STATIC ANALYSIS OF C-S SHORT CYLINDRICAL SHELL UNDER INTERNAL LIQUID PRESSURE USING POLYNOMIAL SERIES SHAPE FUNCTION

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Abstract

*The static analysis of C-S short cylindrical shell under internal liquid pressure is presented. Pasternak's equation was adopted as the governing differential equation for cylindrical shell. By satisfying the boundary conditions of the C-S short cylindrical shell in the general polynomial series shape function, a particular shape function for the shell was obtained. This shape function was substituted into the total potential energy functional of the Ritz method, and by minimizing the functional, the unknown coefficient of the particular polynomial shape function was obtained. Bending moments, shear forces and deflections of the shell were determined, and used in plotting graphs for cases with a range of aspect ratios, 1 ≤ L/r ≤ 4. For case 1, the maximum deflection was 8.65*10-4metres, maximum rotation was 3.06*10- 3radians, maximum bending moment was -886.45KNm and maximum shear force was -5316.869KN. For case 2, the maximum deflection was 2.18*10-4metres, maximum rotation was 7.74*10-4radians, maximum bending moment was -223.813KNm and maximum shear force was -1342.878KN. For case 3, the maximum deflection was 9.71*10-5metres, maximum rotation was 3.44*10-4radians, maximum bending moment was -99.463KNm and maximum shear force was -596.779KN. For case 4,the maximum deflection was 5.48*10-5metres, maximum rotation was 1.94*10- 4radians, maximum bending moment was -56.097KNm and maximum shear force was -336.584KN. It was observed that as the aspect ratio increases from 1 to 4, the deflections, bending moments and shear forces decreases, and the shell tends to behave like long cylindrical shell.*

Keywords: *Static analysis, Short Cylindrical Shell, internal liquid pressure, Polynomial series shape function, Boundary condition, Ritz method.*

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1. INTRODUCTION

The feasibility of many modern constructions necessitates lightweight, thin-walled members. The objective of structural engineering has always been to lower as much as possible the cost and thus the quantity of construction material without compromising the integrity of the structure. Thin-walled structures, which include both thin plates and thin shells, satisfy the afore-mentioned objective.

Thin shells as structural elements occupy a leadership position in engineering and especially in civil engineering, since they can be used in the construction of large liquid storage structures, large span roofs, domes, folded plates and so on.

Large cylindrical shell tanks are widely used in the construction of strategic water or oil reservoir all over the world. In order to lower the cost, and to make the management easier, the volume of such tanks tends to be larger, thus short cylindrical shell. Osadebe and Adamou [1] studied Static Analysis of Cylindrical shell under hydrostatic and Ring forces using initial value method.

The moment theory of shells developed by Darkov [2] and Love [4] using the analogy of plates due to Kirchoff [3], in spite of its attractive accuracy, is seldom applied by an average design engineer due to the involved rigorous mathematics. Subsequently, Vlasov [9], and Pasternak [5], by ignoring the effects of longitudinal bending moment, shear forces and torques arrived at the semi-moment theory, which is found to give acceptable results. Timoshenko and Woinowsky-Krieger [6] experimentally verified on cylindrical shell whose ratio of length to radius ranges from 1 to 4.

Pasternak [5] showed that when the load on a cylindrical shell is axisymmetric, the stresses and strains are functions of only one variable along the axis of the cylinder.

This work is concerned with the analysis of C-S short cylindrical shell subjected to internal liquid pressure.

The objective of this study is to establish an analysis of C-S short cylindrical shell that can be utilized in the design of cylindrical shell reservoir tank and to show the distribution of stress and strain under working load and establish the aspect ratio that is most suited for the construction of such economical tank.

The magnitudes and locations of maximum deformations and stresses along the height of the cylindrical shell tanks are indicated and recorded for the purpose of design.

2. GOVERNING DIFFERENTIAL EQUATION OF A CYLINDRICAL SHELL.

Consider a C-S short cylindrical shell with its dimension L, t and r as shown in figure 1 which is subjected to internal liquid pressure.

