

## SPLIT BLOCK DOMINATION IN GRAPHS

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### Abstract

For any graph  $G = (V, E)$ , block graph  $B(G)$  is a graph whose set of vertices is the union of the set of blocks of  $G$  in which two vertices are adjacent if and only if the corresponding blocks of  $G$  are adjacent. A dominating set  $D$  of a graph  $B(G)$  is a split block dominating set if the induced sub graph  $\langle V[B(G)] - D \rangle$  is disconnected. The split block domination number  $\gamma_{sb}(G)$  of  $G$  is the minimum cardinality of split block dominating set of  $G$ . In this paper many bounds on  $\gamma_{sb}(G)$  are obtained in terms of elements of  $G$  but not the elements of  $B(G)$ . Also its relation with other domination parameters is established.

**Keywords:** Dominating set/ Block graphs /split block domination graphs.

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### 1. INTRODUCTION

In this paper, all the graphs considered here are simple, finite, non trivial, undirected and connected. As usual  $p$  and  $q$  denote the number of vertices and edges of a graph  $G$ . In this paper, for any undefined terms or notations can be found in Harary<sup>1</sup>. As usual, the maximum degree of a vertex in  $G$  is denoted by  $\Delta(G)$ . A vertex  $v$  is called a cut vertex if removing it from  $G$  increases the number of components of  $G$ . For any real number  $x$ ,  $\lfloor x \rfloor$  denotes the greatest integer not greater than  $x$ . The complement  $\bar{G}$  of a graph  $G$  has  $V$  as its vertex set, but two vertices are adjacent in  $\bar{G}$  if they are not adjacent in  $G$ . A graph  $G$  is called trivial if it has no edges, if  $G$  has at least one edge then  $G$  is called nontrivial graph. A nontrivial connected graph  $G$  with at least one cut vertices called a separable graph, otherwise a non- separable graph. A vertex cover in a graph  $G$  is a set of vertices that covers all the edges of  $G$ . The vertex covering number  $\alpha_o(G)$  is a minimum cardinality of a vertex cover in  $G$ . A set of vertices in a graph  $G$  is called an independent set if no two vertices in the same set are adjacent. The vertex independence number  $\beta_o(G)$  is the maximum cardinality of an independent set of vertices. The greatest distance between any two vertices of a connected graph  $G$  is called the diameter of  $G$  and is denoted by  $diam(G)$ . A dominating set  $D$  of a graph  $B(G)$  is a split block dominating set if the induced sub graph  $\langle V[B(G)] - D \rangle$  is a disconnected. The split block domination number  $\gamma_{sb}(G)$  of  $G$  is the minimum cardinality of split block dominating set  $G$ . In this paper many bounds on  $\gamma_{sb}(G)$  are obtained in terms of elements of  $G$  but not the elements of  $B(G)$ , also its relation with other domination parameters is established.

We need the following theorems for our further results.

**Theorem A[3]**: For any graph  $G$  with an end vertex  $\gamma(G) = \gamma_s(G)$ .

**Theorem B[3]**: A split dominating set  $D$  of  $G$  is minimal for each vertex  $v \in D$ , one of the following conditions holds.

- i) There exists a vertex  $u \in V - D$ , such that  $N(u) \cap D = \{v\}$
- ii)  $v$  is an isolated vertex in  $\langle D \rangle$
- iii)  $\langle (v - D) \cup \{v\} \rangle$  is connected.

**Theorem C[2]**: If a graph  $G$  has no isolated vertices, then  $\gamma(G) \leq \lfloor \frac{n}{2} \rfloor$

**Theorem D [3]**: For any graph  $G$ ,  $\gamma_s(G) \leq \frac{p \Delta(G)}{\Delta(G)+1}$

### 2. RESULTS

#### Theorem 1

For any connected  $(p, q)$  graph  $G \neq K_p$ , then  $\gamma_{sb}(G) \leq p - \Delta(G) - 1$

**Proof:** For any non trivial connected graph  $G$ , the block graph  $B(G)$  in which each block is complete. Suppose  $B(G) = K_p$ , then by the definition of split domination, the split domination set does not exists. Hence  $B(G) \neq K_p$ .

We consider the following cases.

