DETERMINATION OF THE EQUIVALENT ELASTIC COEFFICIENTS OF THE COMPOSITE MATERIALS BY THE VIBRATORY ANALYSIS

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Abstract

We must first find a behaving law of a material, before using it in the structure. The establishment of this law in the case of a composite material requires the knowledge of his elastic equivalent coefficient. In this work, we will present the vibratory technic with which we can extract the equivalent elastic coefficients of some composite materials. The relationships we have obtained through this analysis permits to evaluate the equivalent elastic coefficients of composite materials as a function of their self throb. Those relationships are first validated with the determination of mechanical characteristics of conventional materials. After that, equivalent elastic coefficients of some composite materials are evaluated. Simulated results we have obtained are discussed in comparison of Voigt and Reuss boundaries.

Keywords: *Composite materials, Vibration, Self throb, Young's Modulus, Finite Elements Method.*

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1. INTRODUCTION

Composite materials are used in many industrial areas automobile, aeronautic, building, medicine between many others. Their mechanical performances depend on the used charge and matrix, but also on the quality of the interface between the components [1, 2, 3]. An important axis in the research on the composite materials concerns the evaluation of their behaving law that is the relationship which links stress and strain. In this work, we propose a rapid technical characterization of composite materials from the vibratory analysis. This technique is based on the determination of relationships between mechanical properties of materials and their vibratory characteristics from the Lagrange motion equations and the analytical methods of the transversal vibrations of structures [4]. In other to validate those relationships, we will calculate at once, known mechanical characteristics of some conventional materials. After, we will evaluate numerically elastic properties of some stratified composite materials. The discussion on the obtained results will be done according to Reuss [5] and Voigt [6] boundaries results.

2. THEORETICAL MODEL

The aim of this work is to determine elastic coefficients of sandwich composite materials [7] having the morphology of those of the following figure 1:

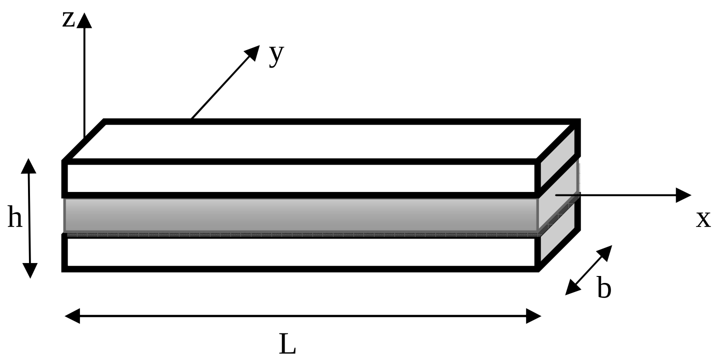

Figure 1: Stratified heterogeneous composite material

Usually when composite materials are used in structures, they are supposed homogeneous. For this reason one can represent them by the following scheme of figure 2:

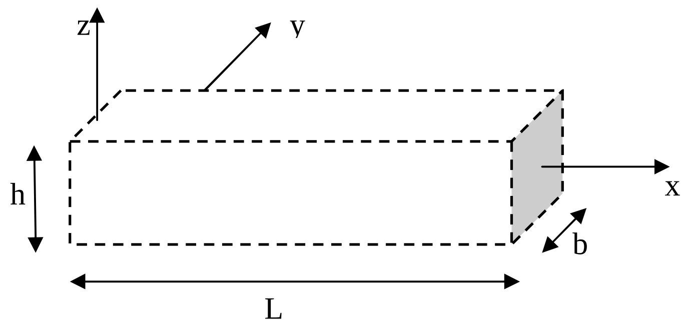

Figure 2: Homogeneous composite material

The structure represented on the figure 2 is coming from a heterogeneous structure, and we are trying to determine its homogeneous elastic coefficients. To solve this problem, we have a lot of methods in literature. The most used of these methods are self-consistent method and homogenization ones [7]. In this work none of them will not be used. We will try to link mechanical properties of a structure to its vibratory properties. Materials of this work are supposed elastics, so we can model them by a spring-mass system which the free motion vibration can be put in the following shape

$$[M]\{\ddot{x}\}+[K]\{x\}=\{0\} \tag{1}$$

Solving these equations passes through the determination of the vibration self throb [8, 9, 10] with the help of the following relationship:

$$\omega^2=\frac{[K]}{[M]} \tag{2}$$

The studied structures will be considered as being beam free-built as represented in the following figure 3:

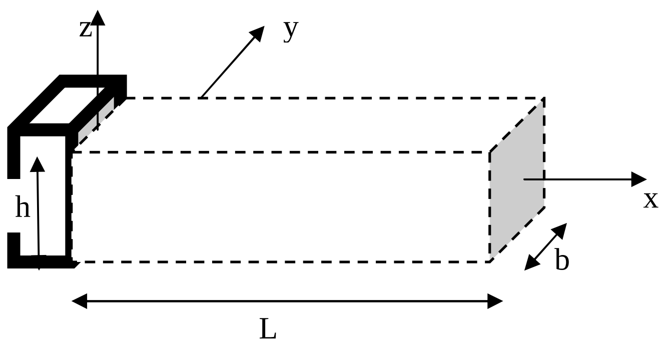

Figure 3: Plane thick beam built in at x=0

Using analytical methods, we can obtain vibration equations of these beams through the technical of Green's resolvante [4] which permits to have the following equation:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}=-\rho A\frac{\partial^2 w(x,t)}{\partial t^2} \tag{3}$$

Solving these equations brings to divide it in two others equations. One depends on time, the other on the space. Solving space equation will take into account the building of boarding condition at x=0 and permit to calculate the λ parameter:

$$\lambda=(2n+1)\frac{\pi}{2} \tag{4}$$

This analytical parameter can be linked to the numerical throb through the relationship:

$$E=\frac{\rho AL^4}{I}\left(\frac{\omega_n^2}{\lambda_n^4}\right) \tag{5}$$

In this relation:

ρ : is the density;
A : the beam area;
L : the length beam;
I : the inertial momentum of the beam according to the rotation axis;
ω : The self throb of the system;
λ : a grandeur which is obtained by the application of the beam bounded limits.

