COMPUTATIONAL MODEL TO DESIGN CIRCULAR RUNNER CONDUIT FOR PLASTIC INJECTION MOULD

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Abstract

Analytical solution quest for viscoelastic shear thinning fluid flow through circular conduit is a matter of great prominence as it directly evolvesmost efficient criteria to investigate various responses of independent parameters. Envisaging this facet present endeavour attempts to develop a computational model for designing runner conduit lateral dimension in a plastic injection mould through which thermoplastic melt gets injected. At outset injection phenomenon is represented by governing equations on the basis of mass, momentum and energy conservation principles [1]. Embracing Hagen Poiseuille flow problem analogous to runner conduit injection the manuscript uniquely imposes runner conduit inlet and outlet boundary conditions along with relative to appropriate assumptions; governing equations evolve a computation model as criteria for designing. To overwhelm Non-Newtonian's abstruse Weissenberg-Rabinowitsch correction factor has been adopted byaccommodating thermoplastic melt behaviour towards the final stage of derivation. The resulting final computational model is believed to express runner conduit dimensions as a function of available type of injection moulding machine specifications, characteristics of thermoplastic melt and required features of component being moulded. Later the equation so obtained has been verified by using dimensional analysis method.

Keywords: *Computational modelling, Runner conduit design, Plastic injection mould, Hagen-Poiseuille flow*

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1. INTRODUCTION

Mathematical models for polymer processingisby and large deterministic *(as are the processes)*typically transport based unsteady *(cyclic process)* and distributed parameter. Particularly complex thermoplastic melt injection mould system was broken into clearly defined subsystems for modelling. Runner dimension plays a vital role in the idealization of an injection mould and hence deriving a computational model to determine runner dimension was of immense need. There have been a fairly large number of Newtonian apprehensions for which a closed form analytical solution are prevailing. However, for non-Newtonian apprehension fluids such as thermoplastic melts exact solutions are rare. In general, non-Newtonian melt injection behaviours are more complicated and subtle compared to Newtonian fluid circumstances[2].Developing a model for complex process like thermoplastic melt (which is non-Newtonian highly viscoelastic, shinning type fluid) injection through runner conduit (circular conduit) requires a clear objective definition. Hereto the sole objective of this derivation is to obtain a runner diameter design criteria as a function of injection moulding machine specifications used for the purpose of injection, type of thermoplastic melt being injected and features of the component being moulded as known parameters.

The physical process of thermoplastic melt injection through runner conduit in a typical plastic injection mould is represented by a set of expressions, which insights adept acquaintance of in-situ physical phenomena that occurs within actualprocessing[3]. Mathematical modelling involves assembling sets of various mathematical equations, which originates from engineering fundamentals, such as the material, energy and momentum balance equations [4]. Herein representative mathematical equations attempts to computationally model the interrelations that govern actual processing situate[5]. More complex the mathematical model, the more accurately it mimics the actual process [6]. Towards obtaining an analytical solution we must first simplify the balance equations, although the resulting equations are fundamental, rigorous,nonlinear, collective, complex and difficult to solve [2]. Therefore, the resulting equations are sufficiently simplified by considering appropriate assumptions that correspond to those the actual processing interrelations between variables and parameters. These assumptions are geometric simplifications, initial conditions and physical assumptions, such as isothermal systems, isotropic materials as well as material models, such as Newtonian, elastic, visco-elastic, shear thinning, or others [6]. Finally boundary conditions like velocity and temperature profiles are applied to simplifying the resulting equations completely [7]. Further meticulous rearranging of the functions leads us to a complete computational model that enables design engineers to confidently design superior moulds at bonus costs thus idealizing the process from mould design perspective [8].

2. GOVERNING EQUATIONS

The phenomenon ofactual melt injection through the runner conduit is represented by governing equations that discriminately appreciate compressibility, unsteadiness and non-Newtonian factors. Hence are highly rigorous, nonlinear, comprehensive, complex and difficult to solve. Few explicit assumptions are made herein are:

(a) Thermoplastic viscosity remain consistent

(b)Body forces are neglected when compared to that of viscous forces

(c) Thermoplastic melt thermal conductivity is considered constant.