**Case1:** Assume  $G$  is a tree. Then every block of  $G$  is an edge. Let  $S = \{B_1, B_2, B_3, \dots, B_n\}$  be the blocks of  $G$  and  $M = \{b_1, b_2, b_3, \dots, b_n\}$  be the block vertices in  $B(G)$  corresponding to the blocks  $B_1, B_2, B_3, \dots, B_n$  of  $S$ . Let  $\{B_i\} \subset S$  such that for all  $B_i$  is an non end blocks of  $G$ . Then  $\{b_i\} \subseteq V[B(G)]$  which are cut vertices corresponding the set  $\{B_i\}$ . Since each block is a complete in  $B(G)$ . Then every

vertex of  $V[B(G)] - \{b_i\}$  is adjacent to atleast one vertex of  $\{b_i\}$ . Clearly  $|b_i| = \gamma_b(G)$ . Also each  $b_i$  is a cut vertex, which also gives  $|b_i| = \gamma_{sb}(G)$ . Since for a tree  $p = q + 1$  each block in tree  $T$  is an edge then  $P = B_n + 1$ .

Hence  $P - \Delta(G) - 1 \geq |b_i|$  which gives  $P - \Delta(G) - 1 \geq \gamma_{sb}(G)$

**case 2 :** Suppose  $G$  is not a tree. Then there exists atleast one block which is not an edge. Let  $B_1, B_2, B_3, \dots, \dots, B_n$  be the blocks of  $G$  and  $b_1, b_2, b_3, \dots, \dots, b_n$  be the corresponding block vertices in  $B(G)$ . Let  $\{b_i\} \subseteq V[B(G)]$  in which  $\forall b_i$  which is adjacent to at least one vertex of  $V[B(G)] - \{b_i\}$ . But every vertex of  $b_i$  is a cut vertex in  $B(G)$ . Since each block of  $B(G)$  is a complete graph. The set  $\{b_i\}$  gives a minimal split dominating set in  $B(G)$ . Hence  $|b_i|$  gives a split domination number  $\gamma_{sb}(G)$  in  $B(G)$ . Since  $P - \Delta G - 1 \geq |b_i|$  then  $\gamma_{sb}(G) \leq P - \Delta G - 1$  as required.

**Theorem 2**

For any graph  $G$  with  $n - blocks, B(G) \neq K_p$  then

$$\gamma_{sb}(G) \leq n - 2$$

**Proof :** Suppose  $B(G)$  is a complete graph, then by definition split domination does not exists in  $B(G)$ . Hence  $B(G) \neq K_p$ . Suppose  $G$  be any graph with  $S = \{B_1, B_2, B_3, \dots, \dots, B_n\}$  be the blocks of  $G$  and  $M = \{b_1, b_2, b_3, \dots, \dots, b_n\}$  the block vertices in  $B(G)$  corresponding to the blocks  $B_1, B_2, B_3, \dots, \dots, B_n$  of  $S$ .

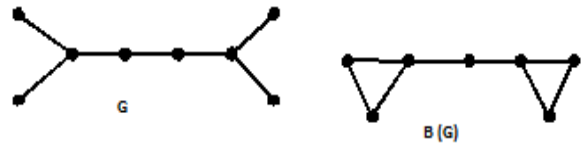
Let  $H = \{B_1, B_2, B_3, \dots, \dots, B_i\}$  be the set of all non end blocks in  $G, 3 \leq i \leq n$  which generates a set  $M_1 = \{b_1, b_2, b_3, \dots, \dots, b_i\}$  in  $B(G), \forall b_i$  in  $B(G)$  are cut vertices. Now we consider  $M_1^1 \subseteq M_1$ , such that  $V[B(G)] - M_1^1$  gives a disconnected block graph and every vertex of  $V[B(G)] - M_1^1$  are adjacent to at least one vertex of  $M_1^1$ . Hence  $M_1^1$  is a minimal split dominating set of  $B(G)$ .

In  $B(G), |V[B(G)]| = n$  and  $\forall v_i \in M_1^1$  are at a distance at most two which gives  $|V[B(G)] - 2 \geq |M_1^1|$ . Hence  $\gamma_{sb}(G) \leq n - 2$ .

**Theorem 3**

If  $\gamma_{sb}(G) = \gamma(G)$  then  $n - 2 \geq \gamma(G)$  where  $n$  is the number of blocks of  $G$  but the converse of the above need not be true.

