MINIMUM SHEAR REINFORCEMENT FOR OPTIMUM DUCTILITY OF REINFORCED CONCRETE BEAMS

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Abstract

Failures in reinforced concrete (RC) structures under transverse shear forces are proved to be catastrophic with the presence of web reinforcement. The minimum web reinforcement recommended by several codes of practice has been intended to maintain adequate strength and deflection ductility after the formation of diagonal cracking and to contain widening of the diagonal cracking. However, the expressions for estimating the minimum shear reinforcement in the codes of practice are based on the experimental data base observed on testing of small size beams made of normal strength concrete (NSC). Such code provisions need to be reinvestigated on large size beams made of high strength concrete (HSC). Further, there has been lack of consensus on the quantity of minimum shear reinforcement to be provided by different codes of practice, as they differ significantly in respect of HSC members. In this paper, many factors influencing the minimum shear reinforcement required in RC beams have been studied. An expression has been proposed incorporating a wide range of parameters. A comparison of the minimum shear reinforcement predicted by the proposed expression has been made with the codes of practice. The influence of shear reinforcement on the ductility of RC beams of varying sizes has been investigated. The optimum shear reinforcement index has been found to be somewhere between 0.45 and 0.5. Ductility of RC beams increases with increasing the shear reinforcement index. Small size beams exhibited significant ductility for the given shear reinforcement index.

Keywords: *Minimum shear reinforcement, reserve strength, ductility, RC beams, HSC*

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1. INTRODUCTION

It is known from earlier study that the web reinforcement in the form of stirrups does not influence the diagonal cracking strength significantly. However, the presence of shear reinforcement alters the behaviour of RC beams and enhances shear capacity by modifying the shear transfer mechanisms through improved dowel action, restraining crack propagation, minimizing bond splitting and increasing the contribution of concrete in compression zone. In the design of RC structures, use of minimum shear reinforcement is mandated when the factored shear force exceeds one-half of the design shear strength of concrete. The objective of specifying minimum shear reinforcement by the codes of practice is to prevent sudden failure at the formation of first diagonal cracking, to control widening of cracks at service loads and also to ensure adequate ductility before failure. Some of the major codes of practice such as ACI, CANADIAN and AASHTO specify the minimum shear reinforcement as a function of concrete strength while it is only a function of yield strength of shear reinforcement and independent of compressive strength of concrete by IS and BS codes of practice.

2. REVIEW OF LITERATURE

A brief review of literature carried out by authors on minimum shear reinforcement is presented. Lin and Lee [1] reported that an increase of tension reinforcement increases the strength but decreases the ductility of the beams. Further an increase in compression steel reinforcement and concrete strength enhances the ductility of the beams effectively, and the beams provided with high strength shear reinforcement exhibit the same cracking resistance as those with normal strength shear reinforcement. In another study, Lin and Lee [2] concluded that the factors affecting the ductility of RC beams are a/d ratio, spacing of stirrups and strength of shear reinforcement. Also it was concluded that by increasing the quantity and the strength of shear reinforcement does not have apparent influence on the diagonal cracking strength. However, increasing the strength of concrete and strength of shear reinforcement increases the ultimate strength, whereas the decrease in the shear-span-depth ratio and the spacing of stirrups increases the ultimate strength of beams. For beams with small a/d ratio, the effectiveness of the shear reinforcement is much reduced.

Xie et al. [3] carried out experimental investigation on 15 RC beams for understanding the ductility under shear dominant loading with and without web reinforcement. The variables considered in the study are; compressive strength of concrete ranging between 40 and 109 MPa, shear span-to-depth ratio varied between 1.0 and 4.0 and the quantity of shear reinforcement varied between 0.0 and 0.784%. The post peak response was characterised through the shear ductility of RC beams. Based on the comparison of two large scale beams tested by Johnson and Ramirez [4] and the tests on reduced size specimens, Robert Frosch [5] concluded that the beam size did not affect the post cracking behaviour or the shear strength provided by the stirrups. However, from the analysis of the test results from the previous studies, Johnson and Ramirez [4] concluded that the overall reserve strength after the diagonal tension cracking diminished with the increase in the compressive strength of concrete, f'c for beams designed with the current provisions of minimum shear reinforcement. This situation would be more critical for beams with larger a/d ratios and smaller quantity of longitudinal reinforcement. Yoon et al. [6] concluded that for HSC members the crack spacing is a function of the spacing of longitudinal and transverse reinforcement. The spacing of the shear cracks increases as the member size increased and hence the Canadian Standards Association [7] (CSA) predicts lower shear stress at failure for large size members.

