BATCH ARRIVAL RETRIAL QUEUING SYSTEM WITH STATE DEPENDENT ADMISSION AND BERNOULLI VACATION

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Abstract

A single server batch arrival retrial queue with server vacation under Bernoulli schedule is considered. Arrivals are controlled according to the state of the server. The necessary and sufficient condition for the system to be stable is derived. Explicit formulae for the stationary distributions and performance measures of the system in steady state are obtained. Numerical examples are presented to illustrate the influence of the parameters on several performance characteristics.

Keywords: *Retrial queue, batch arrival, state dependent admission control, Bernoulli vacation.*

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1. INTRODUCTION

Retrial queues have the feature that arriving customers finding no free servers must leave the service area and repeat their demands for service after a random time. A customer is said to be in orbit between two retrials. Nowadays, retrial queues have become increasingly important in the analysis of computer and communication networks. For recent papers on retrial queues, see [1], [3], [4], [5] and [13].

In recent years queues with server vacation have emerged as an important area of research due to their various application in production systems, communication systems, computer networks and etc. Some comprehensive studies on the recent results for a variety of vacation models can be found in [6], [7], [10], [11], [12], [14].

In many queuing situations, the customer's arrival rate varies according to the server state idle, busy and on vacation. Altman et al. [2] considered the state dependent $M/G/1$ type queuing analysis for congestion control in data networks. Madan and Abu-Dayyeh [7] and Madan and Choudhury [8] have investigated classical queuing system with restricted admissibility of arriving batches and Bernoulli server vacation. This paper examines the state dependent retrial queuing system with bulk arrival and server vacation. The similar situation of retrial can be realized in on-line ticket booking centres.

2. SYSTEM DESCRIPTION

Consider a single server infinite capacity queuing facility with batch arrival. One of the arriving customers begins his service immediately if the server is available and the remaining customers leave the service area to join the orbit.

The arrival epochs occur in accordance with a Poisson process with rate λ and the number of arrivals at each epoch is a random variable X having distribution $P[X=n]=c_n$ and moments $\overline{C}_n$, $n \ge 1$. Successive inter retrial times of any customer in orbit is generally distributed with distribution function $A(x)$ and Laplace transform $A^*(s)$. The service time is a random variable with distribution function $B(x)$, Laplace transform $B^*(s)$ and finite moments $\mu_n, n \ge 1$.

After completion of each service, the server may take a vacation with probability θ or may continue to be in the system with complementary probability. The vacation times are generally distributed with distribution function $V(x)$, Laplace transform $V^*(s)$ and finite moments $v_n, n \ge 1$.

The arriving batches are allowed to join the system with state dependent admission control policy. Let α_1, α_2 and α_3 be the assigned probabilities for an arriving batch to join the system during the period of idle, busy and vacation times respectively.

The hazard rate function of retrial time, service time and vacation time are defined as

$$\eta(x) = \frac{a(x)}{1-A(x)};\ \mu(x) = \frac{b(x)}{1-B(x)};\ \beta(x) = \frac{v(x)}{1-V(x)}.$$

3. THE JOINT DISTRIBUTIONS

The stage of the system at time t can be described by the Markov process $\{(M(t); t \geq 0\} = \{J(t), N(t), \xi_i(t); t \geq 0, i = 0, i = 0,1,2\}$ where $J(t)$ denotes the server state 0,1,2 according as the server being idle or busy or on vacation and $N(t)$ denotes the number of customers in the retrial queue at time t. If $J(t) = 0$ and $N(t) > 0$, then $\xi_0(t)$ represents the elapsed retrial time, if $J(t) = 1$ and $N(t) > 0$, $\xi_1(t)$ corresponds to the elapsed service time of the customer at time 5, and if $J(t) = 2$ and $N(t) > 0$, $\xi_2(t)$ corresponds to the elapsed vacation time t.

