

# DIGITAL SIGNAL PROCESSING TECHNIQUES FOR LTI FIBER IMPAIRMENT COMPENSATION

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## Abstract

Coherent detection is one of the active research areas for the development of high speed, high spectral efficient optical communication network. Digital signal processing is the important technique for compensating the fiber transmission impairments because of number of advantages such as signal can be amplified, delayed, splitted and manipulated without degrading the signal quality. This paper presents DSP compensation algorithms for linear time invariant (LTI) impairment such as chromatic dispersion (CD) and polarization mode dispersion (PMD) in optical fiber communication. We presented a mathematical framework for compensation of LTI fiber impairments. This paper also focuses the different compensation methods both in time and frequency domain for chromatic dispersion compensation. These DSP techniques confirm that coherent detection with high data rates will become feasible in future for compensating transmission impairments.

**Keywords:** Coherent Detection, Chromatic Dispersion, Polarization Mode Dispersion

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## 1. INTRODUCTION

One of the important research areas for the development of high speed, high spectral efficient optical networks for long distance transmission is the coherent detection. Coherent receiver allows information symbols to be decoded by recovering the electric field, which leads to improved power and spectral efficiency. In direct detection, light intensity is converted into electrical signal by photo detector. Due to this, phase information is totally lost. In order to overcome the above problem, coherent detection is used in which the received optical signal is mixed with local optical carrier and then intensified light signal being detected in the photo diode. Coherent optical communication begins in 1990s. Kahn [1] et al demonstrated a coherent 4 Gb/s binary PSK (BPSK) using narrow line width Laser and optimized phase locked loops. Another Scheme, known as 310 Mb/s QPSK was demonstrated in [2].

In 1990s, with the invention of EDFA, coherent optical systems were rapidly developing, which enables transmission of data over long distances with the rapid increase of traffic, which carries a number of multiplexed signals, such as data, voice, music, video etc. There is a need for development of high spectral efficiency system to use existing optical fiber installation. The goal of long haul optical fiber system is to transmit the high data throughput over the long distance without signal regeneration.

Digital signal processing is used at the receiver for compensation of fiber impairments [3]. Digital compensation can be done at the receiver after the optical signal has been converted to electric signal. If the baseband signal is sampled above the Nyquist rate, digitized signal represents the full content of analog electric signal, which enables digital signal processing compensation. DSP has advantages, such as signals can be delayed, split, amplified and manipulated without degradation in signal quality.

Sun et al [4] demonstrated a real time coherent detection of polarization multiplexed 40 Gb/s QPSK. The systems 100 Gb/s and above are currently being developed. DSP algorithm will become complex as the bit rate, constellation size and transmission distances are increased. Fiber impairment compensation using coherent detection and DSP was presented in [3]. In this paper, DSP compensation algorithms for chromatic and polarization mode dispersion are reviewed. Both time domain and frequency domain techniques all presented. A technique for linear equalization for compensating both CD and PMD is proposed.

The organization of this paper is as follows. Review on optical transmission system and DSP is presented in section II. Coherent optical systems with DSP, Signal propagation in optical fiber and LTI (linear time invariant) model are presented in Section III. Chromatic Dispersion Compensation and PMD Compensation techniques all presented in Section IV.

Finally conclusion drawn from the paper and scope for further work is presented in the last Section.

## 2. REVIEW ON OPTICAL TRANSMISSION SYSTEM AND DSP

An optical transmission system can be represented as shown in Fig.1



Fig.1 A typical optical transmission system

Where  $E_{TX}$  is the transmitted signal,  $H(\omega)$  is the channel transfer function and  $E_{RX}$  is the received Signal.

The goal of a DSP System is to implement  $H^{-1}(\omega)$ . This will be interpreted as the combination of all the linear effects during propagation of light through fiber. Schematic block diagram for DSP implementation is shown in Fig.2

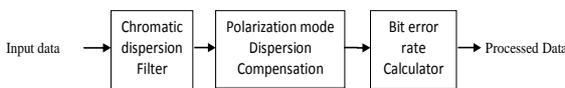


Fig.2 DSP implementation scheme

In order to compensate for these impairments, the received electrical signal is applied with a series of algorithms in order to minimize the Bit Error Rate (BER). BER is the main evaluation criteria for estimating the quality of a digital communication system.