Roller and Russell [8] from their experimental work concluded that the minimum quantity of shear reinforcement specified in the codes must be increased with the increase of compressive strength of concrete. The equation proposed by Yoon et al. [6] was re-evaluated by Ozcebe et al. [9], which concluded that the quantity of shear reinforcement could be 20% smaller than that of the minimum shear reinforcement specified by the ACI code [10]. The experimental investigations conducted by Angelakos [11] showed that the minimum quantity of the shear reinforcement by the ACI code [10] resulted in inadequate safety margins.

3. SHEAR REINFORCEMENT PROVISIONS

The design provisions for the minimum reinforcement in shear specified by various codes of practice are presented in Table 1.

Table-1: Minimum Shear Reinforcement by various Codes of Practice

S. No	Code	Equation	Eq.
1	ACI	$\frac{A_{sv}}{bS_v} \geq \frac{\sqrt{f_c}}{16f_y} \geq \frac{0.33}{f_y}$	(1)
2	AASHTO [12]	$\frac{A_{sv}}{bS_v} \geq \frac{\sqrt{f_c}}{12f_y}$	(2)
3	CSA	$\frac{A_{sv}}{bS_v} \geq \frac{\sqrt{f_c}}{16.67f_y}$	(3)
4	IS 456 [13], BS 8110 [14]	$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87f_y}$	(4)

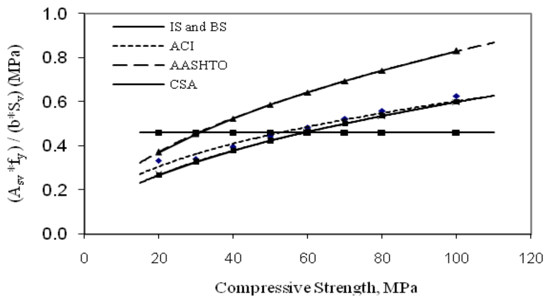

Fig1. Reinforcement Index vs. Compressive Strength

Using these provisions the variation of shear reinforcement index, r*fy (r = Asv/bSv) with compressive strength of concrete is shown in Fig. 1. It can be observed that the quantity of minimum shear reinforcement is a function of the compressive strength of concrete according to the provisions by ACI, AASHTO and CSA codes. But it is a function of the strength of the shear reinforcement alone as per BS and IS codes. Also, the variation is much higher for HSC beams compared to NSC beams. The shear reinforcement is represented in terms of shear reinforcement index, which is the product of shear reinforcement ratio and the yield strength of the shear reinforcement.

4. DETERMINATION OF MINIMUM SHEAR REINFORCEMENT

The minimum web reinforcement provided by different codes of practice is intended mainly to ensure that the capacity of a member after the diagonal cracking exceeds the load at which the inclined cracking occurs or in other words for a beam with a given geometry and properties of materials the minimum quantity of shear reinforcement is necessary to increase the shearing strength of the beam to a particular shear force 'V' greater than that corresponds to cracking strength, Vcr. The studies carried out by Johnson and Ramirez [4] showed that the post cracking strength decreases with increase in the compressive strength of concrete. It would be more critical with the increase in the shear-span-to-depth ratio and decrease in the longitudinal reinforcement. Hence, it is clear that the minimum shear reinforcement must be a function of the shear-span-to-depth ratio and the longitudinal reinforcement along with the compressive strength of concrete.

