RISK ASSESSMENT OF A HYDROELECTRIC DAM WITH PARALLEL REDUNDANT TURBINE

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ABSTRACT

This paper deals with the risk assessment of a hydroelectric dam. Hydroelectric dam produces electric power with the help of water collected in a pond. Here, in this paper, the author has been taken one parallel redundant turbine to improve system's overall performance. On failure of any one turbine, the whole system works in reduced efficiency state. The whole system can fail due to failure of any of its subsystems. All failures follow exponential time distribution whereas all repairs follow general time distribution. In order to risk assessment of the system, we have obtained reliability function, M.T.T.F. and availability function for considered system. A particular case, when all repairs follow exponential time distribution, and long-run behavior of system have also been computed to improve practical utility of the model. Graphical illustration followed by a numerical computation has also been appended at the end to highlight important results of present study.

Key Words: *Reliability, Mean time to failure, exponential time distribution, Inclusion of supplementary variables etc.*

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1. INTRODUCTION

The whole system consists of five major parts and these are named here as:

(i) Subsystem A : Reservoir, intake gate
(ii) Subsystem B : Penstock
(iii) Subsystem C : Turbine
(iv) Subsystem D : Generator
(v) Subsystem E : Power house.

The system has been shown in fig -1(a). The subsystem A takes the water and sends it to subsystem B (Penstock). By the movement of this water, we rotate the turbine and by rotation of turbine, generator produces electricity. The so produced electric power can be stored at power house and also can further distribute to various territories.

Since, the system under consideration in Non-Markovian, the author has used supplementary variables to formulate mathematical model of the system. This model has been solved further by taking help of Laplace transform.

2. ASSUMPTIONS

This paper is based on following assumptions:

(1) Initially the whole system is good and operable.
(2) After failure, repair facilities can be provided immediately.
(3)All failures follow exponential time distribution and are S-independent.
(4) All repairs follow general time distribution and are perfect.
(5) Nothing can fail from a failed state.
(6) On failure of any one turbine (subsystem C), the whole system works in reduced efficiency

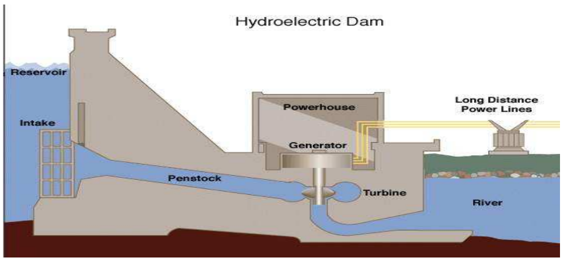


Fig-1(a): System Configuration of Hydro-electric Dam

3. LIST OF NOTATIONS USED:

x	:	Failure rates of subsystems *A, B, C, D* and *E* respectively.
$\psi_1(x)\Delta$ etc.	:	First order probability that subsystem A can be repaired in the time interval $(x, x+\Delta)$, conditioned that it was not repaired upto time *x*.
$\psi_{3,3}(u)\Delta$	:	First order probability that both the units of subsystem C can be repaired in the time interval $(u, u+\Delta)$, conditioned that they were not repaired upto time u.
$P_0(t)$	:	Pr {at time t, whole system is operable}.

$P_1(x,t)\Delta$ etc : Pr {at time t, system is failed due to failure of subsystem A}. Elapsed repair time lies within $(x, x+\Delta)$.

$P_3(z,t)\Delta$ Pr {at time t, system is degraded due to failure of any one component of subsystem C}. Elapsed repair time lies within $(z, z+\Delta)$.

$P_{3,1}(x,t)\Delta$ *etc* Pr{at time t, system is failed due to failure of subsystem A while one component of subsystem C has already failed}. Elapsed repair time of subsystem A lies within $(x, x+\Delta)$.

$\overline{P}(s)$: Laplace transform (L.T.) of function P(t).

M.T.T.F. : Mean time to failure.

$S_i(j)$: $\psi_i(j)\exp.\left\{-\int \psi_i(j)dj\right\}$, $\forall\, i$ and j.

$D_i(j)$: $\left[1-\overline{S}_i(j)\right]/\,j$, $\forall\, i$ and j.

