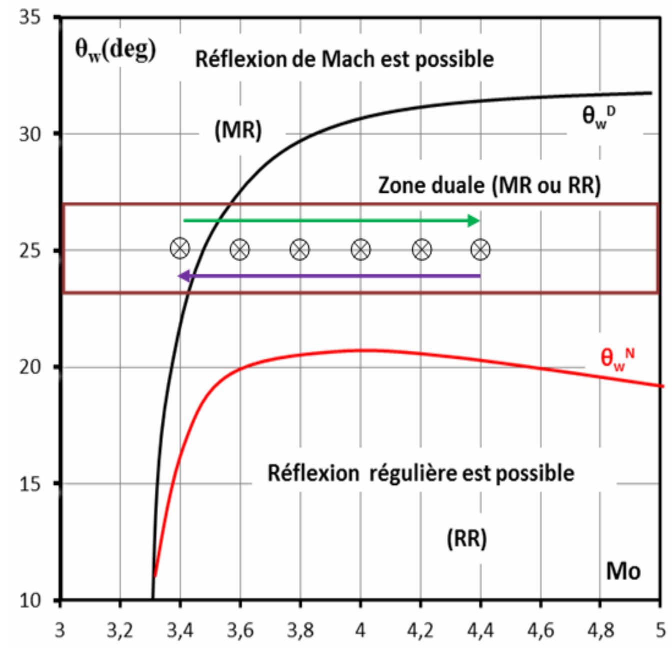


Fig- 5: Criterion of transition in the (θ, Mo) plan

The sequence of Figure 6. Presents iso density curves. Initially, a regular reflection (RR) was obtained. This configuration has been obtained starting from a uniform field equal to Mach 4.4. Then, the Mach number is decreased and every time a stable stationary solution has been reached, based on the initial field converged to the previous number of Mach. The calculations were performed until a Mach reflection (MR), and were then repeated in reverse. The transition from regular reflection to Mach reflection occurs for a Mach number equal to 3.4. for this configuration, only the criterion of detachment is cut.

Figures clearly show the sudden shock of an almost normal appearance. A further reduction of the Mach number would only increase gradually up to this Mach disk. Conversely, when the Mach number increases, the configuration (MR) remains in the dual zone beyond the criterion of detachment. The height of the Mach disk gradually decreases but does not vanish as the criterion of Von Neumann is never reached.

In comparing the results obtained by the fluent code with other A. Durand et al [1] [2] D. J. Azevedo et al. [10]. For the same Mach number, we note that we obtain, according to the direction of travel, a regular reflection or a Mach reflection. A comparison of the heights and positions, Mach disks observed experimentally, numerically and analytically, was compared. Figures (7) and (8). Show the evolution of these dimensionless quantities, depending on the upstream Mach number. The agreement between the numerical experimental, analytical and is relatively good.

