

COMPARATIVE ANALYSIS OF SINGULARITIES OF 3-PRS AND 3-PRS PARALLEL MANIPULATORS USING GRASSMANN-CAYLEY ALGEBRA

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Abstract

Grassmann-Cayley Algebra (GCA) is a symbolic approach to Plücker coordinates lines. It is used to write expressions for geometric incidences in projective geometry. GCA gives a useful way to analyze mechanism singularity without algebraic coordinate expression. Since it has great powerful tools for coordinate's free representation, geometric interpretation and singularity analyzing in real time computing, this method is implemented in the present research. The goal of this study is finding the singularities conditions of 3-PRS Parallel Manipulators (PMs) based on GCA when the position of the actuators is firstly on prismatic joints and secondly on revolute joints. Geometric method of reciprocal screw is used to formulate the Global Wrench System (GWS) of the PMs. GWS represents the determinant of Jacobian matrix which rows are Plücker coordinate lines. The degeneracy analysis of determinant of this matrix based on the dependency notion of Plücker coordinate lines is performed in coordinate free expressions and provides double and single singularities conditions, respectively for 3-PRS and 3-PRS PMs, which contains all generals and specials cases. The keys elements of transition between the rows of Jacobian matrix and singularity condition are the introduction of Plücker lines, superbrackets, join and meet operators. The vanishing points of these superbrackets decomposition are analyzed to describe the singularity conditions associated with different motions. The results indicate that for the 3-PRS and 3-PRS PMs, a common singularity occurs when the three constraints forces of the mobile platform lie on the common plane and intersect at a unique point. It is suggested that both position and orientation of the actuator have to be integrated in the conceptual design stage in order to optimize the rigidity frame of the mechanism.

Key Words: Singularity, 3-PRS Parallel Manipulators, Grassmann-Cayley Algebra, Projective space, Wrench graph.

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1. INTRODUCTION

Robot singularity has always been a classical problem for mechanical designers. Singularity is commonly referred to a position in the robot's workspace where the joints no longer work as independent controlling variables. The level of difficulty involved in computing the determinant $\det(J)$ of the Jacobian matrix (J) associated with the Plücker coordinate lines, is closely related to the complexity of the robot mechanism. The more complicate its architecture, the more difficult $\det(J)$ is to be calculated. Parallel singularity occurs when the robot's mobile platform gains extra degree of freedom (dof) and becomes uncontrollable. To overcome this problem, several methods have proposed over the past years. Among these methods, the Grassmann-Cayley Algebra (GCA) is probably one of the most appealing since it provides sufficient tools to properly solve the abovementioned problem due to its merits. First of all, GCA is suitable for analyzing the rigidity of the framework of the architecture and for scene analysis. It's a bridge which allows interaction between symbolic and numerical computation [1-11]. Secondly, it has a powerful coordinate free representation which means GCA has a high capability to work and make representation shortly in coordinate free

manners. A reciprocal translation between geometric entities and algebraic expressions can be obtained [1-17]. The third motivation is its geometrical generic interpretation: GCA has a suitable language for interpreting geometrically singularity of parallel manipulators (PMs). The tools such as superbrackets have ability to analysis the constraints applied on the moving platform and compare manipulators at the conceptual design stage [1-5; 10-15; 18-19]. Another interesting feature of GCA is that it has the ability on real time computing. The computational time needed to calculate the singularity equation in GCA is fairly short and low cost. The solution provided by GCA is a single condition that contains all of the special cases in which the robot is singular [1-4; 9-11; 15]. To prevent the parallel singularity, and the robot's performance deterioration, it is necessary to obtain, firstly the J related to its instantaneous motion and secondly calculate $\det(J)$ degeneracy.

Because 3-PRS (prismatic, revolute and spherical) robots have a wide industrial application, they have been intensively investigated over the past few years using different approaches [56-58]. For example, Tsai et al [20] in order to detect the undesirable configurations of 3-PRS PMs compared both Bezout's elimination method and optimization techniques while Li and Xu [21] used screw

theory and its reciprocal to represent the mobility of 3-PRS PMs and investigated algebraically the singularity condition after a numerical calculus of J . Other approaches of solving singularity of PMs included the application of the algebraic properties of Lie group [22], kinematics models [23], non-pure rotational and non-pure translational J [24], numerical analysis [25], analytical solution based on MATLAB programming [26], analytical expression in singularity locus of general Stewart–Gough PMs [27], discretization of the J [28], etc...