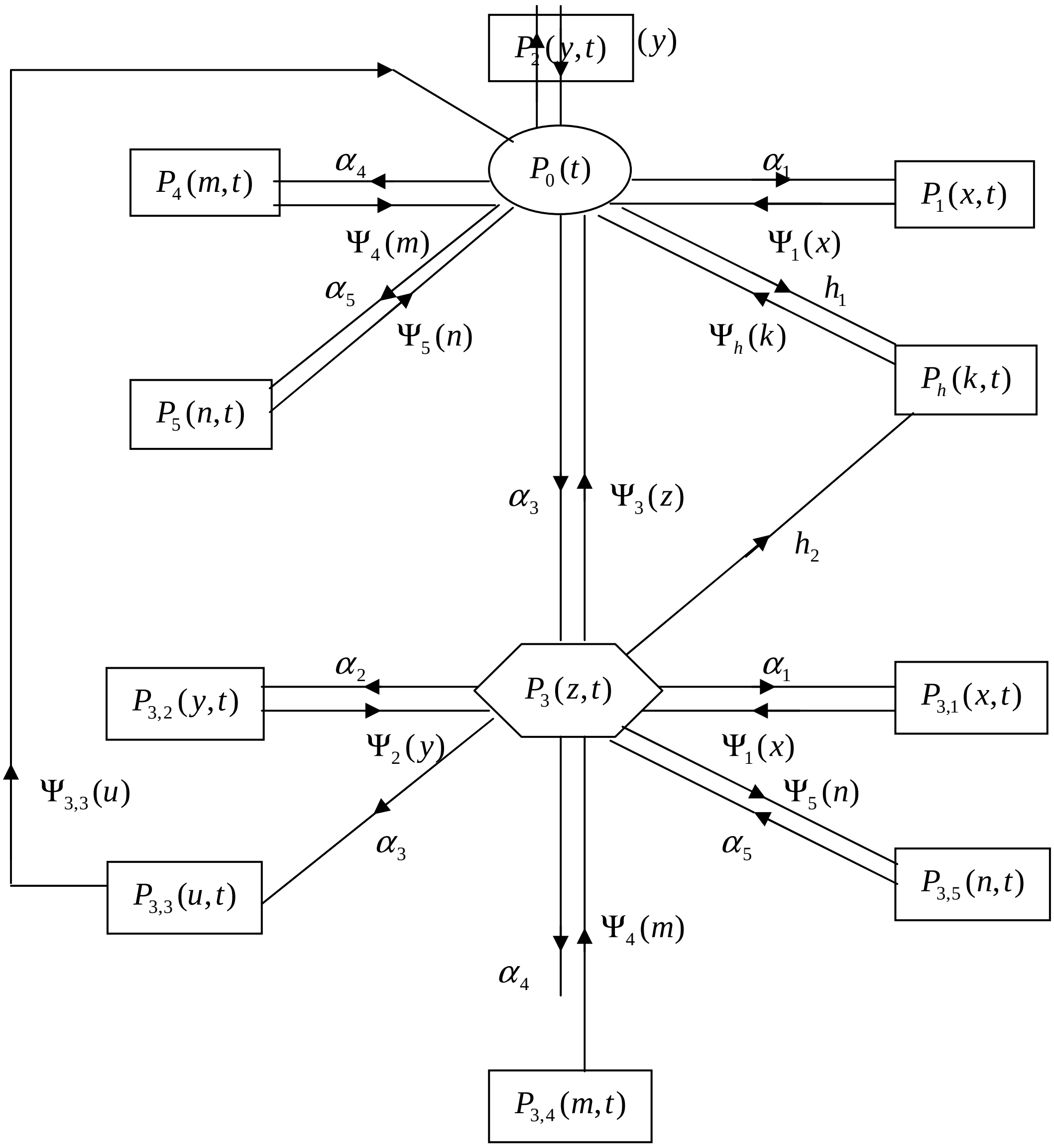
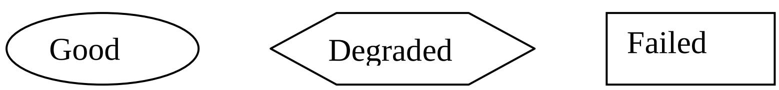


Fig-1(b): Transition-state diagram

States:



4. FORMULATIONOFMATHEMATICAL MODEL

Using continuity argument and limiting procedure, we obtain the following set of difference-differential equations, governing the behaviour of considered system, discrete in space and continuous in time:

$$\left[\frac{d}{dt}+\sum_{i=1}^{5}\alpha_i+h_1\right]P_0(t)=\int_0^\infty P_1(x,t)\psi_1(x)dx+\int_0^\infty P_2(y,t)\psi_2(y)dy+\int_0^\infty P_3(z,t)\psi_3(z)dz \quad (1)$$

$$+\int_0^\infty P_4(m,t)\psi_4(m)dm+\int_0^\infty P_5(n,t)\psi_5(n)dn+\int_0^\infty P_h(k,t)\psi_h(k)dk+\int_0^\infty P_{3,3}(u,t)\psi_{3,3}(u)du$$

$$\left[\frac{\partial}{\partial j}+\frac{\partial}{\partial t}+\psi_i(j)\right]P_i(j,t)=0\text{ , } i=1,2,4,5 \text{ and } j=x,y,m,n \text{ respectively.} \quad \ldots(2)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\sum_{i=1}^{5}\alpha_i+h_2+\psi_3(z)\right]P_3(z,t)=0 \quad \ldots(3)$$

$$\left[\frac{\partial}{\partial j}+\frac{\partial}{\partial t}+\psi_i(j)\right]P_{3,i}(j,t)=0\text{ , } i=1,2,4,5 \text{ and } j=x,y,m,n \text{ respectively.} \quad \ldots(4)$$

$$\left[\frac{\partial}{\partial u}+\frac{\partial}{\partial t}+\psi_{3,3}(u)\right]P_{3,3}(u,t)=0 \quad \ldots(5)$$

$$\left[\frac{\partial}{\partial k}+\frac{\partial}{\partial t}+\psi_h(k)\right]P_h(k,t)=0 \quad \ldots(6)$$

Boundary conditions are:

$P_i(0,t)=\alpha_i P_0(t)$,i = 1, 2, 4 and 5 …(7)

$$P_3(0,t)=\alpha_3P_0(t)+\int_0^\infty P_{3,1}(x,t)\psi_1(x)dx+\int_0^\infty P_{3,2}(y,t)\psi_2(y)dy \quad \ldots(8)$$

$$+\int_0^\infty P_{3,4}(m,t)\psi_4(m)dm+\int_0^\infty P_{3,5}(n,t)\psi_5(n)dn$$

$P_{3,i}(0,t)=\alpha_i P_3(t)$, i = 1, 2, 4 and 5 …(9)

$$P_{3,3}(0,t)=\alpha_3P_3(t) \quad \ldots(10)$$

$$P_h(0,t)=h_1P_0(t)+h_2P_3(t) \quad \ldots(11)$$

Initial conditions are:

$P_0(0)=1$, otherwise zero. …(12)

5. SOLUTION OF THE MODEL

Taking Laplace transforms of equations (1) through (11) subjected to initial conditions (12) and then on solving them one by one, we obtain the following probabilities of states depicted in fig-1(b):

$$\overline{P}_0(s)=\frac{1}{B(s)} \quad \ldots(13)$$

$$\overline{P}_i(s)=\frac{\alpha_iD_i(s)}{B(s)} \text{ for i = 1, 2, 4 and 5} \quad \ldots(14)$$

$$\overline{P}_3(s)=\frac{A(s)}{B(s)} \quad \ldots(15)$$

$$\overline{P}_{3,i}(s)=\frac{\alpha_iA(s)D_i(s)}{B(s)} \text{ for i = 1, 2, 4 and 5} \quad \ldots(16)$$

$$\overline{P}_{3,3}(s)=\frac{\alpha_3 A(s)D_{3,3}(s)}{B(s)} \qquad \dots(17)$$

$$\overline{P}_h(s)=\frac{1}{B(s)}\left[h_1+h_2A(s)\right]D_h(s) \qquad \dots(18)$$

where, $$A(s)=\frac{\alpha_3 D_3\left(s+\sum_{i=1}^{5}\alpha_i+h_2\right)}{1-\left[\sum_{i=1}^{5}\alpha_i\overline{S}_i(s)-\alpha_3\overline{S}_3(s)\right]D_3\left(s+\sum_{i=1}^{5}\alpha_i+h_2\right)} \qquad \dots(19)$$

and

$$\begin{aligned}B(s)&=s\left[1+\alpha_1D_1(s)+\alpha_2D_2(s)+\alpha_4D_4(s)+\alpha_5D_5(s)\right]+\alpha_3+h_1\\&-\left[h_1+h_2A(s)\right]\overline{S}_h(s)-\alpha_3A(s)\overline{S}_{3,3}(s)\\&-\left[\alpha_3+A(s)\left(\alpha_1\overline{S}_1(s)+\alpha_2\overline{S}_2(s)+\alpha_4\overline{S}_4(s)+\alpha_5\overline{S}_5(s)\right)\right]\overline{S}_3\left(s+\sum_{i=1}^{5}\alpha_i+h_2\right)\end{aligned} \qquad \dots(20)$$

