A SIMPLIFIED METHOD OF DESIGNING A PHASE-LEAD COMPENSATOR TO IMPROVE THE M-S-D SYSTEM'S PHASE MARGIN

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Abstract

Compensators are used to alter the response of a control system in order to accommodate the set design criteria. This is done by introducing additional poles and zeros to the system. Improving the mass-spring-damper phase margin is an essential so as to improve its performance. The paper aims to describe short steps to design a phase-lead compensator for the mass-spring-damper system to achieve the desired level of phase margin for system.

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1. INTRODUCTION

The compensator is an additional device or component used to improve the system performance. Phase-lead compensator alters the transient response of the system. Phase-lag compensator alters the steady state characteristics of the response. The cascade compensator is usually used in series with the plant to achieve the required phase margin of the system [1]. Figure (1) shows the block diagram of a cascaded compensator with the plant. The cascade compensator's transfer function is denoted as Gc(s), and the plant as G(s). Input and output are denoted as R(s) and C(s) respectively.

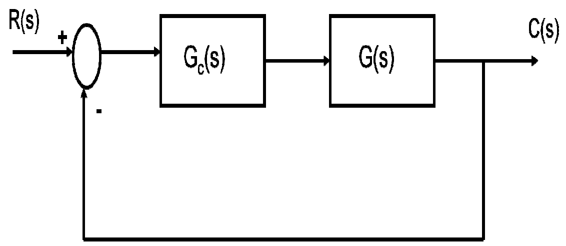

Figure (1) Cascade compensator in the system

The open loop transfer function of the system is given in equation (1). Equation (2) computes the overall closed loop system transfer function [2].

$$\text{Open-loop} = Gc(s).G(s) \qquad (1)$$

$$\text{Closed-loop}=\frac{Gc(s).G(s)}{1+Gc(s).G(s)} \qquad (2)$$

The plant G(s) used in the paper was the mass-spring-damper system. The problem could be simplified in how the phase margin of the system will be increased to improve the system performance.

2. M-S-D TRANSFER FUNCTION

A typical arrangement of the mass-spring-damper system (ignoring the friction force) is shown in figure (2). To derive the transfer function, the differential equation was written using Newton's law as[3]:

$$M\frac{dx^2}{dt^2}+b\frac{dx}{dt}+kx = F \qquad (3)$$

Where M= Mass
b = damping constant
k = spring stiffness constant
x = displacement

Typical values used for M, b, and k are:

M= 10 kg, b= 11 n.s/m and k = 1 n/m.

With a force (F) applied to the mass, the transfer function (G(s)) of the system taking the force as input and the displacement x as output will be:

$$G(s) = \frac{100}{10S^2+11S+1} \qquad (4)$$

This a second order system with a gain of 100.

The phase margin of the system could be found using the Bode plot[4]. A phase margin (PM) of about 450 is suitable for the system to have a stable performance.

The design problem is how to improve the phase margin of the system?

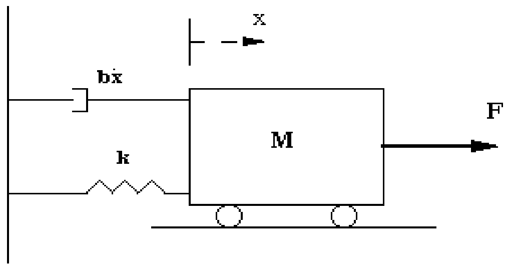

Figure (2) Mass-spring-damper system

3. COMPENSATOR DESIGN STEPS

At first the current phase margin of the system (without compensation) was investigated using the Bode plot. Matlab command (margin) was used for checking the current phase margin. This is shown in figure (2).

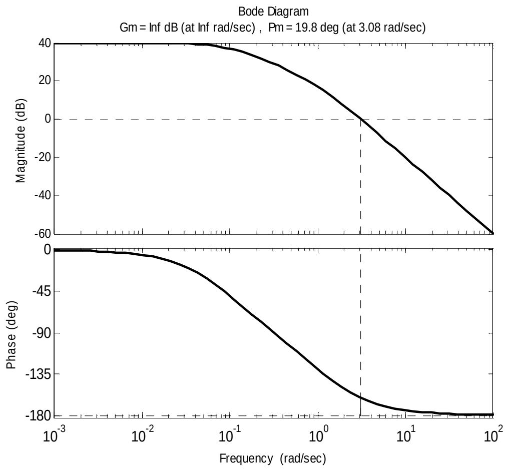

Figure (2) Phase margin of the system before compensation

Figure (2) shows that the phase margin of the mass-spring-damper system equal to 19.8 degree ≈ 200.