Some challenges are still encountered in designing 3-PRS PMs. Even if their singular positions could be successfully detected by software simulation [29], parallel robot singularity and its associated motion still are an interesting research field for robot designers due to the complex nature of the topic. Joshi and Tsai [30] used screw theory and its reciprocal to present the Jacobian matrix of 3-UPU and 3-RPS PMs which provides information about both architecture and constraint singularities while X. Ding et al [31] proposed recently a novel deployable prism mechanism. The singularity motion associated to his parallel manipulator was studied by screw theory method and provided two singularities in the whole mechanism when it was deploying.

It has been shown that singularity analysis of parallel manipulator cannot be performed without manipulating the J associated with the instantaneous motion of the moving platform and/or the actuated joint of the manipulator [32-34]. J.P Merlet [35] confirmed that calculating the determinant is only the first stage, the second stage is finding the poses that cancel the determinant is the most difficult.

In this paper we focus on the poses that cancel the determinant using GCA . This method allows us to analyze the singularity in coordinate-free manners and make a geometric interpretation for the vanishing point of the superbrackets decomposition. The superbrackets are alternative manners to calculate $\det(J)$.

In this research, the comparative analysis of 3-PRS PMs and 3-PRS PMs based on the degeneracy of each Global Wrench System (GWS) using the dependency of Plücker coordinate lines is performed in coordinate free expression to obtain respectively a double and a single singularity conditions including all special cases. The work is organized as follows: Sect.2 recalls some mathematical background of GCA used in projective space such as a concept of lines, reciprocal screw theory, and GWS before join-meet operators. In Sect.3, the 3-PRS PMs and the adopted representation of the mechanism are described. Geometrically, GWS which is similar to J is determined by the reciprocal screw theory before the symbolic approach of Plücker coordinate lines in Sect.4 while in Sect.5, the singularity conditions and associated motions are discussed. Finally the conclusion is given at Sect.6.

2. MATHEMATICAL BACKGROUND AND FUNDAMENTAL CONCEPTS

In this section we will give a brief overview of GWS in projective space before a symbolic approach of Plücker coordinate lines in GCA . More details on GCA and superbrackets decomposition should refer to [4, 7, 16, 19, 52-55].

2.1 Projective Space

The projective space refers to the affine space augmented by the points at infinity where any pair of parallel lines can be said to meet at the unique point at infinity on the plane Π_∞ . Toms [36] recalled three important axioms from projective geometry: firstly for any given two distinct points in a plane, there is a unique line that lies on both of them, secondly for any given two distinct lines in a plane, there is a unique point that lies on both of them and at last all algebraic representation of projective geometry must be homogeneous. In projective space any point (line) has four (six) homogeneous coordinates and each line is called Plücker line which can be extended to coordinates for the screws and the duality twist-wrenches in application for kinematics of robot manipulators [17,37-38]. Screw theory has been an important tool in mechanics of robotics [39-42]. A finite line, L , is represented by its Plücker coordinates vector F :

$$F = (s; r \times s) \quad (1)$$

where s is the line direction, $(r \times s)$ represents the moment of L with respect to the origin and r is the position vector of any point on L . An instantaneous screw axis is a Plücker coordinate vector line with its associated pitch in a given position, then the screw axis of PMs is described by:

$$\hat{\$} = [s; (s_0 \times s + hs)]^T \quad (2)$$

with, s the unit vector along the screw axis, s_0 the position vector of a point on the screw axis with respect to a reference frame, h the pitch of the screw. A zero pitch screw $\hat{\$}_0$ and an infinite pitch screw $\hat{\$}_\infty$ can be respectively identified with a Plücker coordinate vectors of a finite line and a line at infinity:

$$\hat{\$}_0 = [s; (s_0 \times s)]^T \quad (3)$$

$$\hat{\$}_\infty = [0; s]^T \quad (4)$$

A twist is a screw representing the instantaneous motion (linear and angular velocity) of a rigid body. A body instantaneously undergoing a pure rotation about an axis l is a twist of zero pitch $\hat{\$}_0$. A body instantaneously undergoing a pure translation along an axis l is a twist of infinite pitch $\hat{\$}_\infty$. A wrench is a screw representing a combination of a force and a couple acting on a rigid body. A body subject to a pure force along the axis l is a wrench of zero pitch

screw $\hat{\$}_0$. A body subject to a pure couple is a wrench of infinite pitch screw $\hat{\$}_\infty$.

2.2 Geometrical Method for Reciprocal Screw Theory

Since n number of independents kinematic chains form (n) screw space which is composed of (n) system screws, all screws which are reciprocal to a (n) system screws form a $(6-n)$ system [14, 30, 44-51]. Two zero pitch screws $\hat{\$}_0$ and $\hat{\$}'_0$ are reciprocal to each other if and only if their axes are coplanar. A zero pitch screw $\hat{\$}_0$ is reciprocal to an infinite pitch screw $\hat{\$}_\infty$ if only if their directions are orthogonal to each other. Two infinite pitch screws $\hat{\$}_\infty$ and $\hat{\$}'_\infty$ are always reciprocal to each other. [30, 46-49].

2.3 Global Wrench System (GWS)

GWS is a platform wrench system which is obtained by the combination of the actuation wrench system (W_n) and the constraint wrench system (F_i) [18]. The constraint wrench system of l_i is identified as a reciprocal screw system to all the joint screws of l_i . Its depends of the architecture of the manipulator and defines how the mobile platform is forced by l_i . while the actuation wrench system of l_i is depend on the operating mode and define how the actuators act on the *dof* of the effectors. It is obtained my locking the actuators of l_i before determining all reciprocal screw to the joint screws except that of the actuated *link* _{n} of l_i . The GWS is described as:

$$GWS = (W_n, F_i) = (\hat{\$}_a^1, \dots, \hat{\$}_a^n, \hat{\$}_c^1, \dots, \hat{\$}_c^{6-n}) \quad (5)$$

$$\text{with } W_n = \underbrace{\hat{\$}_a^1, \dots, \hat{\$}_a^n}_{(n)\text{system-actuated}} \text{ and } F_i = \underbrace{\hat{\$}_c^1, \dots, \hat{\$}_c^{6-n}}_{(6-n)\text{system-constraint}}$$

The $\det(J)$ is written in the *GWS* form as:

$$\det(J) = [GWS] = \left[\underbrace{W_1, \dots, W_n}_{(n)\text{system}}, \underbrace{F_1, \dots, F_i}_{(6-n)\text{system}} \right] \quad (6)$$

2.4 Grassmann-Cayley Algebra based on Plücker coordinates lines

Let V^n be n -dimensional vector space over the field R , W be k -dimensional subspace of V^n with $\{w_1, w_2, \dots, w_k\}$ a basis of W and P a Plücker coordinate vector of W . A

symbolic level of P without specific coordinate is also a Plücker coordinate vector P in the \mathcal{O}_n^k -dimensional vector space V^k and is written as [5]:

$$P = w_1 \vee w_2 \vee \dots \vee w_k = w_1 w_2 \dots w_k \quad (7)$$

In coordinate free expression P is called an extensor of step k . Let be S a set of finites points $\{w_1, w_2, \dots, w_k\}$ defined now in the $(k-1)$ dimensional projective space P over the field R where $w_i = (x_{1,i}, x_{2,i}, \dots, x_{k,i})^T$. Each w_i is represented by it homogeneous coordinate form k -tuple with $1 \leq i \leq k$. The bracket of these vectors is defined as the determinant of matrix having w_i as its columns and described as:

$$[w_1, w_2, \dots, w_k] = \det \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,k} \\ \vdots & \vdots & \dots & \vdots \\ x_{k,1} & x_{k,2} & \dots & x_{k,k} \end{bmatrix} \quad (8)$$

This determinant in *GCA* language represents the symbolic approach of Plücker coordinate lines which is similar to the superbrackets and is used to analyze the singularity without algebraic coordinates.