Using the relationship between the diagonal cracking and the ultimate shear strength by authors [15] and also from the condition (Vc + Vs) > Vcr, in Eq. 5, an expression for the minimum shear reinforcement has been developed in Eq. 10.

$$V_c + V_s \geq V_{cr} \tag{5}$$

The relationship between the diagonal cracking strength and the ultimate shear strength [15] is as shown in Eq. 6,

$$V_{cr} = \left(v_u \frac{\sqrt[3]{a/d}}{2\rho^{1/6}} \right) bd \quad (6)$$

The shear strength of concrete for the reinforced concrete members as per the ACI code [10] is,

$$V_c = \left(\frac{0.17\sqrt{f_c'}}{\gamma} \right) bd \quad (7)$$

Taking the lower bound value for the tensile strength of concrete as a function of the compressive strength in [16] proposed an expression for the ultimate shear strength of concrete which is given by Eq. 8,

$$v_u = 0.5\sqrt{f_c'} \quad (8)$$

Taking,

$$V_s = A_{sv} f_y \left(\frac{d}{s_v} \right) \quad (9)$$

From Eq. 5 substituting for vu, Vs, Vcr and Vc we get,

$$\therefore \left(\frac{A_{sv}}{bs_v} \right) f_y = \sqrt{f_c'} \left(0.25\frac{\sqrt[3]{a/d}}{\rho^{1/6}} - 0.22 \right) \geq 0.33\sqrt{f_c'} \quad (10)$$

The minimum reinforcement is a function of compressive strength of concrete, shear span to depth ratio and percentage flexural reinforcement. For the beams with the shear span-to-depth ratio, a/d > 3, the minimum shear reinforcement predicted by the ACI, CSA and AASHTO provisions is not conservative and further investigations are to be carried out for the higher shear span-to-depth ratios with compressive strength of concrete greater than 60 MPa. The model proposed in this study predicts satisfactorily the minimum shear reinforcement for all types of beams with various compressive strengths of concrete, shear-span-to-depth ratio and longitudinal reinforcement.

5. VARIATION OF MINIMUM SHEAR REINFORCEMENT

The minimum shear reinforcement evaluated using the model developed in Eq. 10 in the present study for cube concrete compressive strength of 20, 40 and 60 MPa for different percentages of longitudinal reinforcement are shown in Figs. 2 to 4 respectively in beams with different shear span-to-depth ratios.

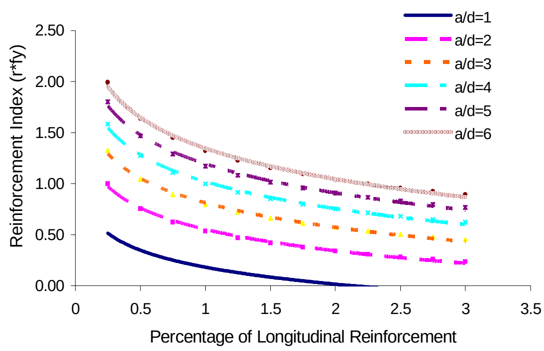

Fig2. Reinforcement Index vs. % Longitudinal Reinforcement for 20 MPa Concrete

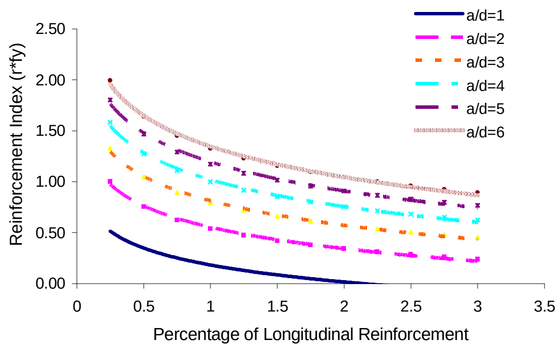

Fig3. Reinforcement Index vs. % Longitudinal Reinforcement for 40 MPa Concrete

As shown in Figs. 2 to 4 using 20, 40 and 60 MPa compressive strength of concrete, the reinforcement index decreases with increase in the percentage longitudinal reinforcement. The longitudinal reinforcement provided in the tension region of the beam can improve the shear strength of RC beams due to dowel action of the flexural tensile reinforcement. Due to the dowel action of tension reinforcement, the quantity of shear reinforcement required to resist the transverse forces has been observed to be decreased.