## 3. SYSTEM MODEL AND SIGNAL PROPAGATION

### 3.1 System Model:

A general coherent optical system with DSP is shown in Fig.3

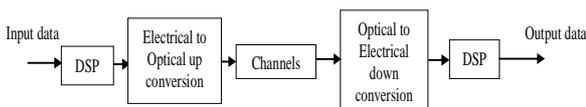


Fig.3 Coherent optical system with DSP

### 3.2 Signal Propagation:

The main aim of a communication systems is to propagate a signal  $E(t, z)$  from the transmitter to receiver, so that, the signal at the receiver  $E(t, z_{end})$  will be close approximation to the transmitted signal  $E(t, 0)$ . The propagation of light in optical fiber is described by the non-linear Schrodinger equation in [3] as follows:

$$\begin{aligned} \frac{\partial E}{\partial z} = & \left( -\frac{1}{2}\alpha - \beta_1 \frac{\partial}{\partial t} - j \frac{1}{2!} \beta_2 \frac{\partial^2}{\partial t^2} + \frac{1}{3!} \beta_3 \frac{\partial^3}{\partial t^3} \right) E \\ & + j\gamma \left[ |E|^2 I - \frac{1}{3} (E^H \sigma_3 E) \sigma_3 \right] E \\ = & (\hat{D} + \hat{N}) E \end{aligned} \tag{1}$$

Where  $\hat{D}$  and  $\hat{N}$  are the linear and nonlinear operators, and  $E(z, t) = [E_1(z, t) E_2(z, t)]^T$  is the Jones vector of the electric field. The  $2 \times 2$  matrices  $\alpha, \beta_1, \beta_2$  and  $\beta_3$  are the fiber's loss, group velocity, dispersion and dispersion slope.  $\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$  is a Pauli spin matrix [3]

### 3.3 Linear Time Invariant Model:

For LTI Model of OFC system, we keep non-linear operator  $\hat{N}$  as Zero. Hence, Schrodinger's equation become

$$\frac{\partial E}{\partial z} = \hat{D}E \tag{2}$$

By taking Fourier Transform of equation (2) we obtain

$$E(\omega, z) = H_{fiber}(\omega) E(\omega, 0) \tag{3}$$

Where  $H_{fiber}(\omega)$  is fiber's frequency response and  $E(\omega, z) \& E(\omega, z)$  are the Fourier Transform of fiber input and output respectively. For long haul transmission, attenuation in fiber is compensated using EDFA or Raman amplifier. In optical amplifier noise electric field is generated by spontaneous emission. This noise electric field can be modeled using additive white Gaussian noise. Hence, the system can be modeled using base band electric fields at the transmitter and receiver.

Let  $x_1, t = [x_1(t) x_2(t)]^T$  and  $y_1, t = [y_1(t) y_2(t)]^T$  are the baseband, analog electric fields at the transmitter and

receiver respectively. They are related as  $y(t) = h(t) * x(t) + n(t)$  where \* denotes convolution. In frequency domain, we get

$$Y(\omega) = H(\omega) X(\omega) + N(\omega)$$

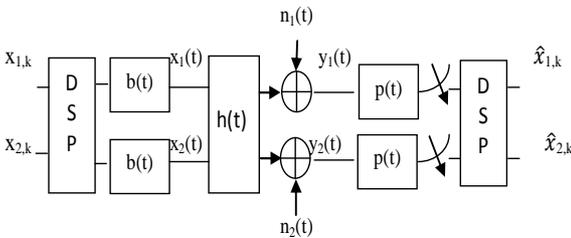


Fig. 4 A coherent optical system as Linear Time Invariant model

Where b(t) is the impulse response of electrical to optical conversion and p(t) is the impulse response of optical to electrical conversion. b(t) includes the frequency responses of D/A converters, Mach-Zehnder (MZ) modulators and optical multiplexing elements, while p(t) includes the responses of optical de-multiplexing elements, the coherent photo receiver and A/D converters.

To obtain equalization algorithms for linear impairment, the channel is modeled as [3] and is given below

$$H(\omega) = H_{CD}(\omega) H_{PMD}(\omega)$$

#### 4. COMPENSATION METHODS FOR LINEAR TRANSMISSION IMPAIRMENTS:

##### 4.1 Chromatic Dispersion Compensation:

Chromatic dispersion arises due to propagation of speed of light as a function of wavelength, so that different spectral components of signal exhibit relative delay causing signal distortion.

In the absence of non linearities, the resulting system becomes linear as given by equation 2. Hence, we write the expression, that shows the effect of Chromatic Dispersion on the signal  $E(t, z)$

$$\frac{\partial}{\partial Z} E(t, z) = -j\beta_2 \frac{\partial^2}{\partial t^2} E(t, z) \tag{4}$$

Where ‘z’ is propagation distance, t is the time variable,  $\beta_2$  is the group velocity dispersion. Group velocity results, because of different spectral components of signal travel with different velocity.