2.5 Join and Meet Operators

The symbols \vee and \wedge represent respectively the operators join and meet [1-3]. The join of two or more extensors represent the operation of (union) joining the associated vector subspaces while meet is for the intersection of subspaces. These tools are useful in the interpretation of singularity conditions Robot Manipulators. Instantaneously the wrench space of parallel combination of motion is the sum of the wrench spaces of the composing constraints. Then in *GCA* language for kinematics chains the wrench space of the parallel connection of kinematic chains is the support of the join of the extensors that represent the wrench spaces of the chains, provided that their wrench extensors are linearly independent [8]. In consequence, the system Eq. (8) is linearly dependent when:

$$w_1 \vee w_2 \vee \dots \vee w_k = [w_1, w_2, \dots, w_k] = 0 \quad (9)$$

3. DESCRIPTION AND ADOPTED REPRESENTATIONS OF 3-PRS PARALLEL MANIPULATORS

3.1 Description of 3-PRS Parallel Manipulators

The chosen architecture (Fig-1) is similar to Tsai's [20].

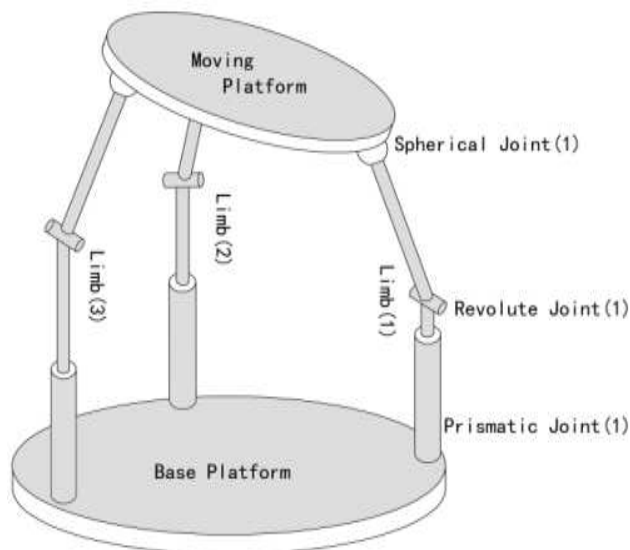


Fig -1: 3-PRS Parallel Manipulators designed by Tsai

The moving platform (M) centered on $O'(x', y', z')$ is connected to a fixe base (F) centered on $O(x, y, z)$ by three fully-parallel limbs l_i . Each limb l_i consists of a prismatic joint (P) fixed on (F) at point P_i , followed by a revolute joint (R) at a point (R_i) about which rotates a fixed-length link and the connecting link to the end-effector is a spherical joint (S) at point S_i on (M) (see Fig-2).

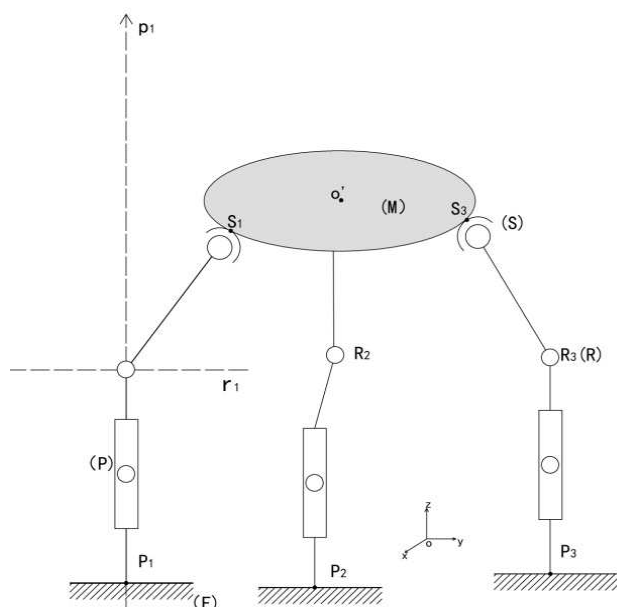


Fig -2: Mechanism architecture

Each independent l_i has five degrees of freedom express as one translation -one rotation- and one spherical which consist of three intersecting and non-coplanar rotations