For the process $\{M(t); t \geq 0\}$, define the probabilities

$$I_0(t) = P\{J(t) = 0, N(t) = 0\}$$

$$I_n(t,x)dx = P\{J(t) = 0, N(t) = n, x \leq \xi_0(t) \leq x + dx\}, n \geq 1$$

$$W_n(t,x)dx = P\{J(t) = 1, N(t) = n, x \leq \xi_1(t) \leq x + dx\}, n \geq 0$$

$$V_n(t,x)dx = P\{J(t) = 2, N(t) = n, x \leq \xi_2(t) \leq x + dx\}, n \geq 0$$

Let $I_0, I_n(x), W_n(x)$ and $V_n(x)$ are the limiting densities of $I_0(t)$, $I_n(t,x)$, $W_n(t,x)$ and $V_n(t,x)$.

Define the probability generating function

$$I(z,x) = \sum_{n=1}^{\infty} I_n(x) z^n; \; W(z,x) = \sum_{n=0}^{\infty} W_n(x) z^n;$$

$$V(z,x) = \sum_{n=1}^{\infty} V_n(x) z^n \text{ and } c(z) = \sum_{n=1}^{\infty} c_n z^n$$

4. ERGODICITY CONDITION

Let $\{\tau_n, n \in N\}$ be the sequence of epochs of the service completion times or vacation termination times. The sequence of random vectors $Q_n = \{J(\tau_n^+), N(\tau_n^+)\}$ forms a Markov chain, which is the embedded Markov chain for our queuing system with state space S = {0, 1, 2} x {0, 1, 2, ...}

Theorem1

$\{Q_n, n \geq 1\}$ is ergodic if and only if $\overline{C}_1[1 - A^*(\alpha_1\lambda)] + \lambda\overline{C}_1[\alpha_2\mu_1 + \alpha_3\theta v_1] < 1$

Proof

$\{Q_n, n \geq 1\}$ is an irreducible and aperiodic Markov chain. To prove ergodicity, we shall use Foster's criterion : An irreducible and aperiodic Markov chain is ergodic if there exists a non negative function $f(j), j \in N$ and $\varepsilon > 0$ such that the mean drift $\Psi_j = E[f(Q_{n+1}) - f(Q_n) \mid Q_n = j]$ is finite for all $j \in N$ and $\Psi_j \leq -\varepsilon$ for all $j \in N$, except perhaps a finite number.

Take $f(j) = j$. Then we have

$$\Psi_j = \begin{cases} \overline{C}_1[1 - A^*(\alpha_1\lambda)] + \lambda\overline{C}_1[\alpha_2\mu_1 + \alpha_3\theta v_1] - 1 & \text{if } j = 1,2,3,.... \\ \lambda\overline{C}_1[\alpha_2(1-\theta) + \alpha_3\theta v_1] - 1 & \text{if } j = 0 \end{cases}$$

Clearly, the inequality $\overline{C}_1[1 - A^*(\alpha_1\lambda)] + \lambda\overline{C}_1[\alpha_2\mu_1 + \alpha_3\theta v_1] < 1$ is a sufficient condition for ergodicity. The same inequality is also necessary for ergodicity. We can guarantee the non-ergodicity of the Markov chain $\{Q_n, n \geq 1\}$, if it satisfies Kaplan's condition, namely $\Psi_j < \infty$ for all $j \in N$ and there exists $j_0 \in N$ such that $\Psi_j \geq 0$ for $j \geq j_0$. In our case, Kaplan's condition is satisfied because there exists $k \in N$ such that $r_{ij} = 0$ for $j < i - k$ and $i > 0$, where $R = (r_{ij})$ is the one step transition matrix of $\{Q_n, n \geq 1\}$. Then the inequality $\overline{C}_1[1 - A^*(\alpha_1\lambda)] + \lambda\overline{C}_1[\alpha_2\mu_1 + \alpha_3\theta v_1] < 1$ implies the non ergodicity of the Markov chain.