**Proof :** Suppose  $\gamma_{sb}(G) = \gamma(G)$  then by theorem (2)  $\gamma_{sb}(G) \leq n - 2$ . Since  $\gamma_{sb}(G) = \gamma(G), \therefore n - 2 \geq \gamma(G)$ . The following graphs show that the converse of the above is true. i.e., if  $n - 2 \geq \gamma(G)$  then  $\gamma_{sb}(G) = \gamma(G)$



Total number of blocks in  $G, n = 7, \gamma(G) = 2, \gamma_{sb}(G) = 2$ .

But the converse of the above theorem is not always true. This can be shown by the following graph.



Total number of blocks in  $G, n = 5, \gamma(G) = 2, \gamma_{sb}(G) = 1$  therefore, even though  $n - 2 \geq \gamma(G), \gamma_{sb}(G) \neq \gamma(G)$ .

**Theorem 4**

For any graph  $G, B(G) \neq K_p$ , then  $\gamma_{sb}(G) \leq \frac{P \cdot \Delta(G)}{2 + \Delta(G)}$ .

**Proof :** We consider only those graphs which are not  $B(G) = K_p$ . let  $D$  be a  $\gamma_s$  - set of  $B(G)$  using theorem (B) it follows that for each vertex  $v \in D$  there exist a vertex  $u \in V[B(G)] - D$ . such that  $v$  is adjacent to  $u$ . Since each block in  $B(G)$  is a complete, this implies that  $V[B(G)] - D$  is a dominating set of  $B(G)$ .

Thus

$$\gamma(G) \leq |V[B(G)] - D|, \gamma(G) \leq P - \gamma_{sb}(G), \text{ since by theorem } \gamma_s(G) \leq \frac{P \cdot \Delta G}{1 + \Delta(G)}$$

$$\text{which gives } \gamma_{sb}(G) \leq \frac{P \cdot \Delta(G)}{2 + \Delta(G)}$$

**Theorem 5**

For any graph  $G, B(G) \neq K_p$ , then  $\gamma_{sb} \leq \alpha_0(G)$  where  $\alpha_0$  is a vertex covering number.

**Proof :** Suppose  $B(G)$  is a complete graph. By definition, split domination does not exists in  $B(G)$ . Hence  $B(G) \neq K_p$ . Let  $T$  be a maximum independent set of vertices in  $B(G)$ , then  $T$  has atleast two vertices and every vertex in  $T$  is adjacent to some vertex in  $V[B(G)] - T$ . This implies that  $V[B(G)] - T$  is a split

dominating set of  $B(G)$ . Let  $S = \{B_1, B_2, B_3, \dots, B_n\}$  be the set of blocks in  $G$ . We consider the following case.

**case 1 :** Suppose each block is an edge in  $\{S\}$ . Let  $S_1 = \{B_i\}, 1 \leq i \leq n$  be a subset of  $T$  then  $\forall e_i \in E(G)$  are incident to atleast one vertex of  $S_1$ . clearly  $S_1 \subseteq V[B(G)] - T$ . Then  $|S_1| \leq |V[B(G)] - T|$  which gives  $\gamma_{sb} \leq \alpha_0(G)$ .

**case 2 :** Suppose there exists atleast one block which is not an edge, then there exists a block which contains atleast three vertices. From the above case 1,  $\alpha_0(G)$  is increasing more than that of the graph as in case 1, clearly  $\gamma_{sb}(G) \leq \alpha_0(G)$ .

**Theorem 6**

For any connected graph  $G, B(G) \neq K_p$ , then  $\gamma_{sb}(G) \leq \gamma(G)$

**Proof :** For split domination, we consider the graphs with the property  $B(G) \neq K_p$ . Let  $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$  and  $H \subset V(G)$  be a minimal dominating set of  $G$  such that  $|H| = \gamma(G)$ . For  $\gamma_{sb}(G)$  of any graph we consider the following cases.

**case 1:** Suppose each block of  $G$  is an edge then in  $B(G)$  each block is complete. Let  $H = \{b_1, b_2, b_3, \dots, b_i\}$  be the set of cut vertices in  $B(G)$  which corresponds to the non end blocks of  $G$ . We consider  $H_1 \subseteq H$  such that  $|H_1| = \gamma_{sb}$  which gives  $|H| > |H_1|$ . Hence  $\gamma_{sb}(G) < \gamma(G)$ .