The design aims to increase the phase margin to 450. The next step done was to calculate the phase margin difference between the current phase and the desired phase margin namely 450 - 200 = 250.

A phase margin of 50 is added as a safety margin 5]. The phase margin to be compensated will be 300. The compensator standard form of the phase- lead compensator takes the form shown in equation (5) as shown below [6].

$$\frac{1+\propto TS}{1+Ts} \qquad \propto > 1 \qquad (5)$$

Where α = p/z , the ratio of pole to zero.

Note that if $\alpha < 1$ the general form of the compensator represents a phase-lag compensator.

Values of α and T must be found to complete the phase-lead compensator design.

To find α value the sine rule[7] was used as shown in equation (6)

$$\sin(PM) = \frac{\alpha-1}{\alpha+1}. \qquad (6)$$

Thus $\sin(3) = \frac{\alpha-1}{\alpha+1}$ from which $\alpha = 3$. Now the transfer function takes the form shown in equation (7).

$$\frac{1+3TS}{1+Ts} \qquad (7)$$

The next step is to find the value of T. We have the magnitude of the compensator at maximum phase shift equal to -20*Log (α) dB [6]. Thus

$$-20*\mathrm{Log}(3) = 9.54 \text{ dB} \qquad (8)$$

The max magnitude of the phase Lead occurs half way between poles and zeros frequencies therefore[7]:

$$\omega m = \frac{1}{T\sqrt{\alpha}} \qquad (9)$$

ωm meets the Half of the magnitude of the max phase-lead shift.

The half magnitude = (0.5)(9.54) = -4.77 dB . From Bode plot this value meets ωm = 4 rad/s .substituting this value in equation (9) yields:

$$T = \frac{1}{4\sqrt{3}} = 0.14 \qquad (10)$$

Substituting values of α and T in the standard form of the compensator shown in equation (5) yields the transfer function of the required phase-lead compensator as shown in equation (11).

$$Gc(s) = \frac{1+0.42S}{1+0.14S} \qquad (11)$$

Referring to figure (1) this compensator is inserted in cascade before the M-S-D transfer function.

The loop transfer function as explained in equation (1) is found as:

$$G(s).Gc(s) = \frac{42S+100}{1.4S^3+11.54S^2+11.14S+1} \qquad (12)$$

The Bode plot of the system's loop transfer was found as shown in figure (3).

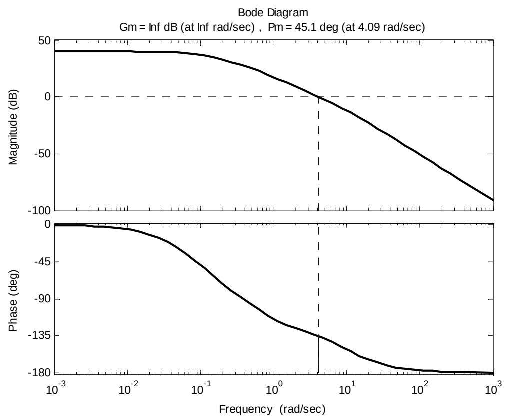

Figure (3): The 450 required PM of the system

Note that the required phase margin was obtained.

4. RESULT DISCUSSION

The steps followed in the design obtained exactly the required phase margin of M-S-D system. Matlab command (margin) was very useful in the design procedure. The steps taken in the design procedure are summarized as:

Step 1: Check the frequency response of the system before starting the compensation procedure and note the phase margin.

Step 2: Find the difference between the system phase margin and the required phase margin.

Step 3: Add five degree to the difference as safety margin and find the required phase margin to be compensated.

Step 4: Use the sine law to find α then find the half of the maximum magnitude from -20*Log α and find the corresponding frequency ωm and use equation (9) to find T.

Step 5: Construct the compensator, insert it in cascade with the system, and check the desired phase margin of the system' loop.

The step response of the compensated system can also be plot to study the transient response characteristics. The compensator has added a new pole to the two poles of the M-S-D system. The three poles were found by using the Matlab command (pole).

P1 = -7.1429
P2 = -1.0000
P3 = -0.1000.

Note that since the first pole (-7.1429) is located to the left of the other two poles, the characteristics of the transient response will get better.

5. CONCLUSIONS

In this paper the transfer function of the mass-spring-damper system was modelled and its phase margin was checked. The phase margin was probably narrow. A lead compensator was designed and added to the system to increase its phase margin to 450. After the compensator was designed and inserted in cascade with the system, the desired phase margin was obtained. PID controller may designed for the system to improve its closed loop characteristics.

6. REFERENCES

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