2.1 Equation of Continuity (Mass Balance)

$$\frac{\partial\rho}{\partial t}+\frac{\rho U_r}{r}+\frac{\partial(\rho U_r)}{\partial r}+\frac{1}{r}\frac{\partial(\rho U_\theta)}{\partial\theta}+\frac{\partial(\rho U_\xi)}{\partial\xi}=0 \quad (1)$$

$$\frac{\partial\rho}{\partial t}+\nabla.\left(\rho\vec{U}\right)=0 \quad (2)$$

Substituting $\nabla.\left(\rho\vec{U}\right)=\rho\nabla.\vec{U}+\nabla\rho.\vec{U}$, in equation (2),

$$\frac{\partial\rho}{\partial t}+\rho\nabla.\vec{U}+\nabla\rho.\vec{U}=0 \quad (3)$$

$$\frac{d\rho}{dt}+\rho\nabla.\vec{U}=0$$

$$\frac{d(\log\rho)}{dt}+\nabla.\vec{U}=0$$

$$\nabla.\vec{U}=-\frac{d(\log\rho)}{dt} \quad (4)$$

2.2 Equations of Motion

$$\rho\left(\frac{\partial U_r}{\partial t}+U_r\frac{\partial U_r}{\partial r}+\frac{U_\theta}{r}\frac{\partial U_r}{\partial\theta}+U_\xi\frac{\partial U_r}{\partial\xi}-\frac{U_\theta^{\,2}}{r}\right)=-\frac{\partial P}{\partial r}+\left(\frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r}+\frac{1}{r}\frac{\partial(\tau_{\theta r})}{\partial\theta}+\frac{\partial(\tau_{\xi r})}{\partial\xi}-\frac{\tau_{\theta\theta}}{r}\right) \quad (5)$$

$$\rho\left(\frac{\partial U_\theta}{\partial t}+U_r\frac{\partial U_\theta}{\partial r}+\frac{U_\theta}{r}\frac{\partial U_\theta}{\partial\theta}+U_\xi\frac{\partial U_\theta}{\partial\xi}+\frac{U_rU_\theta}{r}\right)=-\frac{1}{r}\frac{\partial P}{\partial\theta}+\left(\frac{1}{r^2}\frac{\partial(r^2\tau_{r\theta})}{\partial r}+\frac{1}{r}\frac{\partial(\tau_{\theta\theta})}{\partial\theta}+\frac{\partial(\tau_{\xi\theta})}{\partial\xi}+\frac{\tau_{\theta r}-\tau_{r\theta}}{r}\right) \quad (6)$$

$$\rho\left(\frac{\partial U_\xi}{\partial t}+U_r\frac{\partial U_\xi}{\partial r}+\frac{U_\theta}{r}\frac{\partial U_\xi}{\partial\theta}+U_\xi\frac{\partial U_\xi}{\partial\xi}\right)=-\frac{\partial P}{\partial\xi}+\left(\frac{1}{r}\frac{\partial(r\tau_{r\xi})}{\partial r}+\frac{1}{r}\frac{\partial(\tau_{\theta\xi})}{\partial\theta}+\frac{\partial(\tau_{\xi\xi})}{\partial\xi}\right) \quad (7)$$

The Newtonian constitutive relations are

$$\tau_{rr}=-\mu\left(2\frac{\partial U_r}{\partial r}-\frac{2}{3}\left(\nabla.\vec{U}\right)\right) \quad (8)$$

$$\tau_{\theta\theta}=-\mu\left(2\left(\frac{1}{r}\frac{\partial U_\theta}{\partial\theta}+\frac{U_r}{r}\right)-\frac{2}{3}\left(\nabla.\vec{U}\right)\right) \quad (9)$$

$$\tau_{\xi\xi}=-\mu\left(2\frac{\partial U_\xi}{\partial\xi}-\frac{2}{3}\left(\nabla.\vec{U}\right)\right) \quad (10)$$

$$\tau_{r\theta}=\tau_{\theta r}=-\mu\left(r\frac{\partial}{\partial r}\left(\frac{U_\theta}{r}\right)+\frac{1}{r}\frac{\partial U_r}{\partial\theta}\right) \quad (11)$$

$$\tau_{r\xi}=\tau_{\xi r}=-\mu\left(\frac{\partial U_\xi}{\partial r}+\frac{\partial U_r}{\partial\xi}\right) \quad (12)$$

$$\tau_{\theta\xi}=\tau_{\xi\theta}=-\mu\left(\frac{\partial U_\theta}{\partial\xi}+\frac{1}{r}\frac{\partial U_\xi}{\partial\theta}\right) \quad (13)$$

$$\nabla.\vec{U}=\frac{1}{r}\frac{\partial}{\partial r}(rU_r)+\frac{1}{r}\frac{\partial U_\theta}{\partial\theta}+\frac{\partial U_\xi}{\partial\xi} \quad (14)$$