It is worth noticing that

Sum of equations (13) through (18) = $\frac{1}{s}$ $\qquad \dots(21)$

6.LONG-RUN BEHAVIOR OF CONSIDERED SYSTEM:

Using final value theorem of Laplace transform, viz., $\lim_{t\to\infty} P(t)=\lim_{s\to 0} s\,\overline{P}(s)=P$ (say), provided limit on left exists, we obtain the following long-run behaviour of considered system from equations (13) through (18):

$$P_0=\frac{1}{B'(0)} \qquad \dots(22)$$

$$P_i=\frac{\alpha_i M_i}{B'(0)}\text{ ;for i = 1, 2, 4 and 5} \qquad \dots(23)$$

$$P_3=\frac{A(0)}{B'(0)} \qquad \dots(24)$$

$$P_{3,i}=\frac{\alpha_i A(0)M_i}{B'(0)}\text{ ;for i = 1, 2, 4 and 5} \qquad \dots(25)$$

$$P_{3,3}=\frac{\alpha_3 A(0)M_{3,3}}{B'(0)} \qquad \dots(26)$$

$$P_h=\frac{1}{B'(0)}\left[h_1+h_2A(0)\right]M_h \qquad \dots(27)$$

where, $$A(0)=\frac{\alpha_3\left[1-\overline{S}_3\left(\sum_{i=1}^{5}\alpha_i+h_2\right)\right]}{\alpha_3+h_2+\left(\sum_{i=1}^{5}\alpha_i-\alpha_3\right)\overline{S}_3\left(\sum_{i=1}^{5}\alpha_i+h_2\right)} \qquad \dots(28)$$

$$B'(0)=\left[\frac{d}{ds}B(s)\right]_{s=0} \qquad \dots(29)$$

and $M_i=-\overline{S}_i{}'(0)=$ Mean time to repair i^{th} failure. $\qquad \dots(30)$

7. PARTICULAR CASE:

When all repairs follow exponential time distribution

In this case, setting $\bar{S}_i(j) = \psi_i / (j+\psi_i)$, for all *i* and *j*, in equations (13) through (18), we obtain the following L.T. of different state probabilities of fig-1(b):

$$\bar{P}_0(s) = \frac{1}{E(s)} \quad \ldots(31)$$

$$\bar{P}_i(s) = \frac{\alpha_i}{E(s)(s+\psi_i)}; \qquad \text{For i = 1, 2, 4 and 5} \quad \ldots(32)$$

$$\bar{P}_3(s) = \frac{F(s)}{E(s)} \quad \ldots(33)$$

$$\bar{P}_{3,i}(s) = \frac{\alpha_i F(s)}{E(s)(s+\psi_i)}; \qquad \text{For i = 1, 2, 4 and 5} \quad \ldots(34)$$

$$\bar{P}_{3,3}(s) = \frac{\alpha_3 F(s)}{E(s)(s+\psi_{3,3})} \quad \ldots(35)$$

and $\bar{P}_h(s) = \frac{1}{E(s)}\left[h_1 + h_2 F(s)\right]\frac{1}{(s+\psi_h)}$ …(36)

where, $F(s) = \dfrac{\alpha_3}{\left(s + \sum_{i=1}^{5}\alpha_i + h_2 + \psi_3\right) - \left[\sum_{i=1}^{5}\frac{\alpha_i\psi_i}{s+\psi_i} - \frac{\alpha_3\psi_3}{s+\psi_3}\right]}$ …(37)

and $E(s) = s\left[1 + \frac{\alpha_1}{s+\psi_1} + \frac{\alpha_2}{s+\psi_2} + \frac{\alpha_4}{s+\psi_4} + \frac{\alpha_5}{s+\psi_5}\right] + \alpha_3 + h_1$

$$-\left[h_1 + h_2 F(s)\right]\frac{\psi_h}{s+\psi_h} - \frac{\alpha_3 F(s)\psi_{3,3}}{s+\psi_{3,3}}$$

$$-\left[\alpha_3 + F(s)\left(\frac{\alpha_1\psi_1}{s+\psi_1} + \frac{\alpha_2\psi_2}{s+\psi_2} + \frac{\alpha_4\psi_4}{s+\psi_4} + \frac{\alpha_5\psi_5}{s+\psi_5}\right)\right]\frac{\psi_3}{s + \sum_{i=1}^{5}\alpha_i + h_2 + \psi_3} \quad \ldots(38)$$