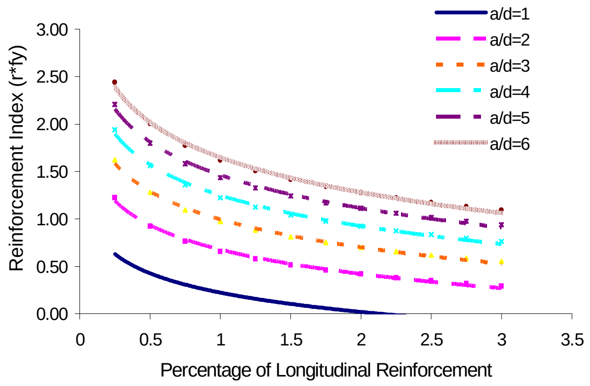

Fig4. Reinforcement Index vs. % Longitudinal Reinforcement for 60 MPa Concrete

The variation of the reinforcement index vs. shear span-to-depth ratio in various beams with concrete compressive strength of 20, 40 and 60 MPa are shown in Figs. 5 to 7 respectively with different percentage flexural tensile reinforcement. The shear reinforcement index increases with increasing the shear span-to-depth ratio. This can be explained by the fact that the shear strength of RC beams, at very low to low shear span-to-depth ratio behaving like a deep and short beams respectively, is very high. The shear reinforcement required is reduced at small a/d ratios. In RC beams with large a/d ratios, the failure tends to be flexure-shear or flexure mode. In such beams, the shear capacity is relatively less and the failures are more like flexural failures. The shear capacity is very small. Hence, at large a/d ratios, the minimum shear reinforcement tends to be increased.

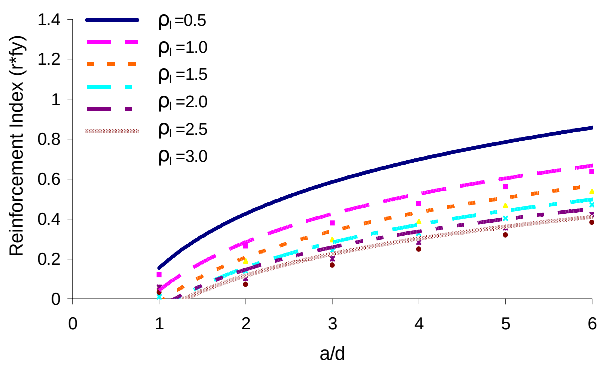

Fig5. Reinforcement Index vs. Shear span-to-depth ratio for 20 MPa Concrete

The minimum shear reinforcement calculated according to the ACI and IS codes corresponding to the ultimate shear strength of the beam, which is equal to the concrete shear strength of 0.48 MPa (for fc'= 60 MPa) and 0.4 MPa respectively. However, the provisions by the codes of practice are independent of the shear-span-to-depth ratio and also the percentage of the longitudinal reinforcement. Figs. 2 to 7 show the variation of the shear reinforcement with percentage flexural reinforcement, ρl and the shear span-to-depth, (a/d) ratio for different compressive strengths of concrete.