Substituting Newtonian constitutive relation Eqn. (8) to (14) in equations of motion Eqn. (5) to (7) we get,

$$\rho\left(\frac{\partial U_r}{\partial t}+U_r\frac{\partial U_r}{\partial r}+\frac{U_\theta}{r}\frac{\partial U_r}{\partial\theta}+U_\xi\frac{\partial U_r}{\partial\xi}-\frac{U_\theta^{\,2}}{r}\right)=-\frac{\partial P}{\partial r}+\frac{\partial}{\partial r}\left[\mu\left(2\frac{\partial U_r}{\partial r}-\frac{2}{3}\nabla.\vec{U}\right)\right]+\frac{2\mu}{r}\left(\frac{\partial U_r}{\partial r}-\frac{1}{r}\frac{\partial U_\theta}{\partial\theta}-\frac{U_r}{r}\right)+\frac{1}{r}\frac{\partial}{\partial\theta}\left[\mu\left(\frac{1}{r}\frac{\partial U_r}{\partial\theta}+\frac{\partial U_\theta}{\partial r}-\frac{U_\theta}{r}\right)\right]+\frac{\partial}{\partial\xi}\left[\mu\left(\frac{\partial U_r}{\partial\xi}+\frac{\partial U_\xi}{\partial r}\right)\right] \quad (15)$$

$$\rho\left(\frac{\partial U_\theta}{\partial t}+U_r\frac{\partial U_\theta}{\partial r}+\frac{U_\theta}{r}\frac{\partial U_\theta}{\partial \theta}+U_\xi\frac{\partial U_\theta}{\partial \xi}+\frac{U_r U_\theta}{r}\right)$$
$$=-\frac{1}{r}\frac{\partial P}{\partial \theta}+\frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{2}{r}\frac{\partial U_\theta}{\partial \theta}+2\frac{U_\theta}{r}-\frac{2}{3}\nabla.\vec{U}\right)\right]$$
$$+\frac{2\mu}{r}\left(\frac{1}{r}\frac{\partial U_r}{\partial \theta}+\frac{\partial U_\theta}{\partial r}-\frac{U_\theta}{r}\right)+\frac{\partial}{\partial r}\left[\mu\left(\frac{1}{r}\frac{\partial U_r}{\partial \theta}+\frac{\partial U_\theta}{\partial r}-\frac{U_\theta}{r}\right)\right]$$
$$+\frac{\partial}{\partial \xi}\left[\mu\left(\frac{1}{r}\frac{\partial U_\xi}{\partial \theta}+\frac{\partial U_\theta}{\partial \xi}\right)\right] \quad (16)$$

$$\rho\left(\frac{\partial U_\xi}{\partial t}+U_r\frac{\partial U_\xi}{\partial r}+\frac{U_\theta}{r}\frac{\partial U_\xi}{\partial \theta}+U_\xi\frac{\partial U_\xi}{\partial \xi}\right)$$
$$=-\frac{\partial P}{\partial \xi}+\frac{\partial}{\partial \xi}\left[\mu\left(2\frac{\partial U_\xi}{\partial \xi}-\frac{2}{3}\nabla.\vec{U}\right)\right]+\frac{\mu}{r}\left(\frac{\partial U_r}{\partial \xi}+\frac{\partial U_\xi}{\partial r}\right)$$
$$+\frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r}\frac{\partial U_\xi}{\partial \theta}+\frac{\partial U_\theta}{\partial \xi}\right)\right]+\frac{\partial}{\partial r}\left[\mu\left(\frac{\partial U_r}{\partial \xi}+\frac{\partial U_\xi}{\partial r}\right)\right] \quad (17)$$

2.3 Energy Balance Equation

$$\rho\frac{d\hat{u}}{dt}+P\left(\nabla.\vec{U}\right)=\nabla.\left(k\nabla T\right)+\overleftrightarrow{\tau}:\nabla\vec{U} \quad (18)$$

From thermodynamic relation we have,

$$\frac{d\hat{u}}{dT}=C_v,\quad \Rightarrow d\hat{u}=C_v dT$$

Hence Eqn. (18) simplifies as

$$\rho\, C_v\frac{dT}{dt}+P\left(\nabla.\vec{U}\right)=\nabla.\left(k\nabla T\right)+\overleftrightarrow{\tau}:\nabla\vec{U} \quad (19)$$

$$\rho\, C_v\frac{dT}{dt}+P\left(\nabla.\vec{U}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial \xi}\left(k\frac{\partial T}{\partial \xi}\right)$$
$$+2\mu\left(\begin{array}{l}\left(\frac{\partial U_r}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial U_\theta}{\partial \theta}+\frac{U_r}{r}\right)^2+\left(\frac{\partial U_\xi}{\partial \xi}\right)^2\\+\frac{1}{2}\left(\frac{\partial U_\theta}{\partial r}-\frac{U_\theta}{r}+\frac{1}{r}\frac{\partial U_r}{\partial \theta}\right)^2+\frac{1}{2}\left(\frac{1}{r}\frac{\partial U_\xi}{\partial \theta}+\frac{\partial U_\theta}{\partial \xi}\right)^2\\+\frac{1}{2}\left(\frac{\partial U_r}{\partial \xi}+\frac{\partial U_\xi}{\partial r}\right)^2-\frac{1}{3}\left(\nabla.\vec{U}\right)^2\end{array}\right) \quad (20)$$

Equation of Entropy

$$T\rho\frac{dS}{dt}=\nabla.\left(k\nabla T\right)+\overleftrightarrow{\tau}:\nabla\vec{U} \quad (21)$$

We can apprehend from Eqn. (19) and (21) that entropy is already present quantitatively in energy equation Eqn. (19) itself.

