EFFECT OF FREE SURFACE WAVE ON FREE VIBRATION OF A FLOATING PLATFORM

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Abstract

Use of floating structures is very common and widespread. Starting from boats, ships and submarines they are used these days for various purposes like platforms for industrial use like offshore oil exploration, civic amenities like floating air strips, defense bases and in some island nations for recreation and habitation. Hence, strength and stability of these platforms are very important. In the present study, the fluid-structure interaction effect on the free vibration frequency of a floating platform is investigated. The fluid is treated as inviscid, incompressible and having a small amplitude irrotational motion. The platform is considered to be of rectangular in size and is made of homogeneous material following Hooke's law. Finite element technique is adopted for the solution of this problem with eight noded brick elements for the fluid and four noded quadrilateral elements for the plate. The interaction problem of the platform and the fluid is solved independently for the platform and the fluid domain by transferring the pressure to the platform from the fluid and the acceleration of the platform to the fluid sequentially. The effects of the surface wave, thickness and aspect ratio of the platform on free vibration frequencies are studied. It is observed that surface wave reduces the free vibration frequencies significantly for frequencies those are due to the bulging modes of the platform.

Key words: *- Inviscid, Irrotational, Hydro elasticity, Eigen frequencies, Eigen value.*

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1. INTRODUCTION

Today floating platforms are used for various purposes like oil exploration, naval bases, air strips, rescue bases, amusement parks and settlement colonies etc. A lot of research has been carried out. Earlier works were mainly based on the motion of rigid plates due to wave action. Later investigations were focused on the flexibility of large floating plates. Thus, hydro elastic analysis took centre stage in the analysis of mat-like floating structures. Breakthrough works by Bishop and Price [1] and Price and Wu [2] led to the full 3-D hydroelasticity theory, where the Green function method is used to model the fluid. The plate is modeled as an elastic thin plate with free edges. The fluid is incompressible, inviscid and its motion is irrotational so that a velocity potential exists. The amplitude of the incident wave and the motions of the VLFS are both small and only the vertical motion of the structure is considered. Eigenfrequencies of a plate either immersed or in contact with a fluid decrease significantly compared to those in vacuum, especially for the fundamental one. This is because the vibration of the plate is transferred to the fluid causing an increase in the kinetic energy of the surrounding fluid. In the early phase of work on this area, Lamb [3] studied the change in natural frequencies of a thin clamped circular plate in an aperture of an infinitely long plane rigid wall in contact with water. Later, Powell and Roberts [4] experimentally verified the work of Lamb and Mclachlan [5] extended Lamb's work to circular plates without any supports. Kwak and Kim [6] and later Kwak [7] obtained the non-dimensional added virtual mass incremental (NAVMI) factors for circular plates placed on a free liquid surface using Hankel's transformation for axisymmetric modes and all other modes. All these works are based on the assumption that mode shapes of the plate remain same both in contact with the fluid and in vacuum. Kwak [8] investigated the effect of water on mode shapes and observed that except the fundamental mode, other modes are influenced by the presence of water. The distortion of mode shapes increases with the increase in mode numbers. The plate was considered to be thin and made of isotropic, homogeneous and linearly elastic material. Kirchhoff's theory for plate vibration and analytical-Ritz method for fluid-structure interaction were adopted for the analysis. The effect of the free surface was studied in some detail by Amabili [9] in which Vibrations of circular plates resting on a sloshing liquid free surface are studied. The fully coupled problem between sloshing modes of the free surface and bulging modes of the plate is solved by using the Rayleigh–Ritz method. The sloshing boundary condition is directly inserted into the eigenvalue problem. The theory is suitable for all axisymmetric plate boundary conditions. The effect of free surface waves on the plate natural frequencies is significant when the fundamental bulging mode of the plate has its natural frequency close to those of the first sloshing modes of the free surface. The natural frequencies and mode shapes for different system parameters are given. Vibration analysis of rectangular plates coupled with fluid was investigated Kerboua [10] developed a mathematical modeling of rectangular plates coupled with fluids which is representative of certain key components of complex structures used in industries such as aerospace, nuclear and naval. The plates can be totally submerged in fluid or floating on its free surface. The mathematical model for the structure is developed using a combination of the finite element method and Sanders' shell theory. The in-plane and out-of-plane displacement components are modelled using bilinear polynomials and exponential functions, respectively. Variation of fluid level is considered in the calculation of the natural frequencies. For the solution of interaction problems, though analytical methods provide better accuracy, their use is limited to either very special or simple cases because of the mathematical complexities involved. However, due to the availability of high speed computational facilities, several numerical techniques may be adopted to obtain a meaningful solution of such complex problems. Among different numerical techniques used, finite element method (FEM) is mostly preferred due to its easy implementation in a wide range of problems. The focus of the present work is to investigate the effect of the flexibility of the plate and fluid depth on eigen frequencies of the plate floating over the fluid considering the effect of the surface wave.