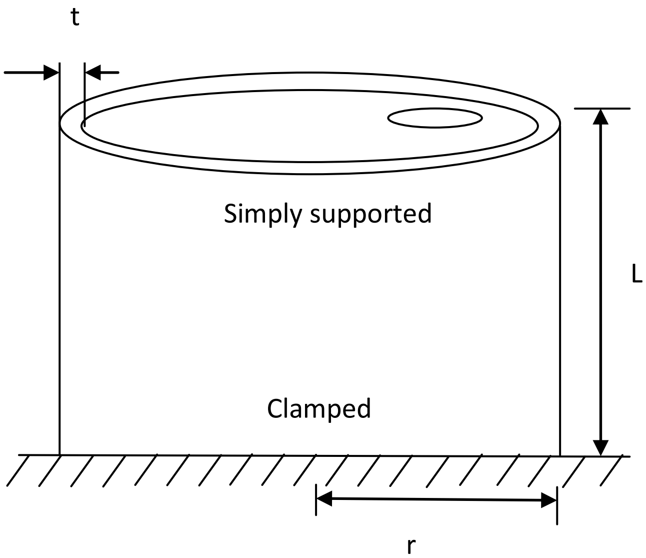

Fig 1: A typical C-S (clamped at one end and simply supported at the other end) short cylindrical shell tank showing the dimensions.

The condition for shortness for an unstiffened cylindrical shell is L/r < 5, where L/r is the aspect ratio.

The governing equation of a cylindrical shell according to the semi-moment theory as used by Timoshenko et al [6]; Ugural [7]; Ventsel and Krauthammer [8] is as stated in equation (1).

$$\frac{d^4w}{dx^4} + 4\beta^4 w = \frac{\gamma}{D}x \qquad (1)$$

Where

$$\beta^4 = \frac{3(1-v^2)}{r^2t^2}$$

γ = Unite weight of the liquid.

Equation (1) is due to Pasternak [5] and is only applicable to cylindrical shell subject to axisymmetric loading.

3. FORMULATION OF POLYNOMIAL SERIES SOLUTION OF SHORT CYLINDRICAL SHELL.

Ventsel and Krauthammer [8] gave cylindrical bending of thin plate equation as given in equation (2) below.

$$\frac{d^4w}{dx^4} = \frac{\gamma x}{D} \qquad (2)$$

Where, $\frac{\gamma}{D} = \Phi$ and $R = \frac{x}{L}$ (3)

Substituting equation (3) into equation (2) gave:

$$\frac{d^4w}{dR^4} = \Phi R L^5 \qquad (4)$$

Integrating equation (4) four times with respect to the dimensionless variable R, and rearranging gave:

$$w = C_0 + C_1R + \frac{C_2R^2}{2} + \frac{C_3R^3}{6} + \frac{\Phi R^5}{120} \qquad (5)$$

Let $C_0 = a_0, C_1 = a_1, \frac{C_2}{2} = a_2, \frac{C_3}{6} = a_3, \frac{\Phi}{120} = a_5$ (6)

Substituting equation (6) into (5) gave:

$$w = a_0 + a_1R + a_2R^2 + a_3R^3 + a_5R^5 \qquad (7)$$

Differentiating equation (7) with respect to R gave:

$$\frac{d^4w}{dR^4} = 120a_5R \qquad (8)$$

Equation (1) can be expressed in terms of R as:

$$\frac{d^4w}{dR^4} + \lambda w L^4 = \Phi R L^5 \qquad (9)$$

Where: $\lambda = 4\beta^4; w =$ Deflection; L = Height of tank.

Substituting equations (7) and (8) into equation (9) gives:

$$120a_5R + a_0L^4\lambda + a_1L^4\lambda R + a_2L^4\lambda R^2 + a_3L^4\lambda R^3 + a_5L^4\lambda R^5 = \Phi L^5 R \qquad (10)$$

Simplifying further we have:

$$w = a_0\left[\frac{120}{(120 + L^4\lambda R^4)}\right] + a_1R\left[\frac{120}{(120 + L^4\lambda R^4)}\right]$$

$$+a_2R^2\left[\frac{120}{(120+L^4\lambda R^4)}\right]$$

$$+a_3R^3\left[\frac{120}{(120+L^4\lambda R^4)}\right]+\left[\frac{\Phi R^5}{(120+L^4\lambda R^4)}\right] \quad (11)$$

$$Where\ b=\frac{1}{(120+L^4\lambda R^4)}\ and\ c=\frac{120}{(120+L^4\lambda R^4)} \quad (12)$$