3. SOLUTION FOR GOVERNING EQUATIONS

Geometrical conditions

a) Analogous to pipe flow transverse velocity components could be considered almost zero *(geometrical constraint)* i.e., $U_r=U_\theta=0$, accordingly transverse pressure gradience would also be zero. Hence $\frac{\partial P}{\partial r}=\frac{\partial P}{\partial \theta}=0$

b) Since runner cross section is axis-symmetric profile, tangential gradience could be considered zero i.e.,
$$\frac{\partial}{\partial \theta}=0$$

c) Only lateral gradience of temperature is considered because radial gradience far exceeds than other two directionsi.e.,
$$\frac{\partial T}{\partial \theta}\approx\frac{\partial T}{\partial \xi}\approx 0,\frac{\partial T}{\partial r}\neq 0$$

Upon substituting above (a) to (c), governing equations reduce to,

$$\frac{d}{dt}\left(\log\rho\right)=-\frac{\partial U_\xi}{\partial \xi} \quad (22)$$

$$\rho\left(\frac{\partial U_\xi}{\partial t}+U_\xi\frac{\partial U_\xi}{\partial \xi}\right)=-\frac{\partial P}{\partial \xi}+\frac{4\mu}{3}\frac{\partial}{\partial \xi}\left(\frac{\partial U_\xi}{\partial \xi}\right)+\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_\xi}{\partial r}\right) \quad (23)$$

$$\rho C_v\frac{\partial T}{\partial t}+P\frac{\partial U_\xi}{\partial \xi}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{4}{3}\mu\left[\left(\frac{\partial U_\xi}{\partial \xi}\right)^2+\frac{3}{4}\left(\frac{\partial U_\xi}{\partial r}\right)^2\right] \quad (24)$$

Substituting Eqn. (22) in Eqn. (23) and Eqn. (24) we get

$$\rho\left(\frac{\partial U_\xi}{\partial t}-U_\xi\frac{d}{dt}\left(\log\rho\right)\right)$$
$$=-\frac{\partial P}{\partial \xi}-\frac{4\mu}{3}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}\left(\log\rho\right)\right)+\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_\xi}{\partial r}\right) \quad (25)$$

$$\rho C_v\frac{\partial T}{\partial t}-P\frac{d}{dt}\left(\log\rho\right)$$
$$=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{4}{3}\mu\left[\left(-\frac{d}{dt}\left(\log\rho\right)\right)^2+\frac{3}{4}\left(\frac{\partial U_\xi}{\partial r}\right)^2\right] \quad (26)$$

3.1 Hagen-Poiscuille Velocity Profile

Thermoplastic melt transportation studies are critical fordesigninglateral dimension of runner conduits as well as injectionbehaviours [9]. Cognising axial velocity of non-Newtonian, shear thinning thermoplastic melt injection can conversely enable conduit dimension determination, because Axial velocity component is a function of conduit radius through which melt is being injected [10]. Since thermoplastic melt injection through circular runner conduit occurs at creep level Reynolds number, the flow is fully developed and laminar. Hence well-established incompressible laminar flow Hagen-Poiscuille velocity profile can be considered analogous to represent velocity for thermoplastic melt injection through circular conduits. Parabolic velocity profile through circular conduits varies from core to wall in such a way that core velocity would be maximum while almost zero at the rigid stationary wall. Accordingly velocity profile would be,

$$U_{\xi}=\frac{-1}{4\mu}\frac{\partial P}{\partial \xi}\left(R^2-r^2\right) \tag{27}$$

Although Eqn. (27) neglects compressibility factor, herein we retain it right from equation of continuity to appreciate actual expandability and compressibility phenomenon through each cycle typically involved in injection moulding. Substituting Eqn. (27) in Eqn. (25) we get,

$$\rho\left(\frac{\partial}{\partial t}\left(\frac{-1}{4\mu}\frac{\partial P}{\partial \xi}\left(R^2-r^2\right)\right)-\left(\frac{-1}{4\mu}\frac{\partial P}{\partial \xi}\left(R^2-r^2\right)\right)\frac{d}{dt}(\log\rho)\right)$$
$$=-\frac{\partial P}{\partial \xi}-\frac{4\mu}{3}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)+\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{-1}{4\mu}\frac{\partial P}{\partial \xi}\left(R^2-r^2\right)\right)\right)$$

$$-\frac{\rho\left(R^2-r^2\right)}{4\mu}\left(\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)+\frac{\rho\left(R^2-r^2\right)}{4\mu}\left(\frac{\partial P}{\partial \xi}\frac{d}{dt}(\log\rho)\right)$$
$$=-\frac{\partial P}{\partial \xi}-\frac{4\mu}{3}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)+\frac{\partial P}{\partial \xi}$$

$$\left(R^2-r^2\right)=-\left(\frac{\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)}\right)$$

$$r^2-R^2=\left(\frac{\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)}\right)$$

$$r^2=\left(\frac{\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)}\right)+R^2 \tag{28}$$