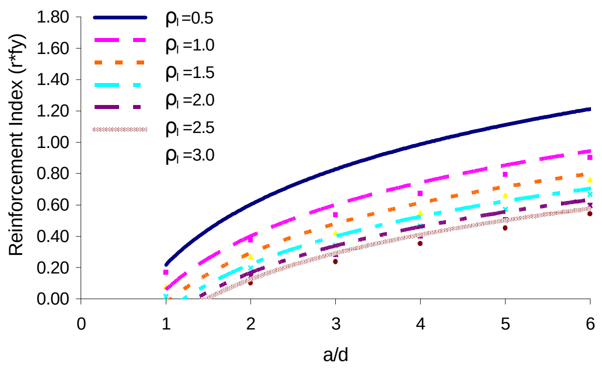

Fig6. Reinforcement Index vs. Shear span-to-depth ratio for 40 MPa Concrete

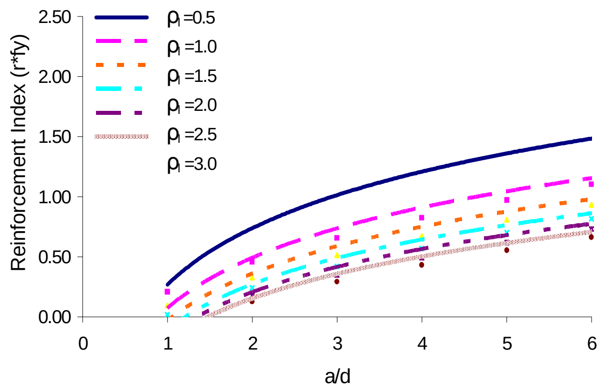

Fig7. Reinforcement Index vs. Shear span-to-depth ratio for 60 MPa Concrete

Fig. 8 shows the variation of the shear reinforcement index with compressive strength for a/d = 3.0 and ρl = 2.4%. It can be observed that the shear reinforcement decreases with increase in the longitudinal tension reinforcement, ρl and increases with increase in the shear span-to-depth ratio, a/d ratio. The objective of codes of practice for minimum reinforcement in shear must also be to ensure minimum ductility apart from ensuring the minimum strength of a member is greater than its diagonal cracking strength. It is also clear that the addition of even small quantity of web reinforcement increases the ductility of a member but one needs to quantify the minimum ductility that has to be developed within the member.

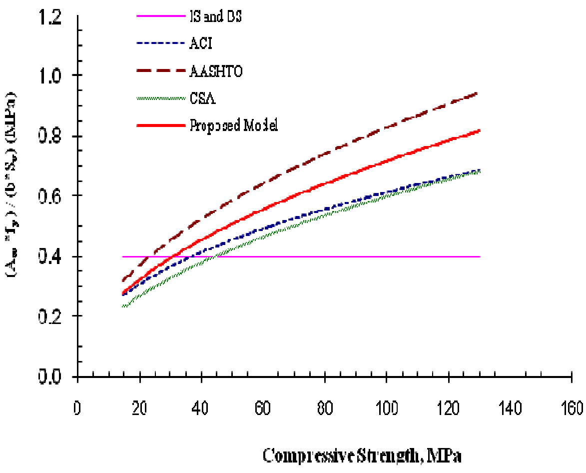

Fig8: Shear Reinforcement Index vs. Compressive Strength (a/d = 3.0, pl = 2.4%).

Table-2: Beam Dimensions

Beam	B mm	D mm	d/b	le mm	le/d	a/d
H20-0.0	150	161	1.1	966	6.0	3.0
H20-0.4	150	161	1.1	966	6.0	3.0
H20-0.6	150	161	1.1	966	6.0	3.0
H20-0.8	150	161	1.1	966	6.0	3.0
H40-0.0	150	345	2.3	2070	6.0	3.0
H40-0.4	150	345	2.3	2070	6.0	3.0
H40-0.6	150	345	2.3	2070	6.0	3.0
H40-0.8	150	345	2.3	2070	6.0	3.0
H60-0.0	150	536	3.6	3216	6.0	3.0
H60-0.4	150	536	3.6	3216	6.0	3.0
H60-0.6	150	536	3.6	3216	6.0	3.0
H60-0.8	150	536	3.6	3216	6.0	3.0

6. EXPERIMENTAL INVESTIGATIONS

6.1. Materials and Beam Dimensions

Portland Pozzolona Cement was used for preparation of the reinforced concrete beams and other companion specimens. The width of the beams was maintained constant at 150 mm maintaining two dimensional similarity. Twelve reinforced concrete beams of three different sizes were cast and tested with depths of beams 200 mm, 400 mm and 600 mm, with the effective depths of 161mm, 345mm and 536mm respectively. The effective span-to-effective depth ratio for all the six beams tested was 6.0. The shear span-to-effective depth ratio was 3.0. The beam dimensions are shown in Table 2. The compressive strength of concrete was varied between from 50 MPa to 60 MPa. A stress-reinforcement index representing the quantity of shear reinforcement, (r*fy) equal to 0.4 was estimated corresponding to the minimum shear reinforcement required as per the IS 456-2000. Two more value of these indices 0.6 and 0.8 were also adopted in this study anticipating the minimum shear reinforcement to be achieved for high strength concrete with large size beams, as the ductility decreases with increase in the strength of concrete and the size of the beam. The beams of depth 200 mm and 400 mm were cast in steel moulds, while 600 mm depth beams were cast in moulds made of well seasoned wood. The beams were demolded after 24 hrs and subsequently cured for 28 days before testing. To obtain the tensile and compressive strength of concrete companion cylinders of 150 mm diameter and cubes of 150 mm size were cast and tested along with the beams.