Since the arrival stream is a Poisson process, it can be shown from Burke's theorem that the steady state probabilities of $\{J(t), N(t), t \geq 0\}$ exist and are positive if and only if $\overline{C}_1[1 - A^*(\alpha_1\lambda)] + \lambda\overline{C}_1[\alpha_2\mu_1 + \alpha_3\theta v_1] < 1$.

From the mean drift Ψ_j= $\overline{C}_1[1 - A^*(\alpha_1\lambda)] + \lambda\overline{C}_1[\alpha_2\mu_1 + \alpha_3\theta v_1] - 1$, for $j \geq 1$ we have the reasonable conclusion that the term $\lambda\overline{C}_1[\alpha_2\mu_1 + \alpha_3\theta v_1]$ represents a batch arrival during service time and on vacation time. The other term $\overline{C}_1[1 - A^*(\alpha_1\lambda)] - 1$ refers to the contribution to the orbit size due to batch arrival during the retrial time excluding the arbitrary customer of the arriving batch whose service commences so that he no longer belongs to the orbit. Similar interpretation can be provided for $j = 0$. the condition $\Psi_j < 0$ assures that the orbit size does not grow indefinitely in course of time.

5. STEADY STATE PROBABILITY GENERATION FUNCTION

The steady state equations that governs the system under consideration are

$$\lambda I_0 = \int_0^\infty V_0(x)\beta(x)dx + (1-\theta)\int_0^\infty W_0(x)\mu(x)dx \quad (1)$$

$$\frac{d}{dx} I_n(x) = -(\lambda+\eta(x))\, I_n(x) + \lambda(1-\alpha_1)\, I_n(x),\ n \geq 1 \quad (2)$$

$$\frac{d}{dx} W_n(x) = -(\lambda+\mu(x))\, W_n(x) + \lambda(1-\alpha_2)\, W_n(x) + \lambda\,\alpha_2\,(1-\delta_{0n})\sum_{k=1}^{n} c_k\, W_{n-k}(x),\ n \geq 0 \quad (3)$$

$$\frac{d}{dx} V_n(x) = -(\lambda+\beta(x))\, V_n(x) + \lambda(1-\alpha_3)\, V_n(x) + \lambda\,\alpha_3\,(1-\delta_{0n})\sum_{k=1}^{n} c_k\, V_{n-k}(x),\ n \geq 0 \quad (4)$$

With boundary conditions

$$I_n(0) = \int_0^\infty V_n(x)\beta(x)dx + (1-\theta)\int_0^\infty W_n(x)\mu(x)dx,\ n \geq 1 \quad (5)$$

$$W_0(0) = \lambda\, c_1\, I_0 + \int_0^\infty I_1(x)\eta(x)dx \quad (6)$$

$$W_n(0) = \lambda\, c_{n+1}\, I_0 + \int_0^\infty I_{n+1}(x)\eta(x)dx + \lambda\,\alpha_1 \int_0^\infty \sum_{k=1}^{n} c_k\, I_{n-k+1}(x)\, dx,\ n \geq 1 \quad (7)$$

$$V_n(0) = \theta \int_0^\infty W_n(x)\mu(x)dx,\ n \geq 0 \quad (8)$$

From the equations (2) – (8) we have,

$$I(z,x) = I(z,0)\, e^{-\alpha_1\lambda x}\,[1-A(x)] \quad (9)$$

$$W(z,x) = W(z,0)\, e^{-\alpha_2\lambda(1-c(z))x}\,[1-B(x)] \quad (10)$$

$$V(z,x) = V(z,0)\, e^{-\alpha_3\lambda(1-c(z))x}\,[1-V(x)] \quad (11)$$

$$I(z,0) = \int_0^\infty V(z,x)\,\beta(x)dx + (1-\theta)\int_0^\infty W(z,x)\,\mu(x)dx - \lambda\, I_0 \quad (12)$$

$$W(z,0) = I(z,0)\,[A^*(\alpha_1\lambda) + c(z)(1-A^*(\alpha_1\lambda))] + \lambda\, I_0 c(z) \quad (13)$$