joints at $S_i(u_i, v_i, w_i)$. For example l_1 has a translational point P_1 fixed on (F) followed by a rotated point R_1 which is connected to the link of the end effector by a spherical point $S_1(u_1, v_1, w_1)$ on (M) where S_1u_1, S_1v_1 and S_1w_1 are respectively three intersecting non-coplanar revolute joint axis at S_1 (see Fig-3).

3.2 Adopted Representations of 3-PRS Parallel Manipulators

(a) The input of the mechanism adopted in this paper consists of two variable positions of actuated joint. Firstly the prismatic joints are actuated and all other joints are passive. Secondly the revolute joints are actuated and all other joints are passive. The order of each limb is respectively represented by the letter $i = 1, 2, 3$.

(b) The axis P_i of prismatic joint and the following axis r_i of revolute joints are orthogonal.

(c) Let be α_i the plane formed by the spherical joint center S_i and the joint axis r_i

(d) Let be β_i the plane which contains the spherical joint

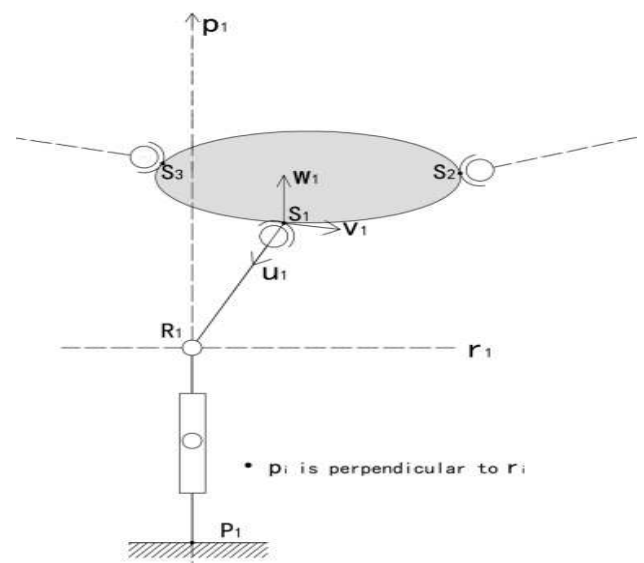


Fig-3: Architecture of l_i

center S_i and perpendicular to the prismatic joint p_i . Consequently:

(e) α_i and β_i intersect each other at a line F_i .

(f) Lines ab , ef and ij represent respectively F_1 , F_2 and F_3 while cd , gh and kl represent respectively r_1 , r_2 and r_3 thus each F_i parallel to r_i (see Fig-4).

The dof of a 3-PRS PMs is expressed as[21]:

$$dof = \lambda(n - g - 1) + \sum_{i=1}^g X_i \quad (10)$$

$dof = 6(8 - 9 - 1) + (5 + 5 + 5) = 3$ Where n , g and X_i are respectively the number of bodies, joints and degree of freedom of the i -th joint of the mechanism.