2. THE MODEL OF INVESTIGATION

A rectangular platform of width $2a$ and length $2b$ is floating on a reservoir of infinite extent as shown in Fig. 1. The fluid is considered to be incompressible and inviscid with small amplitude motion. The effect of the static pressure is not considered in the analysis. It is assumed that the reservoir floor is horizontal and rigid.

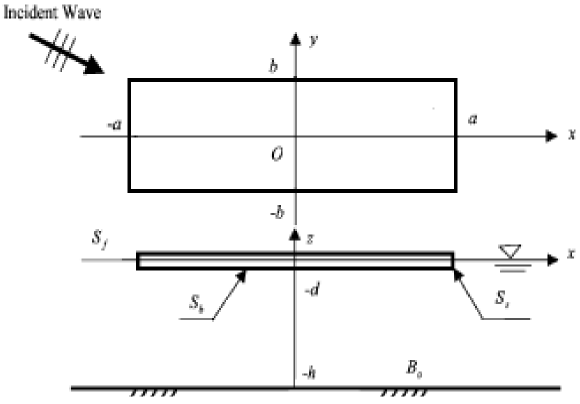


Figure1: Pontoon type of VLFS under wave action

The plate is uniformly thick and its material is homogeneous, isotropic and linearly elastic in nature. In the analysis, the 3-D fluid domain is discretized considering eight noded brick elements and the gate using four noded quadrilateral plate elements. Mindlin's plate bending theory is used to analyze the plate.

3. THEORETICAL FORMULATION

For inviscid, incompressible fluid with a very small magnitude of irrotational fluid flow, the dynamic pressure in the fluid is given as

$$\nabla^2 p(x, y, z) = 0 \qquad (1)$$

∇^2 is the Laplacian operator and p is the dynamic pressure at a point at any instant of time over and above the static pressure. For the problem under consideration of a floating plate (Fig 1) structure, the following boundary conditions are adopted.

3.1 Boundary Conditions

For the problem under consideration of a plate structure floating on a fluid domain, the following boundary conditions are adopted.

The fluid-structure interface (S_{fs}):
The pressure gradient at the fluid-structure interface at any instant of time is related to the acceleration a of the interface as

$$\frac{\partial p}{\partial n} = -\rho_f a \qquad (2)$$

The top free surface (S_f):
The actual free surface is nonlinear in nature. However, a linear approximation of the free surface may be made without much error when the amplitude of the surface wave is relatively small in comparison to the depth.

The free surface boundary condition may be represented as

$$\frac{\partial p}{\partial n} = \frac{\partial p}{\partial z} = -\frac{1}{g}\ddot{p} \qquad (3)$$

The four side surfaces (S_s):
The plate structure vibrates along with the fluid without any cavitations. For small displacement of the plate, it is assumed that pressure waves are perpendicular to the undisturbed position of the plate. It is assumed that there is no pressure change across the side surfaces of the fluid domain considered. So, the boundary conditions are

$$\frac{\partial p}{\partial n}(x, 0, z) = 0 \qquad (4)$$

$$\frac{\partial p}{\partial n}(x,b,z)=0 \qquad (5)$$

$$\frac{\partial p}{\partial n}(0,y,z)=0 \qquad (6)$$

$$\frac{\partial p}{\partial n}(a,y,z)=0 \qquad (7)$$

The bottom surface (S_b):
It is assumed that there is no pressure change across the bottom surface of the fluid domain considered. The boundary condition is represented as

$$\frac{\partial p}{\partial n}(x,y,0)=0 \qquad (8)$$

4. FINITE ELEMENT FORMULATION FOR THE FLUID DOMAIN

The fluid domain is discretized (Fig. 1) as an assemblage of finite elements assuming pressure to be the nodal unknown. The pressure at any point inside an element may be represented as

$$p=\sum_{i=1}^{n} N_i\,\bar{p}_i \qquad (9)$$

N_i, $\bar{p}_i$ are the interpolation functions and nodal pressure values corresponding to node i respectively and n is the number of nodes in the element. Using Galerkin's weighted residual method, the weighted average integral of eqn. 1 may be represented for the whole fluid domain as

$$\sum\int_{\Omega_e} N^T\left(\nabla^2 p\right)d\Omega_e=0 \qquad (10)$$

Here, $\sum$ and Ω_e refer to the summation of all the elements and one element respectively. The weak form of eqn. 10 may be written as

$$\sum\left[\int_{\Omega_e}\left(\nabla N^T.\nabla p\right)d\Omega_e-\int_{\Gamma_e}\left(N^T\frac{\partial p}{\partial n}\right)d\Gamma_e\right]=0 \qquad (11)$$