$$V(z,0) = \theta\, W(z,0)\, B^*(\alpha_2\lambda(1-c(z))) \quad (14)$$

Substituting the expressions of $V(z,x)$ and $W(z,x)$ in terms of $W(z,0)$ in equation (12), we get

$$\begin{aligned} I(z,0) &= \theta\, W(z,0)\, B^*(\alpha_2\lambda(1-c(z)))\, V^*(\alpha_3\lambda(1-c(z))) + (1-\theta) W(z,0) B^*(\alpha_2\lambda(1-c(z))) - \lambda\, I_0 \\ &= W(z,0)\, B^*(\alpha_2\lambda(1-c(z)))\,[1-\theta+\theta\, V^*(\alpha_3\lambda(1-c(z)))] - \lambda\, I_0 \end{aligned} \quad (15)$$

Using the expression $I(z,0)$ in equation (13) and simplifying we obtain

$$W(z,0) = \lambda\, I_0\, A^*(\alpha_1\lambda)\,[1-c(z)]\,/\,D(z) \quad (16)$$

Where

$$D(z) = B^*(\alpha_2\lambda(1-c(z)))\,[1-\theta+\theta\, V^*(\alpha_3\lambda(1-c(z)))]\,[A^*(\alpha_1\lambda) + c(z)(1-A^*(\alpha_1\lambda))] - z \quad (17)$$

Now equations (15) and (14) become

$$I(z,0) = \lambda\, I_0\,\{z - c(z)\, B^*(\alpha_2\lambda(1-c(z)))\,[1-\theta+\theta\, V^*(\alpha_3\lambda(1-c(z)))]\}\,/\,D(z) \quad (18)$$

$$V(z,0) = \lambda\, I_0\, A^*(\alpha_1\lambda)\,[1-c(z)]\,\theta\, B^*(\alpha_2\lambda(1-c(z)))\,/\,D(z) \quad (19)$$

Theorem 2

Using equilibrium state, the joint distribution of the server has the following partial generating functions

$$I(z) = I_0\,[z - c(z)\, B^*(\alpha_2\lambda(1-c(z)))\,[1-A^*(\alpha_1\lambda)]\,[1-\theta+\theta\, V^*(\alpha_3\lambda(1-c(z)))]\}\,/[\alpha_1 D(z)] \quad (20)$$

$$W(z) = I_0\, A^*(\alpha_1\lambda)\,[1-B^*(\alpha_2\lambda(1-c(z)))]\}\,/[\alpha_2 D(z)] \quad (21)$$

$$V(z) = I_0 A^*(\alpha_1\lambda)\theta B^*(\alpha_2\lambda(1-c(z))) [1-V^*(\alpha_3\lambda(1-c(z)))]\}/[\alpha_3 D(z)] \quad (22)$$

$$I_0 = \alpha_1 T_1 / T_2 \quad (23)$$

Where

$$T_1 = 1-\overline{C}_1[1-A^*(\alpha_1\lambda)] - \lambda\overline{C}_1[\alpha_2\mu_1 + \alpha_3\theta v_1]$$

$$T_2 = \alpha_1 T_1 + (1-A^*(\alpha_1\lambda)(A^*(\alpha_1\lambda)\overline{C}_1 - T_1) + \alpha_1 A^*(\alpha_1\lambda)\lambda\overline{C}_1[\mu_1+\theta v_1]$$

Proof

Substituting for $I(z,0)$, $W(z,0)$ and $V(z,x)$ given by equations (9), (10) and (11) and integrating with respect to x from 0 to ∞ we get the results given in equations (20) – (22).

Now, the unknown constant I_0 given in equation (23) can be determined by using the normalizing condition $I_0 + I(1) + W(1) + V(1) = 1$.