Similarly substitute Eqn.(27) in Eqn.(26) we get,

$$\rho C_v\frac{\partial T}{\partial t}=P\frac{d}{dt}(\log\rho)+\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)$$
$$+\frac{4}{3}\mu\left[\left(-\frac{d}{dt}(\log\rho)\right)^2+\frac{3}{4}\left(\frac{\partial}{\partial r}\left(\frac{-1}{4\mu}\frac{\partial P}{\partial \xi}\left(R^2-r^2\right)\right)\right)^2\right]$$

$$\rho C_v\frac{\partial T}{\partial t}=P\frac{d}{dt}(\log\rho)+\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2$$
$$+\frac{r^2}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2$$

$$r^2=-\left(\frac{\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2+P\frac{d}{dt}(\log\rho)+\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)-\rho C_v\frac{\partial T}{\partial t}}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2}\right) \tag{29}$$

Now consideringHagen-Poiscuille temperature profiles for thermoplastic melt injection through circular conduit

$$T_{max}-T_w=\frac{\mu}{4k}\left(U_{max}\right)^2 \tag{30}$$

Eqn. 30featuresmaximum temperature at conduit core with an almost constant streaming gradience, while in actual injection consequent to concurrent cooling melt temperature reduces non-linearly, despite cooling melt streams keep moving ahead. Hence temperature profile is appropriately modified as,

$$T-T_w=\frac{\mu}{4k}(U)^2 \tag{30a}$$

Where $U=\frac{-1}{4\mu}\frac{\partial P}{\partial \xi}\left(R^2-r^2\right)$

T_w = wall temperature

Substituting Eqn. (27) in Eqn. (30a) we get,

$$T=\frac{\mu}{4k}\left(\frac{-1}{4\mu}\frac{\partial P}{\partial \xi}\left(R^2-r^2\right)\right)^2+T_w$$

$$T=\frac{\mu}{4k}\left(\frac{1}{16\mu^2}\left(\frac{\partial P}{\partial \xi}\right)^2\left(R^2-r^2\right)^2\right)+T_w$$

$$T=\frac{\left(R^2-r^2\right)^2}{64\mu k}\left(\frac{\partial P}{\partial \xi}\right)^2+T_w \qquad (31)$$

Now substituting Eqn. (31) in Eqn. (29), we get

$$r^2=-\frac{\left(\begin{array}{l}\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2+\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial}{\partial r}\left(\frac{\left(R^2-r^2\right)^2}{64\mu k}\left(\frac{\partial P}{\partial \xi}\right)^2+T_w\right)\right)\\ +P\frac{d}{dt}(\log\rho)-\rho C_v\frac{\partial}{\partial t}\left(\frac{\left(R^2-r^2\right)^2}{64\mu k}\left(\frac{\partial P}{\partial \xi}\right)^2+T_w\right)\end{array}\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2}$$

As wall temperature variation throughout the cycle is very nominal, it can be considered to be almost constant. Thus applying the condition, equation reduces to,

$$r^2=-\frac{\left(\begin{array}{l}\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2+\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{64\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\frac{\partial}{\partial r}\left(\left(R^2-r^2\right)^2\right)\right)\\ +P\frac{d}{dt}(\log\rho)-\frac{\rho C_v\left(R^2-r^2\right)^2}{64\mu k}\frac{\partial}{\partial t}\left(\left(\frac{\partial P}{\partial \xi}\right)^2\right)\end{array}\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2}$$

$$r^2=-\frac{\left(\begin{array}{l}\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2+\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{64\mu}\left(\frac{\partial P}{\partial \xi}\right)^2 2\left(R^2-r^2\right)(-2r)\right)\\ +P\frac{d}{dt}(\log\rho)-\frac{\rho C_v\left(R^2-r^2\right)^2}{64\mu k}\frac{\partial}{\partial t}\left(\left(\frac{\partial P}{\partial \xi}\right)^2\right)\end{array}\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2}$$

$$r^2=-\frac{\left(\begin{array}{l}\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2+\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\left(r^4-R^2r^2\right)}{16\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\right)\\ +P\frac{d}{dt}(\log\rho)-\frac{\rho C_v\left(R^2-r^2\right)^2}{64\mu k}\frac{\partial}{\partial t}\left(\left(\frac{\partial P}{\partial \xi}\right)^2\right)\end{array}\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2}$$

$$r^2=-\frac{\left(\begin{array}{l}\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2+\frac{1}{r}\frac{\left(4r^3-2R^2r\right)}{16\mu}\left(\frac{\partial P}{\partial \xi}\right)^2+P\frac{d}{dt}(\log\rho)\\ -\frac{\rho C_v\left(R^2-r^2\right)^2}{64\mu k}\frac{\partial}{\partial t}\left(\left(\frac{\partial P}{\partial \xi}\right)^2\right)\end{array}\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2}$$

$$r^2=\frac{\left(\begin{array}{l}\frac{\rho C_v\left(R^2-r^2\right)^2}{64\mu k}\frac{\partial}{\partial t}\left(\left(\frac{\partial P}{\partial \xi}\right)^2\right)-\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2\\ -P\frac{d}{dt}(\log\rho)-\frac{\left(2r^2-R^2\right)}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\end{array}\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2} \qquad (32)$$