Substituting equation (12) into equation (11) gave:

$$w = a_0c + a_1c\,R + a_2cR^2 + a_3cR^3 + \Phi bR^5 \quad (13)$$

$$\text{Say } w = k_0 + k_1R + k_2R^2 + k_3R^3 + kR^5 \quad (14)$$

Where $k_0 = a_0c$; $k_1 = a_1c$; $k_2 = a_2c$;

$$k_3 = a_3c;\ k_5 = \Phi b \quad (15)$$

Equation (14) is the general polynomial shape function for the short cylindrical shell.

4. THE RITZ METHOD.

According to Vintsel and Krauthammer [8], Timoshenko and Woinowsky-Krieger [6], the Ritz equation derived from the principle of theory of elasticity is given as:

$$\Pi = \frac{1}{2}\int_0^L \left(\frac{d^2w}{dR^2}\right)^2 dR + \frac{\lambda}{2}\int_0^L w^2\,dR - \Phi\int_0^L w\,RdR \quad (16)$$

$$\text{Let } w = Af_i \quad (17)$$

Substituting equation (17) into equation (16) gave;

$$\Pi = \frac{A^2}{2}\int_0^L \left(\frac{d^2f_i}{dR^2}\right)^2 dR + \frac{\lambda A^2}{2}\int_0^L (f_i)^2\,dR$$

$$-\Phi A\int_0^L f_i\,RdR \quad (18)$$

5. SHAPE FUNCTION FOR C-S SHORT CYLINDRICAL SHELL

The C-S short cylindrical shell has the following boundary conditions.

$$w_0 = w(R=0) = 0;$$

$$M_0 = \frac{d^2w}{dR^2}(R=0) = 0 \quad (19)$$

$$w_{(1)} = w(R=1) = 0;$$

$$\theta_{(1)} = \frac{dw}{dR}(R=1) = 0 \quad (20)$$

Appling these boundary conditions in equation (14) gave:

$$w = A(R - 2R^3 + R^5) = Af_i \quad (21)$$

That is, fi= $(R - 2R^3 + R^5)$ (22)

Using equation (22) the following integrations were obtained:

$$\int_0^1 \left(\frac{d^2f_i}{dR^2}\right)^2 dR = 16\left(\frac{9}{3} - \frac{30}{5} + \frac{25}{7}\right) = 9.142857 \quad (23)$$

$$\int_0^1 (f_i)^2 dR = \left(\frac{1}{3} - \frac{4}{5} + \frac{6}{7} - \frac{4}{9} + \frac{1}{11}\right) = 0.03694 \quad (24)$$

$$\int_0^1 (f_iR)dR = \left(\frac{1}{3} - \frac{2}{5} + \frac{1}{7}\right) = 0.076191 \quad (25)$$

Substituting equations (23), (24) and (25) into equation (18) gave:

$$\Pi = \frac{9.142857A^2}{2} + \frac{\lambda A^2}{2}(0.03694) - \Phi A(0.076191) \quad (26)$$

Minimizing equation (26) gave:

$$\frac{\partial\Pi}{\partial A} = 9.142857A + 0.03694A\lambda - 0.076191\,\Phi = 0 \quad (27)$$

Making A the subject of equation (27) gave:

$$A = \frac{0.076191\,\Phi}{9.142857 + 0.03694\lambda} \quad (28)$$

Substituting Φ and λ in equation for (28), and simplifying further, gave:

$$A = \frac{2.06249\gamma L^5 r^2}{247.4989 r^2 D + EtL^4} \quad (29)$$

Substituting equation (29) into equation (21) gave:

$$w = \frac{2.06249\gamma L^5 r^2}{247.4989 r^2 D + EtL^4}(R - 2R^3 + R^5) \quad (30)$$