6. MEAN ORBIT SIZE AND MEAN SYSTEM SIZE

Theorem 3

The probability generating function of the number of customer in the orbit is

$$\begin{aligned} P_q(z) = {} & I_0[\theta\alpha_1\alpha_2(1-\alpha_3)A^*(\alpha_1\lambda)B^*(\alpha_2\lambda(1-c(z))) \\ & [1-V^*(\alpha_3\lambda(1-c(z)))] - \alpha_1(1-\alpha_2)\alpha_3 \\ & B^*(\alpha_2\lambda(1-c(z)))A^*(\alpha_1\lambda) \\ & -(1-\alpha_1)\alpha_2\alpha_3[1-A^*(\alpha_1\lambda)]c(z) \\ & B^*(\alpha_2\lambda(1-c(z)))[1-\theta+\theta V^*(\alpha_3\lambda(1-c(z)))] \\ & +\alpha_1\alpha_3 A^*(\alpha_1\lambda) \\ & +\alpha_2\alpha_3 z(1-\alpha_1 - A^*(\alpha_1\lambda))\}/[\alpha_1\alpha_2\alpha_3 D(z)] \end{aligned} \quad (24)$$

The probability generating function of number of customer in the system is

$$\begin{aligned} P_q(z) = {} & I_0[\theta\alpha_1\alpha_2(1-\alpha_3)A^*(\alpha_1\lambda)B^*(\alpha_2\lambda(1-c(z))) \\ & [1-V^*(\alpha_3\lambda(1-c(z)))] \\ & -\alpha_1\alpha_2\alpha_3 A^*(\alpha_1\lambda)B^*(\alpha_2\lambda(1-c(z))) \\ & -(1-\alpha_1)\alpha_2\alpha_3(1-A^*(\alpha_1\lambda)) \\ & c(z)B^*(\alpha_2\lambda(1-c(z))) \\ & [1-\theta+\theta V^*(\alpha_3\lambda(1-c(z)))] \\ & +(1-\alpha_1)\alpha_2\alpha_3 z A^*(\alpha_1\lambda) \\ & (\alpha_1-\alpha_2-\alpha_1 B^*(\alpha_2\lambda(1-c(z))))\} \\ & /[\alpha_1\alpha_2\alpha_3 D(z)] \end{aligned} \quad (25)$$

Proof

The probability generating function for the number of customer in the orbit is

$$P_q(z) = I_0 + I(z) + W(z) + V(z) \text{ and}$$

The probability generating function for the number of customer in the system is

$$P_s(z) = I_0 + I(z) + zW(z) + V(z)$$

Substituting the expressions of $I(z), W(z)$ and $V(z)$, we get the equations as in (24) and (25).

Corollary 1

The mean number of customer in the orbit is

$$L_q = N_2/T_2 + N_1T_3/(T_1T_2) \quad (26)$$

The mean number of customer in the system is

$$L_s = L_q + A^*(\alpha_1\lambda)\alpha_1\lambda\overline{C}_1\mu_1/T_2 \quad (27)$$

Where

$$\begin{aligned} T_3 = {} & \lambda^2\overline{C}_1^2\alpha_2\alpha_3\theta\mu_1 v_1 + \lambda\overline{C}_1^2\alpha_2\mu_1(1-A^*(\alpha_1\lambda)) \\ & +\lambda\overline{C}_1^2\alpha_3\theta v_1(1-A^*(\alpha_1\lambda)) + [\lambda^2\alpha_2^2\overline{C}_1^2\mu_2 \\ & +\theta\alpha_3^2\lambda^2\overline{C}_1^2 v_2 + \overline{C}_2(1-A^*(\alpha_1\lambda))]/2 \end{aligned}$$

$$\begin{aligned} N_1 = {} & (1-\alpha_1)(1-A^*(\alpha_1\lambda))[\overline{C}_1 + \alpha_2\lambda\overline{C}_1\mu_1 \\ & +\alpha_3\lambda\theta\overline{C}_1 v_1] - (1-\alpha_1 - A^*(\alpha_1\lambda)) \\ & +\alpha_1\lambda\overline{C}_1 A^*(\alpha_1\lambda)[\theta(1-\alpha_3)v_1 + (1-\alpha_2)\mu_1] \end{aligned}$$