Thus equating Eqn. (28) and (32) we get,

$$R^2+\left(\frac{\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)}\right)=$$

$$\frac{\left(\begin{array}{l}\frac{\rho C_v\left(R^2-r^2\right)^2}{64\mu k}\frac{\partial}{\partial t}\left(\left(\frac{\partial P}{\partial \xi}\right)^2\right)-\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2\\ -P\frac{d}{dt}(\log\rho)-\frac{\left(2r^2-R^2\right)}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\end{array}\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2}$$

Rearranging the above equation

$$R^2=\left(\frac{\left(\frac{\rho C_v\left(R^2-r^2\right)^2}{64\mu k}\frac{\partial}{\partial t}\left(\left(\frac{\partial P}{\partial \xi}\right)^2\right)-\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2-P\frac{d}{dt}(\log\rho)-\frac{\left(2r^2-R^2\right)}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2}-\left(\frac{\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)}\right)\right)$$

Substituting the condition at the boundary, that is at wall r=R, the above equation simplifies into

$$R^2=\left(\left(\frac{-\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2-P\frac{d}{dt}(\log\rho)-\frac{\left(R^2\right)}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^2}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2}\right)-\left(\frac{\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)}\right)\right)_{r=R}$$

$$R^2=\left(\frac{\left(-\frac{4}{3}\mu\left(\frac{d}{dt}(\log\rho)\right)^2-P\frac{d}{dt}(\log\rho)-\frac{\left(R^2\right)}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\right)\left(\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)-\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)\left(\frac{\partial P}{\partial \xi}\right)^2}{\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\left(\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)}\right)_{r=R}$$

Rearranging the above equation

$$\frac{R^2}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\left(\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)=\left(\begin{aligned}&-\frac{4\mu}{3}\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)^3+\frac{4\mu}{3}\left(\frac{d}{dt}(\log\rho)\right)^2\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\&-P\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)^2+P\frac{d}{dt}(\log\rho)\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\&-\frac{R^2}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^3\left(\frac{d}{dt}(\log\rho)\right)+\frac{R^2}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\&-\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)\left(\frac{\partial P}{\partial \xi}\right)^2\end{aligned}\right)$$

$$\left(\begin{aligned}&\frac{R^2}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\left(\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)\\&+\frac{R^2}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^3\left(\frac{d}{dt}(\log\rho)\right)-\frac{R^2}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\end{aligned}\right)=\left(\begin{aligned}&-\frac{4\mu}{3}\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)^3+\frac{4\mu}{3}\left(\frac{d}{dt}(\log\rho)\right)^2\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\&-P\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)^2+P\frac{d}{dt}(\log\rho)\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\&-\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)\left(\frac{\partial P}{\partial \xi}\right)^2\end{aligned}\right)$$

$$R^2\left(\begin{aligned}&\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^3\left(\frac{d}{dt}(\log\rho)\right)-\frac{1}{4\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\&+\frac{1}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^3\left(\frac{d}{dt}(\log\rho)\right)-\frac{1}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\end{aligned}\right)=\left(\begin{aligned}&-\frac{4\mu}{3}\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)^3+\frac{4\mu}{3}\left(\frac{d}{dt}(\log\rho)\right)^2\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\&-P\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)^2+P\frac{d}{dt}(\log\rho)\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\&-\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)\left(\frac{\partial P}{\partial \xi}\right)^2\end{aligned}\right)$$

$$R^2\left(\frac{3}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^3\left(\frac{d}{dt}(\log\rho)\right)-\frac{3}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)$$

$$=\begin{pmatrix}-\frac{4\mu}{3}\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)^3+\frac{4\mu}{3}\left(\frac{d}{dt}(\log\rho)\right)^2\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\ -P\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)^2+P\frac{d}{dt}(\log\rho)\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\ -\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)\left(\frac{\partial P}{\partial \xi}\right)^2\end{pmatrix}$$

$$R^2=\frac{\begin{pmatrix}-\frac{4\mu}{3}\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)^3+\frac{4\mu}{3}\left(\frac{d}{dt}(\log\rho)\right)^2\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\ -P\frac{\partial P}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)^2+P\frac{d}{dt}(\log\rho)\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\\ -\frac{4\mu}{3\rho}\frac{\partial}{\partial \xi}\left(\frac{d}{dt}(\log\rho)\right)\left(\frac{\partial P}{\partial \xi}\right)^2\end{pmatrix}}{\frac{3}{8\mu}\left(\frac{\partial P}{\partial \xi}\right)^2\left(\left(\frac{\partial P}{\partial \xi}\right)\left(\frac{d}{dt}(\log\rho)\right)-\frac{\partial}{\partial t}\left(\frac{\partial P}{\partial \xi}\right)\right)} \qquad (33)$$