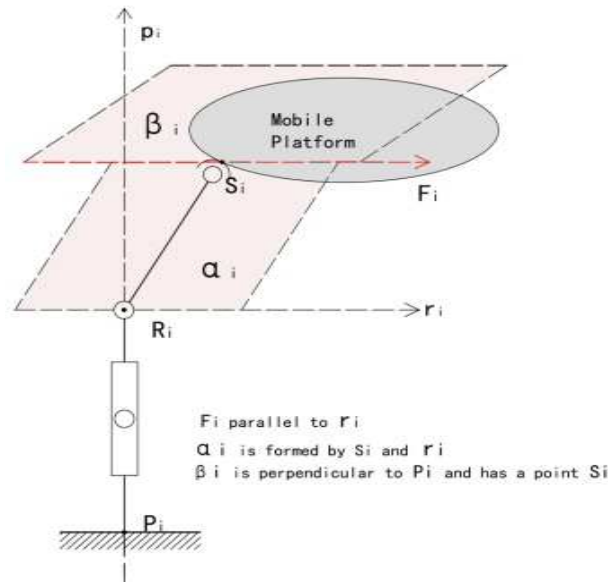


Fig-4: Actuated and constraint wrench system

Since each limb l_i of a 3-PRS PMs consist of five serial kinematic chains, a twist T^i of each limb l_i form 5-system [30, 46-51], see Fig-3 and Eq. (12). Instantaneously the composition of these five serial twists corresponds to their simple addition in the projective space [8].

$$T^i = \left[(\hat{\$}_{pi}) + (\hat{\$}_{ri}) + (\hat{\$}_{ui}) + (\hat{\$}_{vi}) + (\hat{\$}_{wi}) \right] \quad (11)$$

With

$$\hat{\$}_{pi} = [0, p_i]^T; \hat{\$}_{ri} = [r_i, (R_i \times r_i)]; \hat{\$}_{ui} = [u_i, (S_i \times u_i)];$$

$$\hat{\$}_{vi} = [v_i, (S_i \times v_i)]; \hat{\$}_{wi} = [w_i, (S_i \times w_i)]$$

where $p_i, r_i, u_i = S_i u_i, v_i = S_i v_i$ and $w_i = S_i w_i$ are respectively the unit vectors along the prismatic, revolute and the three intersecting non-coplanar revolute joints axis of the spherical joint for the i -th limb (see Fig-3).

4. GWS AND SYMBOLIC APPROACH OF PLUCKER LINES OF 3-PRS PMs

4.1 Constraint Wrench System and Actuated Wrench System of 3-PRS PMs

Constraint wrench system F_i : each limb l_i form a (5)system twist T^i , the reciprocal constraint wrench system to all (5)system twist T^i form a (1)system

constraint wrench of zero pitch [30,44-51].The constraint wrench system F_i of each limb l_i is defined as a line force passing through the center of the spherical joint S_i along the direction parallel to the revolute joint r_i [43] (See Fig-4)

$$F_i = [r_i, (S_i \times r_i)] \quad (12)$$

Actuated wrench system.

When the actuated prismatic joint of l_i is locked, only 4-system twist is valid. Consequently each prismatic actuated wrench, W_i , of each l_i form 2-system wrench and only reciprocal to all passive 4-system twist $\hat{\$}_{ri}, \hat{\$}_{ui}, \hat{\$}_{vi}$ and $\hat{\$}_{wi}$. Then all reciprocal screws lie on a plane α_i (see Fig-4) [43].

When the actuated revolute joint of l_i is locked, only 4-system twist is valid. Consequently each revolute actuated wrench, W'_i , of each l_i form 2-system wrench and only reciprocal to all passive 4-system twist $\hat{\$}_{pi}, \hat{\$}_{ui}, \hat{\$}_{vi}$ and $\hat{\$}_{wi}$. Then all reciprocal screws lie on a plane β_i (see Fig-4) [43].

4.2 The Global Wrench System and Symbolic Approach of Plücker Coordinate Lines of 3-PRS PMs

The GWS explains all characteristic efforts which influence the end-effector and defined as a join of actuated wrench and constraints wrench.

1st case : 3-PRS

$$\det(J_{6 \times 6}) = GWS$$

$$GWS = [\alpha_1, \alpha_2, \alpha_3, F_1, F_2, F_3] \text{ with} \quad (13)$$

$$F_i \in \alpha_i$$

Since any plane can be defined by two different lines and any line is formed by two different points, and according to the adopted representation, the Eq. (13) can be rewrite as:

$$GWS = [ab, cd, ef, gh, ij, kl] \quad (14)$$

In GCA language, the symbolic approach of these six Plücker coordinates lines is described as:

$$P = ab \vee cd \vee ef \vee gh \vee ij \vee kl \quad (15)$$

2nd case : 3-PRS

Similarly:

$$\det(J'_{6 \times 6}) = (GWS)'$$

$$(GWS)' = [\beta_1, \beta_2, \beta_3, F_1, F_2, F_3] \text{ with} \quad (16)$$

$$F_i \in \beta_i$$

Since a point which is not belonging to one line can form one plane with that line, the Eq. (16) can be rewrite as:

$$(GWS)' = [ab, bn, ef, fp, ij, jr] \quad (17)$$

The symbolic approach of these six Plücker lines is:

$$p' = ab \vee bn \vee ef \vee fp \vee ij \vee jr \quad (18)$$

4.3 Wrench Graph of 3-PRS Parallel Mechanism

The graphical representation of GWS is called wrench graph.