Differentiating equation (30) with respect to R gave:

$$\theta = \frac{dw}{dR} = \frac{2.06249\gamma L^5 r^2}{247.4989 r^2 D + EtL^4}(1 - 6R^2 + 5R^4) \quad (31)$$

$$M = -D\left(\frac{d^2w}{dR^2}\right) = -D\left[\frac{2.06249\gamma L^5 r^2}{247.4989 r^2 D + EtL^4}\right](20R^3 - 12R) \quad (32)$$

$$Q = -D\left(\frac{d^3w}{dR^3}\right) = -D\left[\frac{2.06249\gamma L^5 r^2}{247.4989 r^2 D + EtL^4}\right](60R^2 - 12) \quad (33)$$

6. NUMERICAL STUDIES

For the purpose of this work, the deformations and stresses at various points of short cylindrical shells were determined for values of aspect ratios ranging from 1 to 4. The equations of the deformations and stresses of short cylindrical shells of various boundary conditions are presented. The numerical values of the following parameters E, D, L, t, r and γ are substituted accordingly into the formulated solutions.

A short cylindrical shell water reservoir made of Concrete with real life dimensions is adopted for numerical purposes:

For the cases considered, the parameters used are as shown in table 1.

Table 1: Parameters used in the analysis

Case	Aspect ratio(L/r)	Radius (m)	Thickness (m)	Height (m)	Unit weight of Liquid (KN/m3)	Poisson's ratio, υ	Young Modulus E(KN/m2)
1	1	10.00	0.25	10	9.81	0.25	$26*10^6$
2	2	5.00	0.20	10	9.81	0.25	$26*10^6$
3	3	3.34	0.15	10	9.81	0.25	$26*10^6$
4	4	2.50	0.10	10	9.81	0.25	$26*10^6$

7. RESULTS AND DISCUSSION

For the cases considered, the graphs of deflections, rotations, bending moments and shear forces were plotted against the height of the shell as shown in figures (2) to (5) The maximum values of deflection, rotation, bending moment and shear force for each case considered are shown in table 2.

Table 2: The maximum values of deflections, rotations, bending moments and shear forces

Cases	Maximum Deflection(m)	Maximum Rotation(radians)	Maximum Bending moment(KNm)	Maximum Shear force(KN)
1	$8.65 * 10^{-4}$	$3.06*10^{-3}$	-886.145	-5316.869
2	$2.18 * 10^{-4}$	$7.74* 10^{-4}$	-223.813	-1342.878
3	$9.71 * 10^{-5}$	$3.44* 10^{-4}$	-99.463	-596.779
4	$5.48 * 10^{-5}$	$1.94 * 10^{-4}$	-56.097	-336.584

As shown in figure 2, the deflection has a concave-like shape with the peak values at 2/3 of the height of the short cylindrical shell measured from the base for all cases considered.

The slope (rotation) has the maximum values at the top of the shell as shown in figure 3.

From figure 4, it was observed that the bending moment has the maximum values at the base of the C-S short shell.

The values of the shear forces vary along the height of the shell with the maximum values at the base as shown in figure 5.

It was observed that as the aspect ratio increases from 1 to 4, the deflections, rotations, bending moments and shears forces decreases and tends to behave like long cylindrical shell as shown in table 2.

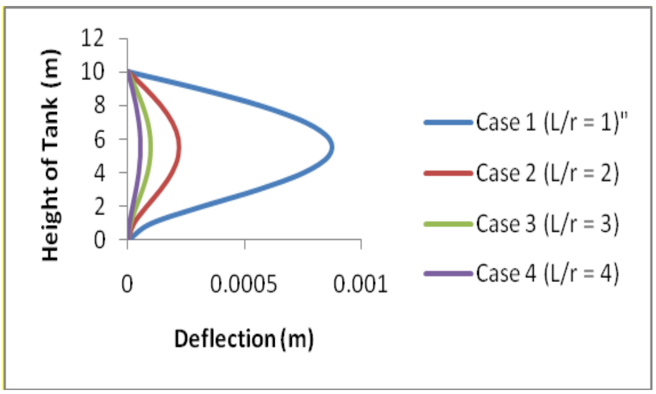

Fig 2: Deflection curves for a C-S short cylindrical shell with various cases of aspect ratios