From Tait Equation we know that,

$$\frac{d}{dt}(\log\rho)=\frac{\begin{pmatrix}0.0894b_4\frac{dT}{dt}+0.0894\left(\frac{-b_3b_4\exp(b_4b_5-b_4T)\frac{dT}{dt}+\frac{dP}{dt}}{(b_3\exp(b_4b_5-b_4T)+P)}\right)\\ -\left(\frac{b_2}{b_1+b_2T-b_2b_5}\right)\begin{pmatrix}1+0.0894\ln b_3+0.0894b_4(b_5-T)\\ -0.0894\ln\left(b_3\exp\left[-b_4(T-b_5)\right]+P\right)\end{pmatrix}\frac{dT}{dt}\end{pmatrix}}{\begin{pmatrix}1+0.0894\ln b_3+0.0894b_4(b_5-T)\\ -0.0894\ln\left(b_3\exp\left[-b_4(T-b_5)\right]+P\right)\end{pmatrix}} \qquad (34)$$

3.2 Weissenberg-Rabinowitsch Correction

Since thermoplastic melt is non-Newtonian type, herein inequality is implicit inabove Newtonian constitutive relations, reasoning wallshear rate differencefor non-Newtonian constitutive relations. Further to equate true non-newtonian viscosity is obtained from Weissenberg-Rabinowitsch correction [11]. Accordingly correct shear rate at the wall for a non-Newtonian thermoplastic could now be calculated from below equation,

$$\dot{\gamma}_R=\dot{\gamma}_a\left[\frac{1}{4}\left(3+\frac{d\ln\dot{\gamma}_a}{d\ln\tau_R}\right)\right] \qquad (35)$$

The term in square brackets is the Weissenberg-Rabinowitsch (WR) correction, by correcting apparent shear rate to true shear rate, viscosity becomes obvious as it is the ratio of shear stress at the wall to true shear rate at the wall of the capillary.

$$\mu=\frac{\tau_R}{\dot{\gamma}_R} \qquad (36)$$

Accordingly viscosity for axisymmetric flow in terms of wall shear stress and apparent shear rate is

$$\mu=\frac{\tau_R}{\dot{\gamma}_a}\left(\frac{3n+1}{4n}\right)^{-1} \qquad (37)$$

Where n= power law index/shear thinning index $n=\frac{d\ln\tau_R}{d\ln\dot{\gamma}_a}$

For n=1 the true and apparent viscosity values are identical. For shear-thinning (Pseudo- plastic) n<1, this means that for aqueous thermoplastic melts, true shear rate would always be greater than apparent shear rate [12]. Thus Eqn (37) would now be,

$$\mu=\mu_0\left(\frac{4n}{3n+1}\right) \qquad (38)$$

Where μ_0 = Apparent viscosity

Thus adopting Weissenberg-Rabinowitsch correction in equation (33) the non-Newtonian behaviour of polymer melt is accommodated.

4. MATHEMATICAL VALIDATION OF THE RUNNER EQUATION USING DIMENSIONAL ANALYSIS

Dimension of all physical parameters being a unique combination of basic constituting physicalquantification can be expressed in terms of the fundamental dimensions *(or base dimensions)* M, L, and T – also these form 3-dimensional vector space.Itmandates strategic relevance to choice of data and isadoptedherein to deduce the credibility of derivedequations and thereon computations. Its most basic benefit beinghomogeneity i.e., only commensurable equations could be substantiated by havingidentical dimensions on either sides. Accordingly dimensional analysis *(also referred as Unit Factor Method)* is adapted to characterising Eqn. (33),

$$\rho=\frac{kg}{m^3}=M^1L^{-3} \qquad T=k=\theta^1$$

$$P = \frac{N}{m^2} = \frac{kg}{m-s^2} = M^1L^{-1}T^{-2}$$

$$b_1 = \frac{m^3}{kg} = M^{-1}L^3 \qquad b_2 = \frac{m^3}{kg-k} = M^{-1}L^3\theta^{-1}$$

$$b_3 = \frac{N}{m^2} = \frac{kg}{m-s^2} = M^1L^{-1}T^{-2}$$

$$b_4 = \frac{1}{k} = \theta^{-1} \qquad b_5 = k = \theta^1 \qquad dt = s = T^1$$

$$\frac{d}{dt}(\log\rho) = T^{-1},\ \xi = m = L^1 \text{ and } \mu = \frac{N-s}{m^2} = \frac{kg}{m-s} = M^1L^{-1}T^{-1}$$