Twelve geometrically similar reinforced concrete beams of three different sizes made of medium strength concrete were cast and tested under three-point loading using a closed loop MTS testing system of 500 kN capacity. The properties of the steel reinforcement used in this programme are shown in Table 3. The compressive strength of concrete in the beams is shown in Table 4. The flexural tensile reinforcement in all the beams was 2.5%. the number of flexural reinforcing bars corresponding to the above percentage reinforcement is shown in Table 4. The spacing of the closed stirrups was varied to study the ductility and minimum shear reinforcement provisions by different codes of practice. The shear reinforcement in the beams was varied in terms of shear reinforcement index. The shear reinforcement index is the product of the shear reinforcement ratio multiplied by the yield strength. Table 4 shows the flexural and shear reinforcement details in the beam.

Table-3: Properties of Web and Longitudinal Reinforcement

Diameter	Area (mm2)	Yield Strength (N/mm2)	Ultimate Strength (N/mm2)
3 mm	6.69	447	491
4 mm	10.17	400	480
5 mm	18.65	479	521
6 mm	28.26	425	530
16 mm	201.00	521	656
20 mm	314.00	595	662

Table-4: Flexural and Shear Reinforcement Details

Beam	fck, MPa	Ast	Asv	r*fy
H20-0.0	50.4	3 -16mm	Nil	0.0
H20-0.4	57.1	3-16mm	3mm @100 c/c	0.4
H20-0.6	50.1	3-16mm	4mm @90 c/c	0.6
H20-0.8	47.0	3-16mm	4mm @70 c/c	0.8
H40-0.0	58.5	6-16mm	Nil	0.0
H40-0.4	57.1	6-16mm	4mm @135 c/c	0.4
H40-0.6	50.1	6-16mm	5mm @200 c/c	0.6
H40-0.8	52.1	6-16mm	5mm @150 c/c	0.8
H60-0.0	58.5	6-20mm	Nil	0.0
H60-0.4	57.1	6-20mm	5mm @300 c/c	0.4
H60-0.6	50.1	6-20mm	6mm @267 c/c	0.6
H60-0.8	52.1	6-20mm	6mm @200 c/c	0.8

6.2. Load vs. Deflection

The variations of load with deflection of beams of depths 200 mm, 400 mm and 600 mm are shown in Figs. 9 to 11, which demonstrate that with increase in SRI the ductility of beams increases. The ultimate loads for 200 mm depth beams with SRI 0.0 and 0.6 were 112 and 150 kN and the corresponding deflections are 2.4 mm and 5.3 mm respectively. Similarly, for 400 mm depth beams with SRI 0.0 and 0.8 the ultimate loads were 240 kN and 270 kN with the corresponding deflections 6.4 mm and 7.4 mm respectively. Also, for the beams of depth 600 mm with SRI of 0.0 and 0.8 the ultimate loads were 340 kN and 460 kN and the deflections at the peak loads were 8.5 mm and 9.8 mm respectively. The comparison of these trends shows that the increase in the deflection at the peak load with the corresponding increase of SRI for 200 mm depth beams seems to be higher. This clearly indicates that the stirrups are more effective in resisting the shear force in the members with smaller depths than the beams of larger depths. On the contrary the failures of the large size beams are more brittle than the small size beams. This phenomenon changes the participation of stirrup for different depths, which needs serious evaluation. Figs. 9 to 11 reveal that the minimum quantity of reinforcement specified by codes may be sufficient for small size beams as adequate ductility has been achieved. However, it is inadequate for the large size beams (depth of 600 mm) made of HSC because the web reinforcement snapped after reaching the ultimate load for r*fy = 0.4 and 0.6 for beams of depth 600 mm.