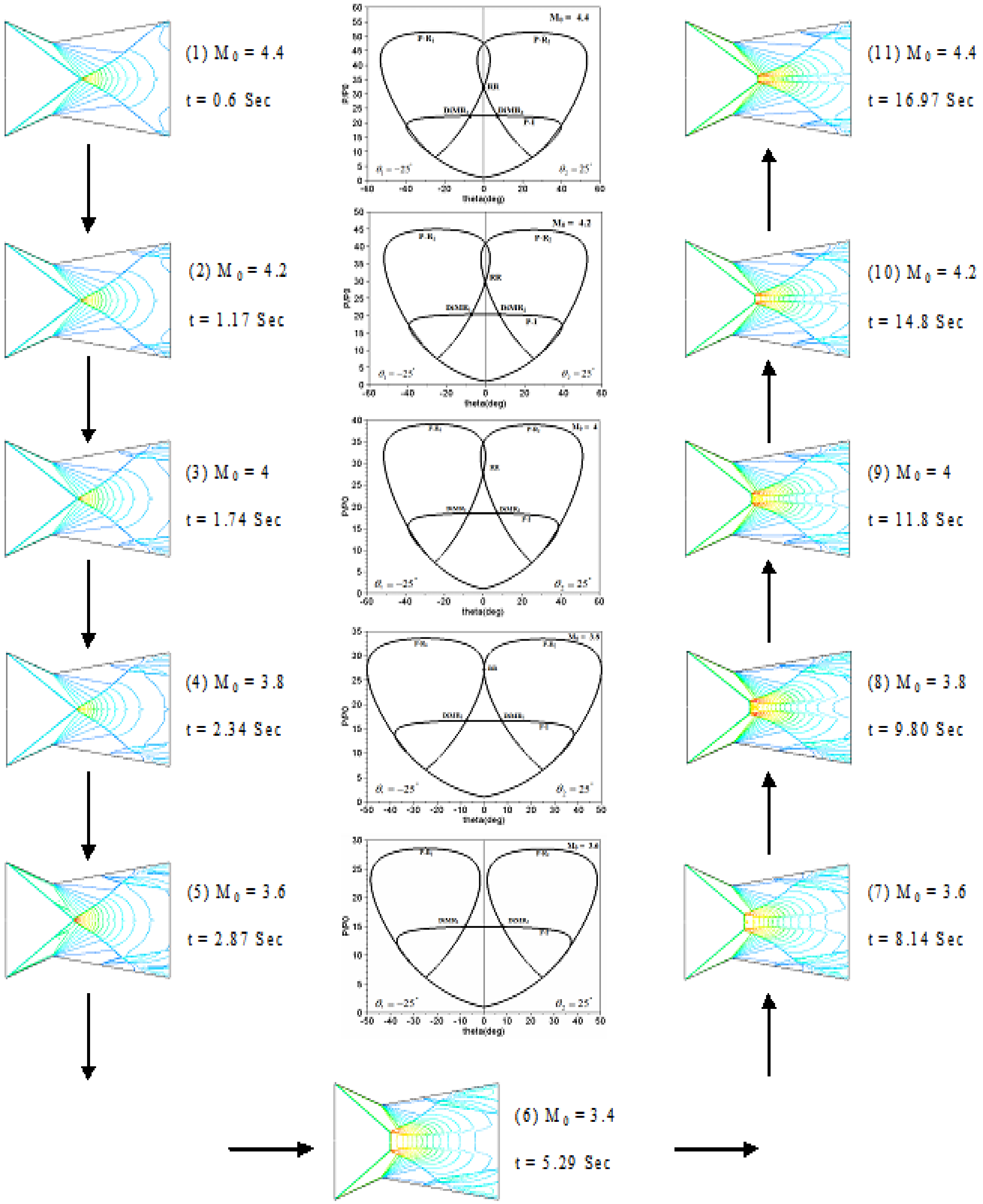


Fig- 6: Hysteresis cycle induced by the variation of the Mach number $\theta = 25^0$

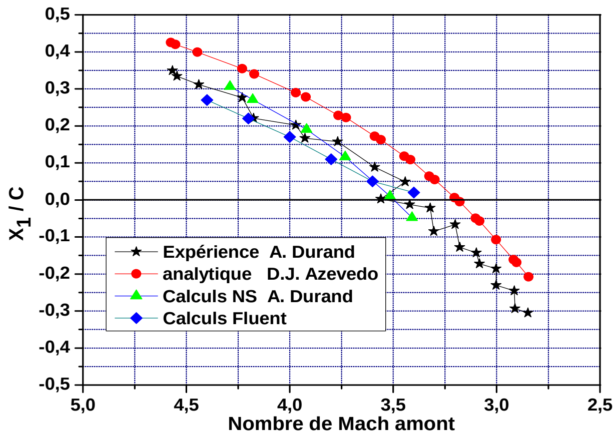


Fig-7: Height comparison of Mach disk experimental, analytical and numerical obtained for an angle of deflection θ = 25°

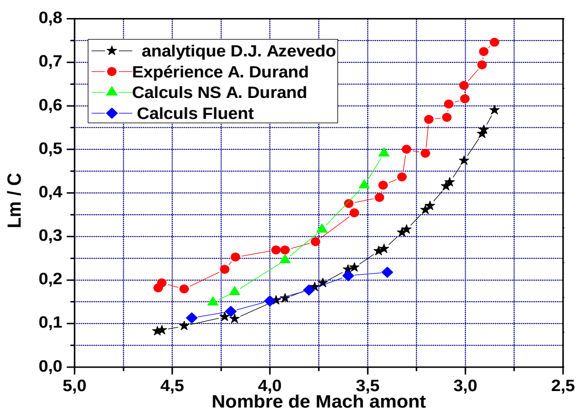


Fig-8: Comparison of the positions of Mach disks obtained experimentally, analytically and numerically for an angle θ = 25°

6. CONCLUSIONS

The transition regular reflection ⇔ Mach reflection was simulated numerically by solving Navier-Stokes, using the code FLUENT calculation for a two-dimensional compressible laminar flow. Through this study we have shown the interest and importance of interaction phenomena of shocks in supersonic nozzles. Thus, numerical simulations in stationary in a 2D nozzle could highlight certain phenomena:

- Hysteresis of the RR-MR transition, due to the memory effect of the flow.
- In accordance with the experiment, the MR solution is more stable than the RR solution.
- The angles of transition from one type to another reflection are different from those found experimentally by A. Durand, and closer to those found analytically by Azevedo.

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