PERFORMANCE ANALYSIS OF PRODUCTION LINE WITH BERNOULLI'S MACHINES

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Abstract

In flexible manufacturing environments, the performance of a production system is often affected by the sequence of operation. While performance evaluation, improvement and lean design of production system have been studied extensively, the joint effect of productivity and quality parameters on operation sequencing remains practically unexplored. Indeed, determining the optimal operation sequence has significant implication from both theoretical and practical perspectives. In this work the frame work of Bernoulli reliability and quality models, we develop effective indicator that area simple and easy to implement in practice to determine the optimal operation sequence that maximize the system production rate.

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1. INTRODUCTION

Performance evaluation, continuous improvement, lean design, and bottleneck identification of production systems have been studied extensively during the last 50 years . In this studies , it is often assumed that all parts are posed according to predetermine sequence of operations. The effect of the operation sequence (OS) on the performance of production systems, however, has not been systematically investigated. Indeed in flexible manufacturing environments, one type of final product can be produced by several sequence of operations. Thus, determining the optimal sequence of operations to be performed by all parts is of importance for the entire production processes. Practical example of operation sequencing can be widely found in product assembly [(3)], as well as in machining processes [(4)].

The development based on a recently developed improvement methodology for production system with quality quantity couple operation where by increasing the probability to complite a job during a cycle time leads to decreasing job quality. Production lines with unreliable machines usually contain finite capacity buffers intended to attenuate mutual perturbations of the machines due to breakdowns. It is well known that the capacity of the buffers should be as small as possible, that is, lean.

In the present work the production machine considered are unreliable machine and have non perfect quality. The buffers used between machines have finite capacity. In the several production lines the machines follow Bernoulli reliability and quality model.

The production system with unreliable machines, non perfect quality and finite buffer are considered. The serial production lines With M machines having Bernoulli reliability and quality models. According to these models, machine m_i, i ϵ {1,…..,M }, when neither blocked nor starved, produces a part during a cycle time with probability p_i and fails to do so with probability (1 - p_i); in addition, for each part produced by this machine, it is of good quality with probability g_i and is defective with probability (1- g_i). Parameters p_i and g_i are referred to as the efficiency and quality of machine m_i, respectively. The Bernoulli reliability model is applicable to manufacturing operations where the unscheduled downtime is comparable to the cycle time (e.g., assembly and painting operations, conveyor pallet jams, etc.). The Bernoulli quality model is applicable when the defects are due to random and uncorrelated events (e.g., dust and scratches, etc.).

2. MODELING OF PRODUCTION LINE

p_1,g_1 N p_2g_2
CR_{1-2} -----→◯----→[]----→◯----→PR_{1-2}
m_1 ↓ b m_2 ↓
SR^1_{1-2} SR^2_{1-2}

Fig: 1.1 sequence $m_1 – m_2$

→◯→[]→◯→[]→…….→◯→[]→◯→
m_1 b_1 m_2 b_2 $m_{M−1}$ $b_{M−1}$ m_M

Fig: 1.2 Serial production line.

The following assumptions are consider for a production system with M machine shown in Fig. (i)The system consists of M machines arranged serially, and M−1 buffers separating each consecutive pair of machines.(ii)The machines have identical cycle time Tc. The time axis is slotted with the slot duration Tc. Machines begin operating at the beginning of each time slot.(iii)Each buffer is characterized by its capacity, $N_i < \infty, 1 \le i \le M-1$.(iv)Machine i is starved during a time slot if buffer i−1 is empty at the beginning of the time slot. Machine 1 is never starved for parts.(v) Machine i is blocked during a time slot if buffer i has Ni parts at the beginning of the time slot, and machine i+1 fails to take a part during the time slot. Machine M is never blocked by ready goods buffer.(vi)Machines obey the Bernoulli reliability model, that is, machine i, i = 1, . . . ,M, being neither blocked nor starved during a time slot, produces a part with probability *pi* and fails to do so with probability 1− pi. Parameter pi is referred to as the efficiency of machine i.

3. MATHEMATICAL EXPRESSION FOR PERFORMANCE MEASURES

Production Rate (PR): The production rate of a flexible manufacturing system may be defined as the average number of parts produced by the downstream machine per cycle time.

Consumption Rate (CR): It is defined as the average number of raw material consumed by the upstream machine per cycle time.

Scrap Rate (SR): Scrap rate means the average number of parts scrap by the machines per cycle time.