Taking Fourier Transform of equalization (4) we get

$$E(\omega, z) = E(\omega, 0) e^{j\beta_2 z \omega^2} \tag{5}$$

It is observed from this frequency domain representation, that chromatic dispersion introduces a distortion on the phase of the signal spectrum without changing the spectral power distribution and at the end of propagation the pulse is broadened. The parameter  $\beta_2$  gives the time delay between two different spectral components separated by a certain frequency interval.

The relation between  $\beta_2$  and dispersion coefficient D is given by

$$D = -\frac{2\pi}{\lambda^2} c \beta_2$$

Using this relation, equation (4) becomes

$$\frac{\partial}{\partial Z} E(t, z) = j \frac{D\lambda^2}{4\pi c} \frac{\partial^2}{\partial t^2} E(t, z)$$

$$E(\omega, z) = E(\omega, 0) e^{-j \frac{D\lambda^2}{4\pi c} z \omega^2}$$

$$E(\omega, z) = E(\omega, 0) H(\omega, z)$$

$H(\omega, z)$ , is the frequency response of the system, called as transfer function of chromatic dispersion.

$$H(\omega, z) = e^{-j \frac{D\lambda^2}{4\pi c} z \omega^2} \tag{6}$$

By taking Inverse Fourier Transform we get

$$h(t, z) = \sqrt{\frac{c}{jD\lambda^2 z}} e^{\frac{j\pi c t^2}{D\lambda^2}} \tag{7}$$

From literature [3], filters for dispersion compensation in frequency and time domain is obtained by inverting the sign of equations (6) and (7).

##### 4.2 CD Compensation in time domain:

In time domain, CD compensation can be obtained from convolution with FIR filter. FIR filter has input output equitation as given below:

$$y(n) = b_0 x(n) + b_1 x(n-1) + \dots + b_n x(n-N)$$

where  $x(n)$  is the input signal  $b_i$  are the filter weights or tap weights,  $N$  is the filter length and  $y(n)$  is the output.

### 4.2.1 Savory Method

Savory [5] derived expression for tap weights and filter coefficients by truncating the impulse response to a finite duration. This can be implemented using FIR filter with the given number of taps and the amplitudes

$$a_k = \sqrt{\frac{jct^2}{D\lambda^2 z}} e^{-\frac{j\pi c\tau^2 k^2}{D\lambda^2 z}} \quad -\frac{N}{2} \leq k \leq \frac{N}{2}$$

$$N = \left\lceil \frac{|D|\lambda^2 z}{ct^2} \right\rceil + 1$$

Where  $N$  is the number of taps,  $\tau$  is the sampling period and  $\lceil x \rceil$  is the integer part rounded.

### 4.2.2 Adaptive Filters:

An adaptive algorithm makes use of iterative procedure for correcting the weights of the filter in order to minimize the mean error between the output  $\hat{y}(n)$  and desired output  $y(n)$ .

LMS (least mean squares) algorithm and constant modulus algorithms are the two techniques, which uses these adaptive algorithms. Adaptive filtering scheme is shown below in fig.5

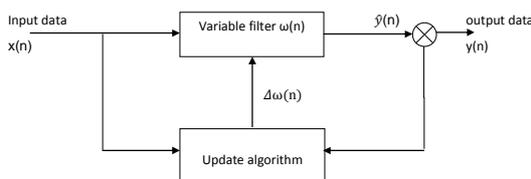


Fig. 5 Adaptive filtering technique

#### 4.2.2.1 LMS algorithm:

It is a technique, which uses iterative procedure, that makes successive correction and the weight of the filter in order to minimize the MSE (mean square error) between the output  $\hat{y}(n)$  and the desired output  $y(n)$

$$\hat{y}(n) = \sum_{k=d}^{M-1} \omega_k * x(n-k) = \omega^H x(n)$$

$$e(n) = y(n) - \hat{y}(n)$$

$$\omega(n+1) = \omega(n) + \mu x(n) e^*(n) \quad (8)$$

Where ‘ $\mu$ ’ is the step size.

Equation (8) can be used for updating the filter weights such that the error between the obtained output and desired output is minimized. The selection of step size ‘ $\mu$ ’ is very important for tap weight updating. Faster convergence can be achieved with high value of  $\mu$  but provides lower precision. Low value of ‘ $\mu$ ’ gives slower convergence with higher precision. A compromise can be made for selection of ‘ $\mu$ ’ to obtain better results.