Consider L.H.S $R^2 = m^2 = L^2$

Consider R.H.S

$$\frac{-\left(\frac{kg}{m-s}\times\frac{\frac{kg}{m-s^2}}{m}\times\frac{1}{s^3}\right)+\left(\frac{kg}{m-s}\times\frac{1}{s^2}\times\frac{1}{s}\times\frac{\frac{kg}{m-s^2}}{m}\right)-\left(\frac{kg}{m-s^2}\times\frac{\frac{kg}{m-s^2}}{m}\times\frac{1}{s^2}\right)+\left(\frac{kg}{m-s^2}\times\frac{1}{s}\times\frac{1}{s}\times\frac{\frac{kg}{m-s^2}}{m}\right)-\left(\frac{\frac{kg}{m-s}}{\frac{kg}{m^3}}\times\frac{1}{m-s}\times\left(\frac{\frac{kg}{m-s^2}}{m}\right)^2\right)}{\frac{m-s}{kg}\times\left(\left(\frac{\frac{kg}{m-s^2}}{m}\right)^2\right)\times\left(\frac{kg}{m^2-s^2}\times\frac{1}{s}-\frac{kg}{m^2-s^2}\times\frac{1}{s}\right)}$$

Upon simplifying

$$\frac{-\left(\frac{kg^2}{m^3-s^6}\right)+\left(\frac{kg^2}{m^3-s^6}\right)-\left(\frac{kg^2}{m^3-s^6}\right)+\left(\frac{kg^2}{m^3-s^6}\right)-\left(\frac{kg^2}{m^3-s^6}\right)}{\left(\frac{kg^2}{m^5-s^6}\right)}$$

$$= \frac{\frac{kg^2}{m^3-s^6}}{\frac{kg^2}{m^5-s^6}} = m^2 = L^2$$

Since LHS = RHS, Runner equation has been verified dimensionally.

CONCLUSIONS

This manuscript features a step by step derivation towards a computational model for determining runner dimension of a plastic injection mould. On the basis of governing equations, Weissenberg-Rabinowitsch correction for non-Newtonian nature of thermoplastic melt Eqn. (33) was derived as a function of thermoplastic melt properties such as viscosity and density, injection moulding machinespecificationssuch as maximum injection pressure and nozzle tip temperature as well as temporal parameter that feature the mould impression in totality owing to processing dynamics. Ultimately we believe Eqn. (33)computational model would offer a definite value of runner dimension that might still diverge from perfect or ideal design owing to computational rigour.

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NOMENCLATURE:

m	Mass	Kg
V	Volume	m^3
P	Pressure	Kgf / m^2
T	Temperature	K
T_w	Wall Temperature	K
K	Thermal conductivity	W/m
A	Cross-section area	m^2
R	Runner radius	m
$\vec{U}$	Linear velocity	m / s
$\vec{U_r}$	Velocity in radial direction	m / s
$\vec{U_\theta}$	Velocity in tangential direction	m / s
$\vec{U_\xi}$	Velocity in arbitrary direction	m / s
a	Acceleration	m / s^2
$\vec{M}$	Linear momentum	Kg – m / s
$\vec{H}$	Angular momentum	Kg – m / s
I	Moment of inertia	$Kg - m^2$
e	Specific total energy	KJ / Kg
$\hat{u}$	Specific internal energy	KJ / Kg
$\sum T$	Resultant torque	N – m
$\sum M$	Resultant moment	N – m
$\sum F$	Resultant force	N / m^2
F_r	Force acting in radial direction	N / m^2
F_θ	Force acting in tangential direction	N / m^2
F_ξ	Force acting in arbitrary direction	N / m^2
$\dot{Q}$	Rate of heat transfer	KW
q	Rate of heat transfer per unit mass	KW
$\dot{W}_v$	Rate of work done by viscous forces	KW
$\dot{W}_p$	Rate of work done by pressure forces	KW
dS	Entropy change	KJ / Kg
C_v	Specific heat at constant volume	KJ / KgK
C_p	Specific heat at constant pressure	KJ / KgK
$\vec{n}$	Unit normal vector	
$\vec{r}$	Position vector	
n	Shear thinning index	

GREEK SYMBOLS

ρ	Density	Kg / m^3
υ	Specific volume	m^3 / Kg
$\vec{\omega}$	Angular Velocity	m / s
α	Angular acceleration	m / s^2
$\dot{\gamma}_R$	True shear rate	$1/s$
$\dot{\gamma}_a$	Apparent shear rate	$1/s$
σ	Surface force	N / m^2
τ	Shear stress	N / m^2
μ	True Viscosity	$N - s / m^2$
μ_0	Apparent viscosity	$N - s / m^2$
ϕ	Viscous dissipation function	