**case2:** Suppose there exists atleast one block with atleast three vertices. Assume  $B_k$  is a block which contains atleast three vertices then  $B_k \in V[B(G)]$ . If  $B_k$  is end block in  $G$ , then  $B_k \in V[B(G)] - D$ , where  $D$  is a minimal dominating set. Otherwise  $B_k \in D$ . Hence in this case  $V[G] \geq V[B(G)]$  and  $V[B(G)] - D$  has more than one component. Hence  $|D| = \gamma_{sb}(G)$  such that  $|H| > |D|$  which gives  $\gamma(G) \geq \gamma_{sb}(G)$ .

**Theorem 7**

For any graph  $G$  with  $n$  - blocks and  $B(G) \neq K_p$ , then  $\gamma_{sb}(G) \leq n + \gamma_s(G) - 4$ .

**Proof :** Suppose  $S = \{B_1, B_2, B_3, \dots, B_n\}$  be the blocks of  $G$ . Then  $M = \{b_1, b_2, b_3, \dots, b_n\}$  be the corresponding block vertices in  $B(G)$  with respect to the set  $S$ . Let  $H = \{v_1, v_2, v_3, \dots, v_n\}$  be the set of vertices in  $G, V(G) = H$ . If  $J = \{v_1, v_2, v_3, \dots, v_m\}$  where  $1 \leq m \leq n$ . such that  $J \subset H$  and suppose there exists a set  $J_1 \subset J$  then  $\{v_i\} \in J_1$  which gives  $H - J_1$  is a disconnected graph. Suppose  $J_1 \cup J$  has the minimum number of vertices, such that  $N(J_1 \cup J) = V(G) - (J_1 \cup J)$  gives a minimal split domination set in  $G$ . Hence  $|J_1 \cup J| = \gamma_s(G)$ . Suppose  $M^1 = \{b_1, b_2, b_3, \dots, b_j\}$  where  $1 \leq j \leq n$  such

that  $M^1 \subset M$  then  $\forall b_i \in M^1$  are cut vertices in  $B(G)$ , since they are non end blocks in  $B(G)$ , and each block is complete in  $B(G)$ . Hence  $M^1$  is a  $\gamma_s$  - set of  $B(G)$ .

clearly  $|M^1| = \gamma_{sb}(G)$ . Now  $|M^1| \leq n + |J_1 \cup J| - 4$  gives the required result.

**Theorem 8**

For any graph  $G$  with  $n$  - blocks,  $B(G) \neq K_p$  then  $\gamma_{sb}(G) \leq \text{diameter}(G) - 2$ .

**Proof :** Suppose  $S = \{B_1, B_2, B_3, \dots, B_n\}$  be the blocks of  $G$  then  $M = \{b_1, b_2, b_3, \dots, b_n\}$  be the corresponding block vertices in  $B(G)$ . Suppose  $A = \{e_1, e_2, e_3, e_4, \dots, e_k\}$  be the set of edges which constitutes the diametral path in  $G$ . Let  $S_1 = \{B_i\}$  where  $1 \leq i \leq n, S_1 \subset S$ . Suppose  $\forall B_i \in S_1$  are non end blocks in  $G$ , which gives the cut vertices in  $B(G)$ . Suppose  $M^1 = \{b_1, b_2, b_3, \dots, b_i\}$  where  $1 \leq i \leq n$  such that  $M^1 \subset M$  then  $\forall b_i \in M^1$  are cut vertices in  $B(G)$ . Since they are non end blocks in  $B(G)$  and each block is complete in  $B(G)$ . Hence  $M^1$  is a  $\gamma_s$  - set of  $B(G)$ . Clearly  $|M^1| = \gamma_{sb}(G)$ . Suppose  $G$  is cyclic then there exists atleast one block  $B$  which contains a block diametrical path of length at least two. In  $B(G)$  the block  $B \in V[B(G)]$  as a singleton and if at most two elements of  $\{A\} \notin \text{diameter}(G)$  then  $|A| - 2 \geq |M^1|$  gives  $\gamma_{sb}(G) \leq \text{diameter}(G) - 2$ .

Suppose  $G$  is acyclic then each edge of  $G$  is a block of  $G$ . Now  $\forall B_i \in S, \exists e_i, e_j, \notin \{A\}$  where  $1 \leq i, j \leq k$  gives  $\text{diameter}(G) - 2 \geq |M^1|$ . Clearly we have  $\gamma_{sb}(G) \leq \text{diameter}(G) - 2$ .