$$\begin{aligned} N_2 = {} & \alpha_1(1-\alpha_3)\theta A^*(\alpha_1\lambda)\alpha_2\lambda^2\overline{C}_1^2\mu_1 v_1 \\ & +\{\alpha_1\alpha_3(1-\alpha_3)\theta A^*(\alpha_1\lambda)\lambda^2\overline{C}_1^2 v_2 \\ & +\alpha_1\alpha_2(1-\alpha_2)A^*(\alpha_1\lambda)\lambda^2\overline{C}_1^2\mu_2 \\ & +(1-\alpha_1)(1-A^*(\alpha_1\lambda))[\overline{C}_2 + \alpha_2^2\lambda^2\overline{C}_1^2\mu_2 \\ & +\theta\alpha_3^2\lambda^2\overline{C}_1^2 v_2]\}/2 + (1-\alpha_1)(1-A^*(\alpha_1\lambda)) \end{aligned}$$

$$[\alpha_2 \lambda \overline{C}_1^2 \mu_1 + \alpha_3 \lambda \theta \overline{C}_1^2 v_1 + \theta \alpha_3 \alpha_2 \lambda^2 \overline{C}_1^2 \mu_1 v_1]$$

Proof

Differentiating $P_q(z)$ and $P_s(z)$ with respect to z and taking limit $z \to 1$ by using L′Hospital rule the expressions for L_q and L_s can be obtained.

7. OPERATING CHARACTERISTICS

Some performance measures for the system are given below.

1. The steady state probability that the server is idle in the empty system is

$$I_0 = \alpha_1 T_1 / T_2$$

2. The steady state probability that the server is idle in the non-empty system is

$$I = [1 - A^*(\alpha_1\lambda)][\alpha_2 \lambda \overline{C}_1 \mu_1 + \theta \alpha_3 \lambda \overline{C}_1 v_1 + \overline{C}_1 - 1] / T_2$$

3. The steady state probability that the server is busy is

$$W = \alpha_1 A^*(\alpha_1\lambda) \lambda \overline{C}_1 \mu_1 / T_2$$

4. The steady state probability that the server is on vacation is

$$V = \alpha_1 A^*(\alpha_1\lambda) \theta \lambda \overline{C}_1 v_1 / T_2$$

5. The probability that the orbit is empty while the server is busy is

$$W_0 = \alpha_1 T_1[1 - B^*(\alpha_2\lambda)] / \{\alpha_2 T_2 B^*(\alpha_2\lambda)[1 - \theta + \theta V^*(\alpha_3\lambda)]\}$$

6. The probability that the orbit is empty while the server is on vacation is

$$V_0 = \theta \alpha_1 T_1[1 - V^*(\alpha_3\lambda)] / \{\alpha_3 T_2 [1 - \theta + \theta V^*(\alpha_3\lambda)]\}$$

7. The probability of orbit being empty is

$$\begin{aligned} E &= I_0 + W_0 + V_0 \\ &= \alpha_1 T_1\{\alpha_2 \alpha_3 B^*(p_2 \lambda)[1 - \theta + \theta V^*(\alpha_3 \lambda)] \\ &\quad + \alpha_3[1 - B^*(\alpha_2 \lambda)] \\ &\quad + \theta \alpha_2 B^*(\alpha_2\lambda)[1 - V^*(\alpha_3\lambda)\} \end{aligned}$$