8. RELIABILITY AND M.T.T.F. EVALUATION:

Form equation (13), we obtain the L.T. of reliability function

$$\bar{R}(s) = \left(s + \sum_{i=1}^{5}\alpha_i + h_2\right)^{-1}$$

taking inverse Laplace transform, we get

$$R(t) = \exp.\left\{-\left(\sum_{i=1}^{5}\alpha_i + h_1\right)t\right\} \quad (39)$$

Also, $M.T.T.F. = \int_0^\infty R(t)dt$

$$= \frac{1}{\sum_{i=1}^{5} \alpha_i + h_1} \quad (40)$$

9. AVAILABILITY OF CONSIDERED SYSTEM:

Availability of considered system can be obtained from equations (13) and (15) as:

$$\overline{P}_{up}(s) = \frac{1}{s + \sum_{i=1}^{5} \alpha_i + h_1} \left[1 + \frac{\alpha_3}{s + \sum_{i=1}^{5} \alpha_i + h_2} \right]$$

takinginverseLaplacetransform,we

obta $P_{up}(t) = \left[1 + \frac{\alpha_3}{h_2 - h_1} \right] \exp.\left\{ -\left(\sum_{i=1}^{5} \alpha_i + h_1 \right) t \right\} - \frac{\alpha_3}{h_2 - h_1} \exp.\left\{ -\left(\sum_{i=1}^{5} \alpha_i + h_2 \right) t \right\}$ (41)

Also, $P_{down}(t) = 1 - P_{up}(t)$ (42)

10. NUMERICAL ILLUSTRATION:

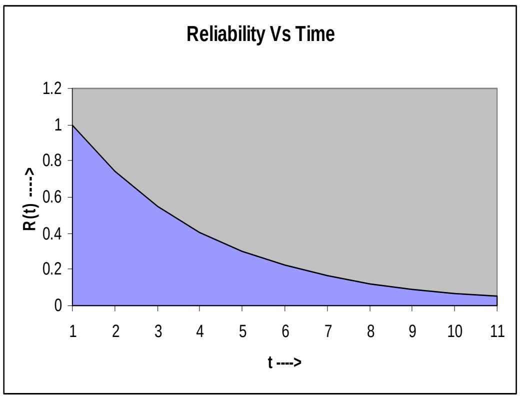
For a numerical illustration we consider the values:

$\alpha_1 = 0.002,\ \alpha_2 = 0.001,\ \alpha_3 = 0.06,\ \alpha_4 = 0.08,\ \alpha_5 = 0.15, h_1 =$

and $t = 0,1,2,---10.$ Using these values in equations (39), (40) and (41), we draw the graphs shown in fig-2, 3 and 4 respectively.

11. RESULTS AND DISCUSSION:

A critical examination of graphs, shown in fig-2, 3 and 4, yields that reliability and availability of considered system decreases smoothly with the increase in values of time *t*. It should be noted that there are no sudden jumps in the values of $R(t)$ and $P_{up}(t)$. Also M.T.T.F. of considered system decreases catastrophically in the beginning but thereafter it decreases approximately in constant manner.



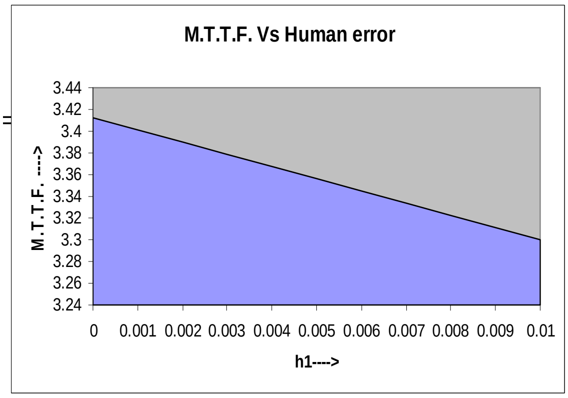


Fig-2: Reliability Vs TimeFig-3:MTTF Vs Human Error

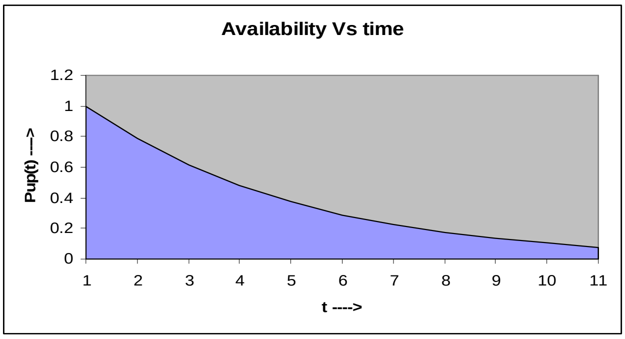


Fig-4: Availability Vs Time

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