#### 4.2.2.2 CMA Algorithm

In order to obtain better convergence compared to LMS algorithms, CMA algorithm is used. In this case filter output is calculated as

$$\hat{y}(n) = \sum_{k=d}^{M-1} \omega_k * x(n-k) = \omega^H(n) x(n)$$

$$e(n) = 1 - |y(n)|^2$$

Filter coefficients for updating can be obtained as

$$\omega(n+1) = \omega(n) + \mu x^*(n) e^*(n) y(n)$$

#### 4.2.3 CD Compensation in frequency domain:

The main advantage of frequency domain approach is that the filtering of input data is obtained by multiplication process rather than convolution. FFT algorithm is most widely used in frequency domain representation. The digital filter weights in frequency domain are obtained as [5]

$$f_k = e^{j\frac{D\lambda^2}{4\pi c} z \omega_k^2}$$

Where  $\omega_k = \frac{2\pi}{T_s N}$ , where  $T_s$  is the sampling period and  $N$  is the length of FFT.

Especially, for large data, FIR filtering is more efficient in frequency domain using either overlap add or overlap save methods [3]. The procedure given below makes use of overlap add method [3].

- The receiver signal is decomposed into smaller windows.
- Zero padding technique should be adopted to make lengths equal
- FFT is computed for each window sequence.

- The output of FFT data is multiplied with filter in frequency domain
- Inverse FFT can be performed on filtered frequency domain signal.

#### 4.2.4 Polarization Mode Dispersion:

Propagating field in long haul single mode fiber is described as consisting of two degenerate modes each corresponding to two orthogonal polarizations. The degeneration is introduced due to the cylindrical symmetries of the optical fiber. Generally optical fibers has physical structure that is not perfectly cylindrical that leads to imperfection and perturbation due to mechanical tension, thermal gradients etc. Due to these imperfections the two fundamental modes see different indices of refraction in the fiber. The two polarization modes are propagating with slightly different group velocity. This phenomenon is called polarization mode dispersion (PMD).

Adaptive PMD and Polarization rotation Equalization: Compensation against the influence of PMD and polarization fluctuation can be performed adaptively by the decision directed LMS filter [5] which are presented below:

$$\begin{bmatrix} x_{out}(n) \\ y_{out}(n) \end{bmatrix} = \begin{bmatrix} \omega_{xx}(n) & \omega_{xy}(n) \\ \omega_{yx}(n) & \omega_{yy}(n) \end{bmatrix} \begin{bmatrix} x_{in}(n) \\ y_{in}(n) \end{bmatrix}$$

Where weight updation can be done using following formula

$$\bar{\omega}_{xx}(n+1) = \bar{\omega}_{xx}(n) + \mu_p \varepsilon_x(n) \bar{x}_{in}^*(n)$$

$$\bar{\omega}_{yx}(n+1) = \bar{\omega}_{yx}(n) + \mu_p \varepsilon_y(n) \bar{x}_{in}^*(n)$$

$$\bar{\omega}_{xy}(n+1) = \bar{\omega}_{xy}(n) + \mu_p \varepsilon_x(n) \bar{y}_{in}^*(n)$$

$$\bar{\omega}_{yy}(n+1) = \bar{\omega}_{yy}(n) + \mu_p \varepsilon_y(n) \bar{y}_{in}^*(n)$$

$$\varepsilon_x(n) = d_x(n) - x_{out}(n)$$

$$\varepsilon_y(n) = d_y(n) - y_{out}(n)$$

Where  $\bar{x}_{in}(n)$  and  $\bar{y}_{in}(n)$  are the complex amplitude values of the input signals and  $x_{out}(n)$  and  $y_{out}(n)$  are the complex amplitudes of the equalizing output signals respectively. The  $\bar{\omega}_{xx}(n)$ ,  $\bar{\omega}_{xy}(n)$ ,  $\bar{\omega}_{yx}(n)$  and  $\bar{\omega}_{yy}(n)$  are the complex tap weights and  $d_x(n)$  and  $d_y(n)$  are the desired signals. The  $\varepsilon_x(n)$  and  $\varepsilon_y(n)$  are the estimated errors

between the output signal and the desired signals respectively and  $\mu_p$  is the step size. PMD compensation can be done after CD compensation.

#### CONCLUSIONS & SCOPE FOR FUTURE WORK

This paper presented DSP Compensation algorithms for LTI fiber impairments namely chromatic dispersion and polarization mode dispersion. DSP techniques provide highly flexible solutions for implementing impairments compensation. The propagation of light in optical fiber is described by Schrodinger equation for LTI model of OFC System keeping non linear term zero. We obtained an expression for frequency response of chromatic dispersion from which impulse response of the system can be calculated. Inverse Fourier transforms to be adopted for compensation. Frequency domain techniques for CD compensation are also prescribed. The main advantage of frequency domain approach is that the filtering of input data is obtained by multiplication process rather than convolution. Non linear fiber impairments compensation techniques using DSP will be the scope for future work.

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