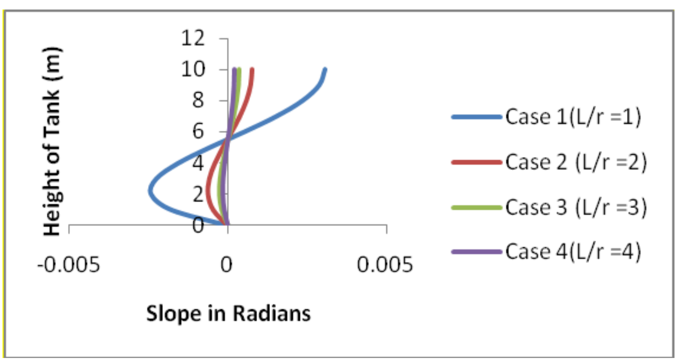

Fig 3: Rotation curves for a C-S short cylindrical shell with various cases of aspect ratios

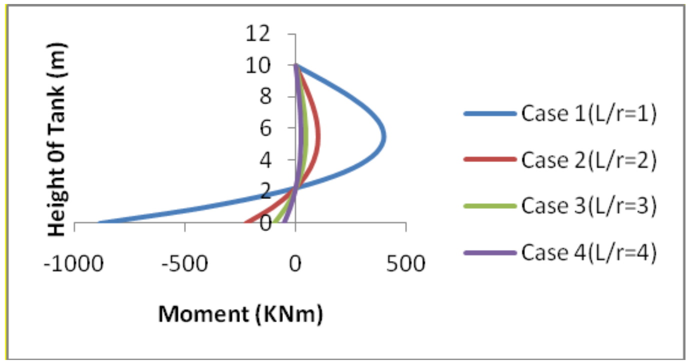

Fig 4: Bending moment diagrams for a C-S short cylindrical shell with various cases of aspect ratios

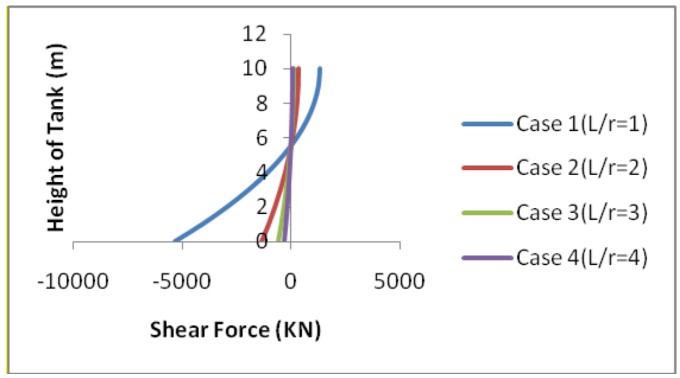

Fig 5: Shear force diagrams for a C-S short cylindrical shell with various cases of aspect ratios

8. CONCLUSIONS

The approach of using the polynomial series in the Ritz method is more convenient for analysis of short cylindrical shell. The knowledge of the point of maximum deflections, bending moment and shear forces along the height of the shell help for adequate reinforcement to be provided at the appropriate point and in the case of stiffening the shell with rings, guide in the position of the rings for optimal design.

It is therefore recommended that this approach could be easily applied in solving short cylindrical shell problems.

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DEFINITION OF NOTATIONS

x is the longitudinal axis of the shell ; w is the deflection function (shape function); r is radius of the shell

L is height of the shell tank; L/r is the aspect ratio; R = x/L is dimensionless parameter;

v is Poisson's ratio;

$\frac{dw}{dR}$ is first derivative of deflection with respect to R
.
M is bending moment.

Π is the Ritz total potential energy functional; t is the thickness of the shell

Q is shear force

θ is slope

$\frac{d^2w}{dR^2}$ is second derivative of deflection with respect to R

γ is unit weight of liquid

$\frac{d^3w}{dR^3}$ is third derivative of deflection with respect to R

$\frac{d^4w}{dx^4}$ is fouth derivative of deflection with respect to x

$\frac{d^4w}{dR^4}$ is fouth derivative of deflection with respect to R

$D = \frac{Et^3}{12(1-v^2)}$ is the flexural rigidity;

$$\beta^4 = \frac{3(1-v^2)}{r^2t^2}$$