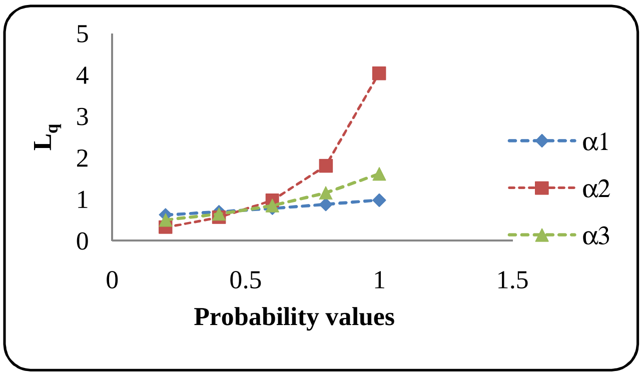
8. NUMERICAL ILLUSTRATION

Numerical results are obtained when the service, retrial and vacation times follow exponential distribution. Table shows the dependence of the performance measures I_0, I, W, V and L_q for the fixed values of $(\theta, \beta, \alpha_1, \alpha_2, \alpha_3, c_1, c_2)$ = (5, 5, 0.8, 0.5, 0.5, 0.5, 0.5). Figures show the effect of α_1, α_2 and α_3 (joining probability during the period of idle, busy and vacation) on the performance measures. L_q the mean number of customer in the orbit for the parameters $(\lambda, \eta, \mu, \theta, \beta, \alpha_1, \alpha_2, \alpha_3, c_1, c_2)$ = (2, 15, 5, 5, 5, 0.8, 0.5, 0.5, 0.5, 0.5).

Table1. Performance measures for the various values of λ, μ and η

λ	μ	η	I_0	I	W	V	L_q
1	10	20	0.7125	0.0281	0.1297	0.1297	0.0895
		30	0.7213	0.0188	0.1299	0.1299	0.0748
		40	0.7258	0.0141	0.1301	0.1301	0.0675
		50	0.7284	0.0113	0.1301	0.1301	0.0632
	30	20	0.7920	0.0271	0.0452	0.1356	0.0644
		30	0.8007	0.0181	0.0453	0.1359	0.0520
		40	0.8051	0.0136	0.0453	0.1360	0.0459
		50	0.8077	0.0109	0.0454	0.1361	0.0423
	50	20	0.8088	0.0269	0.0274	0.1369	0.0608
		30	0.8175	0.0180	0.0274	0.1371	0.0489
		40	0.8218	0.0135	0.0274	0.1372	0.0429
		50	0.8244	0.0108	0.0275	0.1373	0.0394
2	10	20	0.4833	0.0608	0.2280	0.2280	0.3565
		30	0.5015	0.0407	0.2289	0.2289	0.3005
		40	0.5107	0.0306	0.2294	0.2294	0.2738
		50	0.5162	0.0245	0.2296	0.2296	0.2581
	30	20	0.6129	0.0577	0.0824	0.2471	0.2253
		30	0.6307	0.0386	0.0827	0.2481	0.1868
		40	0.6396	0.0290	0.0829	0.2486	0.1682
		50	0.6450	0.0232	0.0829	0.2488	0.1572
	50	20	0.6414	0.0570	0.0503	0.2513	0.2075
		30	0.6591	0.0381	0.0505	0.2523	0.1717
		40	0.6680	0.0286	0.0506	0.2528	0.1543
		50	0.6734	0.0229	0.0506	0.2531	0.1441
3	10	20	0.2949	0.0964	0.3044	0.3044	1.0085
		30	0.3227	0.0647	0.3063	0.3063	0.8163
		40	0.3367	0.0487	0.3073	0.3073	0.7308
		50	0.3451	0.0390	0.3079	0.3079	0.6824
	30	20	0.4562	0.0906	0.1133	0.3399	0.5360
		30	0.4833	0.0608	0.1140	0.3419	0.4400
		40	0.4970	0.0457	0.1143	0.3430	0.3952
		50	0.5052	0.0367	0.1145	0.3436	0.3693
	50	20	0.4931	0.0893	0.0696	0.3480	0.4808
		30	0.5200	0.0599	0.0700	0.3501	0.3956

		40	0.5336	0.0451	0.0702	0.3511	0.3556
		50	0.5418	0.0361	0.0704	0.3518	0.3324



CONCLUSIONS

Retrial queue with batch arrival admission control and Bernoulli vacation has been investigated in this paper. The necessary and sufficient condition for the system to be stable is obtained. The inputs of the parameters on the performance measures are illustrated.

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