1st case : See Fig-5.

Each F_i and r_i are parallel, in projective geometry they intersect each over at infinity at a unique point. Then $b \equiv d, f \equiv h$ and $j \equiv l$ respectively for limbs l_1, l_2 and l_3 . For notation convenience, let designed by the capital letters these points at infinity and rewrite Eq. (14) and Eq. (15)

$$GWS = [aB, cB, eF, gF, iJ, kJ] \text{ and} \quad (19)$$

$$P = aB \vee cB \vee eF \vee gF \vee iJ \vee kJ$$

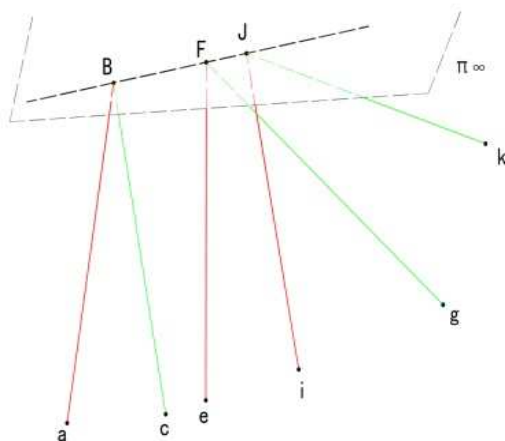


Fig-5: Wrench graph of 3-PRS PMs

2nd case : See Fig-6.

Any line can be represented in projective space by one finite point and one point at infinity, let choose one point at infinity on each of these six Plücker lines. Then Eq. (17) and Eq. (18) can be rewrite as:

$$(GWS)' = [Ab, bN, Ef, fP, Ij, jR] \text{ and} \quad (20)$$

$$P' = Ab \vee bN \vee Ef \vee fP \vee Ij \vee jR$$

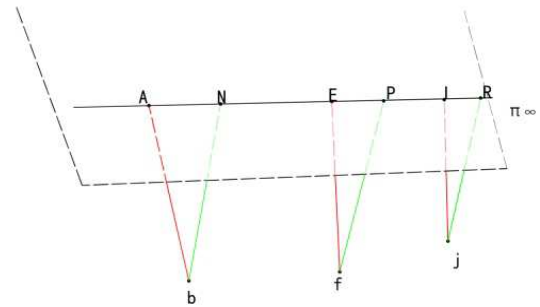


Fig-6: Wrench graph of 3-PRS PMs

4.4 Superbrackets Decomposition of 3-PRS Parallel Mechanism

The superbrackets in Eq. (19) and Eq. (20) are generally developed into 24 combinations of linear monomials [1, 3]. The Graphic User Interface (GUI) provided by Stéphane Caro et al [19] gives the reduced form of the superbrackets decomposition and the singularity conditions.

For 3-PRS PMs:

$$[aB, cB, eF, gF, iJ, kJ] =$$

$$+[aBce][BgFJ][FikJ]$$

$$-[aBcF][BgFJ][eikJ] \quad (21)$$

$$-[aBcg][BeFJ][FikJ]$$

$$+[aBcF][BeFJ][gikJ]$$

For 3-PRS PMs:

$$[Ab, bN, Ef, fP, Ij, jR] =$$

$$-[AbNE][bfPj][fIjR]$$

$$+[AbNf][bfPj][EIjR] \quad (22)$$

$$+[AbNf][bEjf][PIjR]$$

$$-[AbNP][bEjf][fIjR]$$

5. SINGULARITY CONDITIONS AND ASSOCIATED MOTIONS

Kinematically singularities arise when the moving platform becomes uncontrollable and actuators are useless. Algebraically the determinant of Jacobian matrix becomes zero. Geometrically in GCA language the six Plücker coordinate lines are linearly dependent. The superbracket which represents the determinant of the Jacobian matrix vanish. The analysis of the vanishing points of the superbracket is the singularity conditions. The GUI provides the solutions [19]