Work-in-process (WIP): work-in-process defined as the average number of parts in buffer b. i.e., WIP, is given by

$$PR_{1-2} = p_2g_2 [\, 1 - Q\,(p_1g_1 , p_2 , N)]$$

$$CR_{1-2} = \frac{PR_{1-2}}{g_1g_2} = \frac{p_2 [\, 1 - Q\,(p_1g_1 , p_2 , N)]}{g_1}$$

$$SR^1_{1-2} = CR_{1-2}\,(1 - g_1) = \frac{P_2\,(1 - g_1)[\, 1 - Q(p_1g_1,p_2,N)]}{g_1}$$

$$SR^2_{1-2} = CR_{1-2}\, g_1\,(1 - g_2) = p_2(1 - g_2)[1 - Q\,(p_1g_1,p_2,N)]$$

$$SR_{1-2} = SR_{1-2} + SR_{2-1}$$

If $p_1g_1 = p_2 = p$ then the equation of WIP given below

$$WIP = \frac{.\,N\,(N + 1)}{2\,(N + 1 - p)}$$

If $p1g1 \neq p_2$, then the equation of WIP given below:

$$WIP_{1-2} = \frac{p_1g_1 \left[\frac{1 - \alpha^N (p_1g_1,p_2)}{1 - \alpha\,(p_1g_1,p_2)} - N\alpha^N (p_1g_1,p_2)\right]}{P_2 - p_1g_1\, \alpha^N (p_1g_1,p_2)}$$

Where $i - j \in \{ 1 - 2 , 2 - 1\}$ denotes Operation Sequence $m_i - m_j$ and if $x \neq y$ then

$$Q\,(x, y, N) = \frac{(1 - x)[\, 1 - \alpha(x , y)]}{1 - x/y\, \alpha^N (x , y)}$$

If x = y then the equation is :

$$Q\,(x, y, N) = \frac{1 - x}{N + 1 - x}$$

$$\alpha\,(x , y) = \frac{x\,(1 - y)}{y\,(1 - x)}$$

Note that Q (x, y, N) is equal to the probability that the buffer is empty in the steady state of a two machine with Bernoulli line with upstream machine efficiency x, downstream machine efficiency y, buffer capacity N , and no quality issues . For a Bernoulli line define by (i) to (ix) under Operation Sequence $m_i - m_j$, since p_ig_I is the “effective” probability that the upstream machine sends a part to the buffer during a time slot, it follows immediately that $Q\,(p_ig_i,p_j,N)$ is equal to the probability of empty buffer in the steady state for the system consider in this paper.

Idea of the aggregation: Consider the M-machine line and aggregate the last two machines, m_{M-1}, and m_M, into a single Bernoulli machine denoted as m^b_{M-1}, where b stands for backward aggregation (see Figure 2.1). The Bernoulli parameter, p^b_{M-1}, of this machine is assigned as the production rate of the aggregated two-machine line, calculated using the first expression. Next, aggregate this machine, i.e., m^b_{M-1}, with m_{M-2} and obtain another aggregated machine, m^b_{M-2}.The forward aggregation define that the first machine, m_1, with the aggregated version of the rest of the line, i.e., with m^b_2. This results in the aggregated machine, denoted as m^f_2 , where f stands for forward aggregation (see Figure 2.2). This process is continue to the last one which will forward aggregrated. We show below that the steady states of this recursive procedure lead to relatively accurate estimates of the performance measures for the M-machine line.

p_1 N_1 p_2 N_2 p_{M-2} N_{M-2} p_{M-1} N_{M-1} p_M

→◯→[]→◯→[]→……→◯→[]→◯→[]→◯→

m_1 b_1 m_2 b_2 m_{M-2} b_{M-2} m_{M-1} b_{M-1} m_M

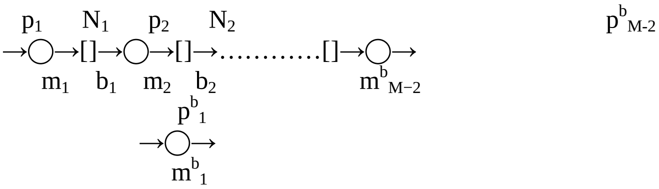


Fig 2.1 Backward aggregation

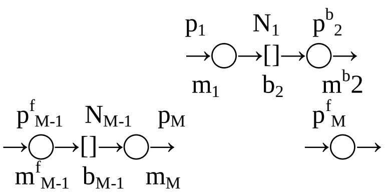


Fig 2.2 Forward aggregation

Blockages and starvations: Since these probabilities must evaluate blockages and starvations of the real, rather than aggregated, machines, taking into account expressions, the estimates of these performance measures, BLi and STi, are introduced as follows:

$BLi = p_i Q(p^b_{i+1};\ p^f_i\ ;N_i);\ i = 1,\ldots\ldots..M - 1;$

$STi = p_i Q(p^f_{i-1};\ p^b_i\ ;N_{i-1});\ i = 2,\ldots\ldots.M:$

4. RESULT AND ANALYSIS OF PERFORMANCE MEASUREMENT:

Table 1 Performance measure of model I. (m_1-m_2)

CASE	p_1	p_2	N	PR	CR	SR	WIP
I	0.3	0.5	2	0.275	0.275	0.55	0.71
	0.4	0.6		0.372	0.372	0.744	0.8
	0.6	0.8		0.576	0.576	1.152	0.91

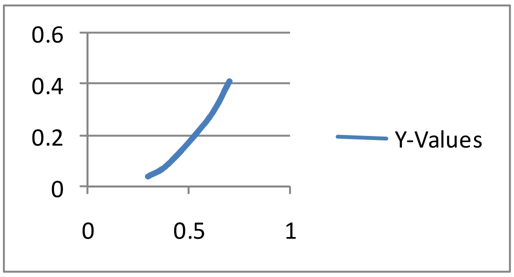
Table 2 Performance measure of model II (m_1-m_2)

CASE	p_1	p_2	g_1	g_2	N	PR	CR	SR	WIP
II	0.3	0.3	0.3	0.5	4	0.04	0.29	0.25	0.38
	0.4	0.4	0.4	0.6		0.09	0.39	0.29	0.55
	0.6	0.6	0.6	0.8		0.28	0.59	0.30	0.93

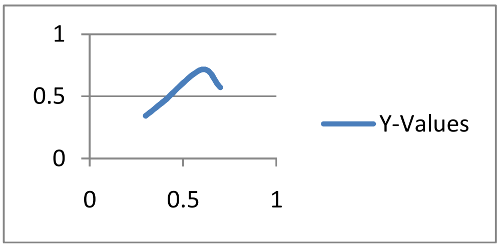
Table 3 Performance measure of model II (m_2-m_1)

CASE	p_1	p_2	g_1	g_2	N	PR	CR	SR	WIP
II	0.3	0.3	0.5	0.3	4	0.04	0.2	0.2	0.7
	0.4	0.4	0.6	0.4		0.09	0.3	0.2	0.9
	0.6	0.6	0.8	0.6		0.2	0.5	0.3	1.4

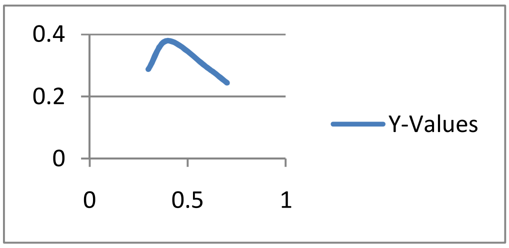
Performance Measure for g1 vs PR, WIP, CR, SR



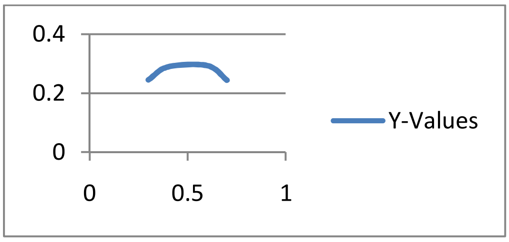
PR vs g1



WIP vs g1



CR vs g1



SR vs g1

CONCLUSIONS AND FUTURE WORK:

When the machine have different efficiency but quality does not exits the higher efficiency machine should be placed upstream to achieved higher production rate. In the same sense the results obtain from previous chapter when the machines have efficiency and quality parameter both, the lower quality machine placed upstream to achieved higher production rate. If the machine efficiency same it is shown that selecting the optimal operation sequence increased the production rate by 6% with typically reduction of work-in-processes by 15%

The future of the system theoretic properties of production system includes: (1)Extension of the results obtained to production system with machines having other reliability and quality model e.g., exponential, Webull, Gamma, Lognormal etc.(2)Generalization of the results for production system with different topologies, e.g., assembly system, closed line, re-entrant line etc.(3)Investigation of the effect of operation sequencing on the trade off among deferent performance measures.(4)Generalization of the results for small volume job-shop production environment with high product varity.

ACKNOWLEDGMENTS

The author is very grateful for the valuable comments and suggestions, especially on the proof of Lemma A.1 by the anonymous reviewers.

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