Γ_e indicates the total boundary surface of the element.
In concise matrix form, this may be represented as

$$[G]\{\bar{p}\}=\{B\} \qquad (12)$$

in which,

$$[G]=\sum\int_{\Omega_e}\left(\nabla N^T.\nabla N\right)d\Omega_e \qquad (13)$$

$$\{B\}=\sum_{s}\int_{\Gamma_e}N^T\frac{\partial p}{\partial n}d\Gamma_e \qquad (14)$$

$\sum_s$ refers to the total boundary surface of the fluid domain.
$\{B\}$ is separated into its components

$$\{B\}=\{B_f\}+\{B_{fs}\}+\{B_b\}+\{B_s\} \qquad (15)$$

At the free surface, since $\frac{\partial p}{\partial n}=-\frac{1}{g}\ddot{p}$

$$\{B_f\}=-\sum_{S_f}\left[\int_{\Gamma_e}\left(N^T N\right)d\Gamma_e\right]\frac{\{\ddot{\bar{p}}\}}{g} \qquad (16)$$

At the fluid structure interface, since $\frac{\partial p}{\partial n}=-\rho_f a$

$$\{B_{fs}\}=-\rho_f\left[R_{fs}\right]\{\bar{a}\} \qquad (17)$$

where,

$$\left[R_{fs}\right]=\sum_{S_{fs}}\int_{\Gamma_e}N^T N_s\,d\Gamma_e \qquad (18)$$

N_s is the shape function of the structure corresponding to nodes of the fluid-structure interface and $\{\bar{a}\}$ is the vector of nodal accelerations.

At the bottom surface of the fluid domain, as the normal pressure gradient vanishes

$$\{B_b\}=0 \qquad (19)$$

At the four side faces of the fluid domain, since $\frac{\partial p}{\partial n}=0$

$$\{B_s\}=0 \tag{20}$$

With the values of the components of $\{B\}$, eqn. 12 may be modified as

$$[E]\{\ddot{\bar{p}}\}+[G]\{\bar{p}\}=-\rho_f\left[R_{fs}\right]\{\bar{a}\} \tag{21}$$

Where,

$$[E]=\frac{1}{g}\sum_{S_f}\left[\int_{\Gamma_e}\left(N^T N\right)d\Gamma_e\right] \tag{22}$$

5. COUPLED MOTION OF THE FLUID AND THE STRUCTURE

Replacing $\{\bar{a}\}$ with $\{-\ddot{\bar{X}}\}$ in eqn. (21) may be written as

$$[E]\{\ddot{\bar{p}}\}+[G]\{\bar{p}\}=\rho_f\left[R_{fs}\right]\{\ddot{\bar{X}}\} \tag{23}$$

The structural free vibration may be represented as

$$[M]\{\ddot{\bar{X}}\}+[K]\{\bar{X}\}=-\left[R_{fs}\right]^T\{\bar{p}\} \tag{24}$$

From eqn. (24)

$$\{\ddot{\bar{X}}\}=-[M]^{-1}\left\{\left[R_{fs}\right]^T\{\bar{p}\}+[K]\{\bar{X}\}\right\} \tag{25}$$

Replacing $\{\ddot{\bar{X}}\}$ in eqn. 23 with that obtained from eqn. 25 and then writing eqns. with rearrangement of terms may be written in matrix form as

$$\begin{bmatrix}[K] & 0\\ 0 & \frac{[E_1]}{\rho_f}\end{bmatrix}\begin{Bmatrix}\{\ddot{\bar{X}}\}\\ \{\ddot{\bar{p}}\}\end{Bmatrix}+$$