1st case : $[aB, cB, eF, gF, iJ, kJ] = 0$ when $[xBFJ] = 0$ or $aBc \wedge ikJ \wedge eF$

2nd case : $[Ab, bN, Ef, fP, Ij, jR] = 0$ when $AbN \wedge bfj \wedge IjR \wedge EPf$

5.1 Interpretation of the Associated Motion

According to our chosen mechanism and adopted representation, each limb l_i is identified by F_i and W_i thus:

1st case . $x = \{a, b, c, e, g, i, k\}$, in singular configuration: $[xBFJ] = 0$ or $aBc \wedge ikJ \wedge eF$

If $[aBFJ] = 0$ then $aB \wedge FJ$ means F collinear to aB or J collinear to aB .Since F and J are respectively the unique intersecting point at infinity of parallel lines eF, gF and iJ, kJ then F collinear to aB or J collinear to aB means aB intersect also eF, gF, iJ and kJ at F as infinity point. Since two parallel lines intersect each other at a unique point at infinity it is obvious that aB is parallel to eF, gF, iJ and kJ .This means F_1 parallel to $F_2W_2F_3W_3$.Similarly if $[cBFJ] = 0$ with the same processes, W_1 Parallel to F_2, W_2, F_3, W_3 .For l_1 , the singularity occurs when F_1 or W_1 parallel to F_2, F_3, W_2, W_3 .It is obvious to deduce for l_i .If $aBc \wedge ikJ \wedge eF$ means the line F_2 crosses the intersection line of the two planes α_1 and α_3

For 3- PRS PMs , the singularity conditions are:

- The robot reach in singularity when the actuated force or the constraint force of any l_k is parallel to either a constraint force or an actuated force of l_k with $k \neq k'$
- Singular motion arises when the common actuated force of l_1 and l_3 crosses the constraint force of l_2 .

2nd case. The singular configuration arises when the four planes $\beta_1, \beta_2, \beta_3$ and bff intersect at a unique point.

For 3- PRS PMs the singularity condition is:

- The common actuated wrench of the three limbs crosses the plane formed by the three constraint forces at a point and then the robot manipulator is in a plane configuration.

Finally a comparative analysis provides that a common singularity condition occurs when the three constraints forces lie on a common plane and intersect at the particular point :the robot manipulator reaches in a plane configuration

6. CONCLUSIONS

This paper presented the singularity analysis of 3-PRS PMs with variable actuated joint using Grassmann-Cayley Algebra. We determined geometrically the GWS by reciprocal screws approach and used an alternative coordinate free method namely GCA to detect the singularity conditions. The graph of the GWS was obtained by introducing the notion of join and meet operator in projective space. An efficient method to investigate singular configuration from GWS was given by GCA. Based on this approach a double and a single singularity conditions

respectively for 3-PRS and 3-PRS PMs were derived. These conditions contained all general and specific critical poses. The comparative analysis of these result suggested that both position and orientation of the actuator have to be integrated in the conceptual design stage in order to optimize the rigidity frame of the mechanism. It was deduced that a common singularity condition occurs when the three constraints forces lie on a common plane and intersect at the particular point where the robot manipulator reaches in a plane configuration.

Even though the current research did not cover all variety and complexity of singularity configuration of 3-PRS PMs, these results may motivated further research question, especially toward the singularity conditions of serial manipulator using the GCA approach since each limb l_i of 3-PRS PMs consists of serial kinematic chains. Elsewhere the powerful tools of GCA are also useful in robotics vision and image modeling.

ACKNOWLEDGEMENT

I'm grateful to Professor Stéphane Caro for his inspiring advice, to the editor and reviewers for their support.

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