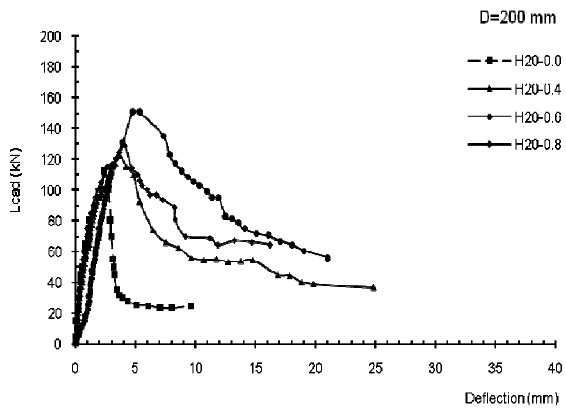

Fig9. Load vs. Deflection in beam HSC-200.

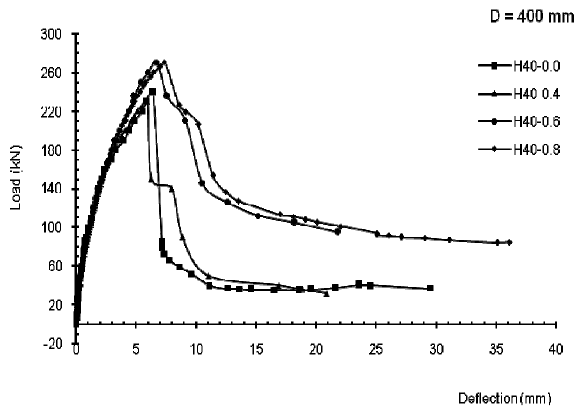

Fig10. Load vs. Deflection in beam HSC-400.

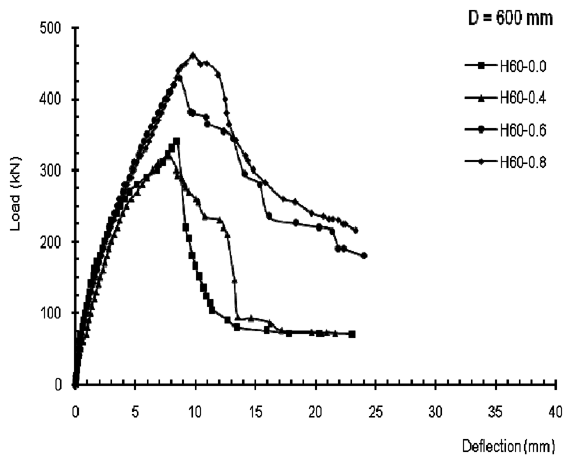

Fig11. Load vs. Deflection in beam HSC-600.

6.3. Modes of Failure

Of all the twelve beams tested, three of them were without web reinforcement failed in shear with a very distinct diagonal crack forming on one side of the beam. The crack widths, crack pattern and also how the cracks were propagated with the application of the load was monitored and also recorded. The cracks were formed symmetrically on either side of the beam with further loading but the final failure was due to one major diagonal crack widening up. The cracks in all the beams extended from the load point to the adjoining support.

6.3.1. H-0.0 Series

The mode of failure for all the three depths of beams without web reinforcement (SRI equal to 0.0) is shown in Fig. 12. The beams developed secondary cracks along the longitudinal reinforcement due to deterioration of the bond and the failure was due to the shear-tension failure. Since sufficient end anchorage was designed, there was no anchorage failure observed. The beams of depth 400 mm and 600 mm depth were provided with longitudinal reinforcement in two layers. The failure pattern in H0.0 beams is shown in Fig. 12.