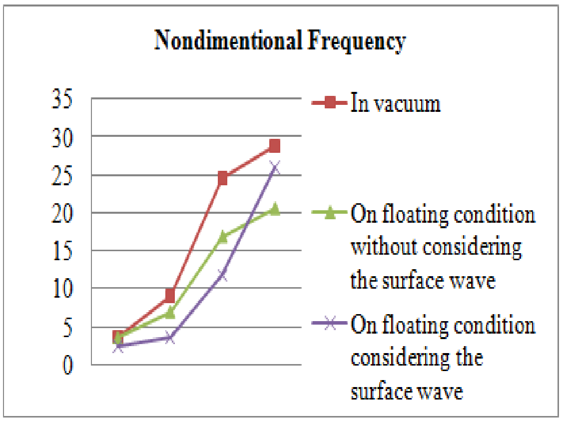
$$\begin{bmatrix}[K][M]^{-1}[K] & [K][M]^{-1}\left[R_{fs}\right]^T\\ \left[R_{fs}\right][M]^{-1}[K] & \frac{[G_1]}{\rho_f}+\left[R_{fs}\right][M]^{-1}\left[R_{fs}\right]^T\end{bmatrix}\begin{Bmatrix}\{\bar{X}\}\\ \{\bar{p}\}\end{Bmatrix}$$

$$=\begin{Bmatrix}\{0\}\\ \{0\}\end{Bmatrix} \tag{26}$$

Eqn. 26 is of the same form as the free vibration equation for structure without damping. This may be used for evaluating the free vibration frequencies of the gate structure and the free surface wave frequencies of the fluid domain.

Table – 1

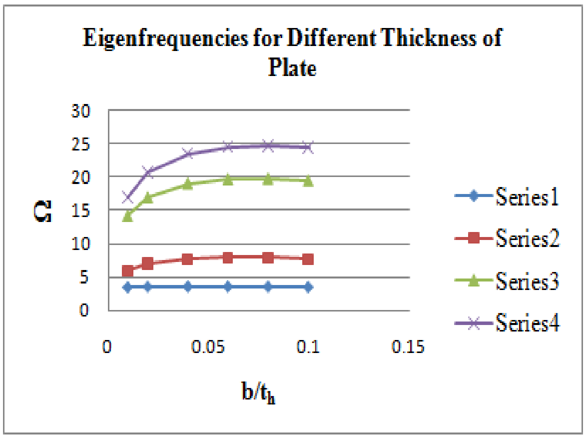
Non dimensional frequency	Ω_1	Ω_2	Ω_3	Ω_4
In vacuum	3.70	9.12	24.60	28.83
On floating condition without considering the surface wave	3.65	7.04	16.95	20.65
On floating condition considering the surface wave	2.41	3.55	11.79	26.04



Eigenfrequencies of a plate of 1.0 m x 1.0 m x 0.02 m floating over water of depth 1.0 m

Table – 2

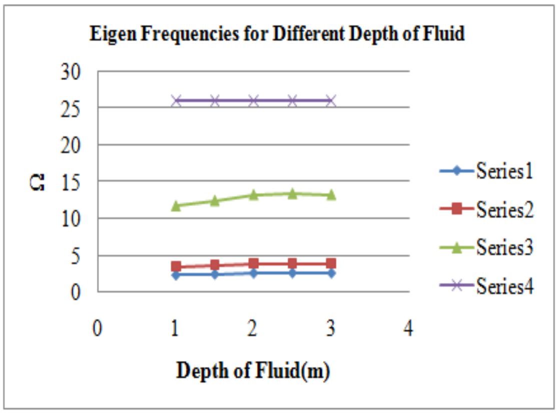
b/th	Ω_1	Ω_2	Ω_3	Ω_4
0.01	3.57	5.95	14.13	16.96
0.02	3.65	7.04	16.95	20.65
0.04	3.67	7.77	19.05	23.55
0.06	3.67	7.93	19.71	24.50
0.08	3.65	7.90	19.77	24.68
0.10	3.60	7.78	19.49	24.46



Eigenfrequencies of a plate of 1.0 m x 1.0 m for different thicknesses floating over water of depth 1.0m

Table – 3

Depth of fluid In metre	Ω_1	Ω_2	Ω_3	Ω_4
1.0	2.41	3.55	11.79	26.04
1.5	2.53	3.71	12.51	26.07
2.0	2.68	3.89	13.31	26.07
2.5	2.70	3.93	13.45	26.07
3.0	2.68	3.88	13.31	26.07



Eigenfrequencies of a plate of 1.0 m x 1.0 m x 0.02m floating over water for different depth of the fluid.

CONCLUSIONS

The changes in eigenfrequencies due to the presence of the fluid are studied considering the effect of the surface wave of the reservoir. It is observed that consideration of the effect of the surface wave decreases the frequencies further. The effect of the flexural rigidity of the platform is investigated by observing the reduction for different thicknesses of the platform. As observed from Table - 2, reduction of the eigenfrequencies decreases with increase in flexibility of the plate. Depth of the fluid domain plays an important role with eigenfrequencies increasing with the depth of the fluid as shown in Table – 3.

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