**Theorem 9**

For any graph  $G, B(G) \neq K_p$  then  $\gamma_{sb} \leq \beta_0(G) - 1$  where  $\beta_0(G)$  is the independence number of  $G$ .

**Proof :** From the definition of split domination,  $B(G) \neq K_p$ . Suppose  $S = \{B_1, B_2, B_3, \dots, B_n\}$  be the blocks of  $G$ . Then  $M = \{b_1, b_2, b_3, \dots, b_n\}$  be the corresponding block vertices in  $B(G)$  with respect to the set  $S$ . Let  $H = \{v_1, v_2, v_3, \dots, v_n\}$  be the set of vertices in  $G, V(G) = H$ . We have the following cases.

**case 1 :** Suppose each block is an edge in  $G$ . Let  $H^1 = \{v_1, v_2, v_3, \dots, v_i\}$  where  $1 \leq i \leq n, H^1 \subset H$  such that  $\forall v_i \in H, \exists e_i \in E(G)$  are joining two vertices of  $H^1$ . Hence each vertex in  $(V(G) - H^1)$  is independent, so  $|V(G) - H^1| = \beta_0(G)$ . Let  $S^1 = \{B_1, B_2, B_3, \dots, B_k\}$  be the set of all non end blocks in  $G$ , then  $M^1 = \{b_1, b_2, b_3, \dots, b_k\}$  be the corresponding block vertices in  $B(G)$  which are cut vertices. Since each block is complete in  $B(G)$  and  $\forall b_k \in M^1$  covers all the

vertices belongs to the corresponding blocks which are complete. Clearly  $|M^1| = \gamma_{sb}(G)$  which gives  $\gamma_{sb}(G) \leq \beta_o(G) - 1$ .

**case 2:** Suppose  $G$  has atleast one block which is not an edge. Assume there exists a block  $B_i$  which is not an edge. Then  $B_i$  contains atleast two independent vertices. From case1 for this graph  $|B_o|$  will be more. In  $B(G)$ ,  $b_i \in V[B(G)]$  is a vertex ( $B_i \leftrightarrow b_i$ ). Again from the above case  $b_i \in M^1$ . If  $b_i$  is the non end block in  $G$  then  $b_i \in M^1$ , otherwise  $b_i \notin M^1$  hence  $|M^1| = \gamma_{sb}(G)$ . Now one can easily verify that  $\gamma_{sb}(G) \leq \beta_o(G) - 1$ .

**Theorem 10**

For any graph  $G, B(G) \neq K_p$ , then  $\gamma_{sb}(G) + \gamma(G) \leq P - 1$   
*Proof :* Suppose  $B(G)$  is a complete graph, by definition, Split domination does not exists in  $B(G)$ . Hence  $B(G) \neq K_p$ . Suppose  $S = \{B_1, B_2, B_3, \dots, \dots, B_n\}$  be the blocks of  $G$ . Then  $M = \{b_1, b_2, b_3, \dots, \dots, b_n\}$  be the corresponding block vertices in  $B(G)$ . Let  $H = \{v_1, v_2, v_3, \dots, \dots, v_n\}$  be the set of vertices in  $G$ .  $J = \{v_1, v_2, v_3, \dots, \dots, v_i\}$ , where  $1 \leq i \leq n$  such that  $J \subset H, \forall v_i \in J$  Which covers all the vertices of  $G$  with minimal cardinality .Hence  $J$  is a minimal dominating set of  $G$  and  $|J| = \gamma(G)$ . Let  $S_1 = \{B_i\}$  where  $1 \leq i \leq n, S_1 \subset S$  and  $\forall B_i \in S_1$  are non end blocks in  $G$ . Then we have  $M_1 \subset M$  which correspondes to the elements of  $S_1$  such that  $M_1$  forms a minimal dominating set of  $B(G)$ . Since each element of  $M_1$  is a cut vertex ,then  $|M_1| = \gamma_{sb}(G)$ . Further  $M_1 \cup J \leq P - 1$  which gives  $\gamma_{sb}(G) + \gamma(G) \leq P - 1$ .