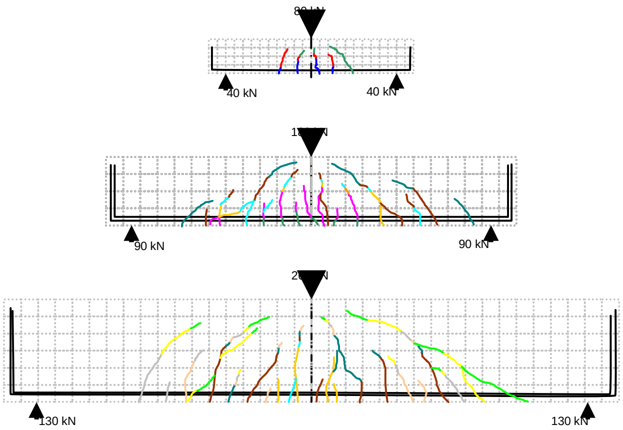

Fig12. Crack Profile for H-0.0 Series beams at 1.7 MPa Stress level

6.3.2 H-0.4 Series

The beams H-0.4 had a web reinforcement corresponding to SRI 0.4 MPa. The beams with 200 mm depth exhibited a ductile behaviour, whereas the beams of 400 mm depth exhibited a very sudden failure before switching on from the load control to the displacement control system during testing. All the stirrups across the diagonal crack passing through it was snapped. Though the failure of all the beams in this series was also due to shear-tension failure but only a single secondary crack along the longitudinal reinforcement was observed unlike in the case of beams without web reinforcement where two distinct secondary cracks were observed.

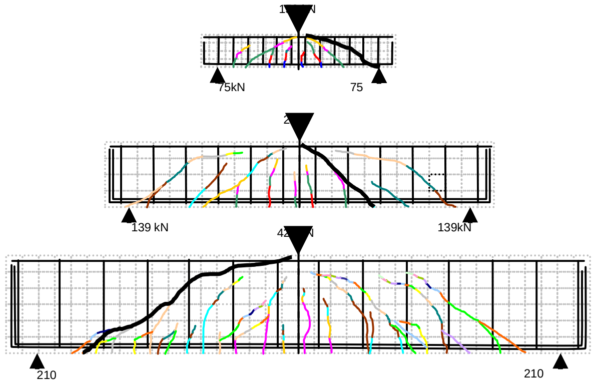

Fig13. Crack Profile for H-0.6 Series beams at Ultimate Stress level

6.3.3. H-0.6 Series

The beam H60 of H-0.6 series exhibited local crushing of concrete in the vicinity of the load point due to the combined action of the shear and compression. Spalling of concrete took place in addition to the diagonal cracking prior to the failure. The beams of depth 200 mm and 600 mm developed the splitting cracks along the longitudinal reinforcement. However, the H40 beams failed in shear-compression mode without any secondary cracking along the longitudinal reinforcement. The failure patterns for this series are shown in Fig. 13.

6.3.4 H-0.8 Series

All the beams in this series failed in shear-tension mode accompanied by local crushing and spalling of concrete.

CONCLUSIONS

1. The minimum shear reinforcement specified by IS and BS code are independent of the compressive strength of concrete, and inadequate for HSC beams. However, ACI, CSA and AASHTO provisions represent as a function of compressive strength of concrete but are independent of a/d ratio and ρ_l.
2. The model proposed in this study predicts reasonably well the minimum web reinforcement for all type of beams based on the compressive strength of concrete, a/d ratio and ρ_l.
3. From the load deflection response for the beams tested with SRI = 0.4 MPa and from the nature of failure observed, it has been found that the provision of SRI of 0.4 MPa is inadequate.
4. For variation in SRI from 0.0 to 0.4, there is no significant enhancement of shear strength for all depths of beams thus indicating that the minimum web reinforcement proposed by the IS and BS codes of practice need to be re-evaluated.

5. Based on the load-deflection response it has been observed that with increase in SRI (rf_y) the shear ductility increases only up to a certain critical limit of 0.6. Therefore, for a given strength of concrete increasing the web reinforcement beyond the critical limit does not improve the ductility significantly.
6. The energy absorption ratio is not enhanced significantly by increasing SRI from 0.0 to 0.4 for the beams of 400mm and 600mm depths compared to the beams of 200mm depth. Participation of shear reinforcement is more effective in small size beams compared to the large size beams.

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Notation

a = shear span
a/d = shear span-to-depth ratio
A_s = area of tension reinforcement
A_{sv} = Area of shear reinforcement
b = breadth of beam
d = effective depth in mm
f_c' = cylindrical compressive strength in MPa
f_y = yield strength of longitudinal steel
f_{vs} = yield strength of shear reinforcement
ρ_l = longitudinal reinforcement ratio
r = A_{sv} / bS_v
S_v = Spacing of stirrups
v_{cr} = diagonal cracking strength
v_u = ultimate shear strength
V_{ult}= factored shear force at the critical section