**Theorem 11**

For any graph  $G, B(G) \neq K_p$  then  $\gamma_{sb} \leq \lfloor \frac{P}{2} \rfloor$

**Proof :** To prove the lower bound for  $\gamma_{sb}(G)$ , the graph  $B(G) \neq K_p$ . Let  $S = \{B_1, B_2, B_3, \dots, \dots, B_n\}$  be the blocks in  $G$  and  $M = \{b_1, b_2, b_3, \dots, \dots, b_n\}$  be the corresponding block vertices in  $B(G)$  with respect to the set  $S$ . Let  $H = \{v_1, v_2, v_3, \dots, \dots, v_n\}$  be the set of vertices in  $G$  such that  $V(G) = H$ . Let  $G$  be a connected and assume that  $x$  is the set of all end vertices adjacent to a vertex  $v$ , where  $|x| = t$ . Then the induced subgraph  $H^1 = \langle H - (x \cup \{v\}) \rangle$  has no isolates and by Theorem (C)  $\gamma_{sb}(H^1) \leq \lfloor \frac{p-t-1}{2} \rfloor$ , If  $p$  is even, say  $p = 2k$ , then  $K = \gamma_{sb}(G) \leq 1 + \lfloor \frac{2k-t-1}{2} \rfloor$  then there exist at most one end vertex adjacent to each  $v \in H$  such that  $t \leq 1$  as required. If  $p$  is odd, say  $p = 2k + 1$ , then  $K = \gamma_{sb}(G) \leq 1 + \lfloor \frac{2k+1-t-1}{2} \rfloor$  and deduce that  $t \leq 2$ . Assume in this case ( $n$  is odd), that  $R$  is the set of vertices that are adjacent to exactly two end vertices, where  $|R| = |M^1|$  where  $M^1 \subset M$  note that each vertex of  $R$  is in any minimal dominating set of  $B(G)$ . Let  $B(G^1)$  be the sub graph formed removing  $R$  and all

end vertices that are adjacent to vertices in  $R$ . The set  $I$  of isolates of  $B(G^1)$  is dominated in  $B(G)$  by  $R$ . The graph  $B(G^1) - I$  is isolate free and has atmost  $2k + 1 - 3r$  vertices. Then  $\gamma[B(G^1) - I] \leq \lfloor \frac{2k+1-3r}{2} \rfloor$ . A dominating set of  $B(G^1) - I$  together with  $R$  denotes  $B(G)$ .

Hence  $K = \gamma_{sb}(G) \leq r + \lfloor \frac{2k+1-3r}{2} \rfloor, r \leq 1$  on the above said two cases, we have  $|M^1| \leq \frac{V(G)}{2}$  which gives  $\gamma_{sb} \leq \lfloor \frac{P}{2} \rfloor$

**Theorem 12**

For any connected graph with  $n - blocks B(G) \neq K_p$  then  $\gamma_{sb}(\bar{G}) \leq n - 2$ .

**Proof :** Let  $G$  be a connected graph with  $n \geq 3$  blocks. Then the number of blocks of  $(\bar{G})$  is less than or equal to  $n$  and hence  $\gamma_{sb}(\bar{G}) \leq \gamma_{sb}(G)$  and from Theorem (2)  $\gamma_{sb}(\bar{G}) \leq n - 2$ .

**Theorem 13**

For any graph  $G, B(G) \neq K_p$ , with  $p \geq 4$  vertics and  $B(\bar{G})$  has no isolates then  $\gamma_{sb}(\bar{G}) \leq P - 3$ .

**Proof :** Let  $G$  be a graph with  $P \geq 4$  vertices and  $B(\bar{G})$  has no isolates. Then the number of blocks of  $B(\bar{G})$  is less than or equal to  $P - 3$ . Hence  $\gamma_{sb}(\bar{G}) \leq P - 3$ .

**Theorem 14**

For any graph  $G, B(G) \neq K_p$ , then

- (i)  $\gamma_{sb}(G) + \gamma_{sb}(\bar{G}) \leq 2(n - 2)$
- (ii)  $\gamma_{sb}(G) \cdot \gamma_{sb}(\bar{G}) \leq (n - 2)^2$ .

**Proof :** From Theorem (2) and Theorem (12) the above result follows

**REFERENCES:**

[1]. Harary.F, Graph Theory, Addition-Wesley, reading mass, (1969)  
 [2]. T.W.Haynes, Stephen T.Hedetniemi, Peter J.Slater, Fundamentals of domination in graphs, Marcel Dekker, Inc, Newyork, pp 41-49 (1998)  
 [3]. V.R.Kulli, Theory of domination in graphs, vishwa international publications, Gulbarga, India. pp 111-118 (2010)