It's that relationship which permits to evaluate elastic equivalent characteristic coefficients of composite materials from their self throbs obtained numerically.

3. SIMULATIONS AND DISCUSSION

Before applying the relation (5) to the numerically conceived composite materials, we will first use this relationship to determine mechanical characteristics of some conventional materials in order to validate it.

3.1 Determination of mechanical characteristics of some conventional materials

3.1.1. Mechanical characteristics of Stell Inox

Young's modulus E of the stell Inox is given in the table 1:

Table1. Young's modulus of Stell Inox

Mode	Frequency (Hz)	E numerical (GPa)	E experimental (GPa) [6]	Error %
1	1248,0	208,26	203	2,6

3.1.2. Mechanical characteristics of Iron

Mechanical characteristic of iron is given in the table 2:

Table2. Young's modulus of Iron

Mode	Frequency (Hz)	E numerical (GPa)	E experimental (GPa) [6]	Error %
1	1271,4	216,14	210	2,92

3.1.3. Mechanical characteristics of Glass

For this material data are summarized in table 3:

Table3. Young's module of Glass

Mode	Frequency (Hz)	E numerical (GPa)	E experimental (GPa) [9]	Error (%)
1	1318,8	76,833	74,000	3,82

According to results in tables 1, 2 and 3, we have found values of elastic modulus of some conventional material for the first vibration mode with an error less than 4%. So we have a good coherence between numerical and experimental results. Hence we hope that we can use the relationship (5) to determine with acceptable accuracy equivalent elastics coefficients of some composite materials. Mechanical characteristics of single conventional materials are from reference [11] and are given in table 4.

Table 4: Mechanical characteristics of some conventional materials

Material	Elastic modulus (GPa)	Poisson's coefficient	Density kg/m3	Elastic limit (MPa)
Steel inox	203	0,28	7850	200,00
Iron	210	0,28	7850	200,00
Steel 45 SCD 6	220	0,28	7850	1450,00
Glass	74	0,25	2600	60,00
Epoxy	4,500	0,4	1200	70,00
Aluminium alloy	70	0,34	2700	30,00

3.2. Evaluation of mechanical characteristics of some composite materials

We have worked with four composite materials:

3.2.1. Composite material n°1

We have considered a thick stratified beam composed of: Alloy aluminium/Iron/Alloy aluminium, with the following millimeter dimensions: 100*[(0,3+1,2+1,5)*10]*10; (Length*thickness*width).

3.2.1.1. Elastic modulus

Different values of the elastic modulus are given in table 5

Table 5: Values of elastic modulus for some models

Composite 1	Reuss method	Vibratory method (frequency: 1233.6 Hz)	Voigt method
Young's modulus E (GPa)	116,67	142,224	154,00

3.2.2. Composite material n°2

It concerns a thick sandwich beam: Alloy aluminium/Stell 45 SCD 6/Alloy d'aluminium, with the following millimeter dimensions: 100*[(0.3+1.2+1.5)*10]*10; (Length*thickness*width).

3.2.2.1. Elastic modulus

The values of elastic modulus are presented in the following table 6:

Table6. Values of elastic modulus for some models

Composite 2	Reuss method	Vibratory method (frequency: 1240.8 Hz)	Voigt method
Young's modulus E (GPa)	118,67	143,88	160,00

3.2.3. Composite material n°3

This time we have a thick sandwich beam based on Glass/Inox Stell/Glass, with the following mm dimensions: 100*[(0,3+1,2+1,5)*10]*10; (Length*thickness*width).

3.2.3.1. Elastic modulus

The elastic modulus values are indicated in table 7:

Table7. Values of elastic modulus for some models

Composite 3	Reuss method	Vibratory method (frequency : 1236,9 Hz)	Voigt method
Young's modulus E (GPa)	119,60	142,22	151,40

3.2.4. Composite material n°4

At last we have considerate a thick sandwich beam composed with: (Epoxy/Glass/Epoxy), with the following mm dimensions: 100*[(0,3+1,2+1,5)*10]*10; (Length*thickness*width).

3.2.4.1 Elastic modulus

The obtained values of elastic modulus are reported in table 8:

Table8: Values of elastic modulus for some models

Composite 4	Reuss method	Vibratory method (frequency : 1186,2 Hz)	Voigt metho d
Young's modulus E (GPa)	10,670	45,90	46,20

The results obtained show that, Young's modulus are between the Voigt and Reuss limits as predicted by the authors of references [7, 12], what makes plausible our numerical results. However, some verifications are needed to definitely validate our results, notably according to the orientation of shell in the matrix and their volume fraction. Also for good validation our resultants must also be compared the experimental ones. In this work we supposed that our composite materials are not damped. But in reality we have to take into account the damping in some real composite materials. So we will perform our modeling.

CONCLUSIONS

The aim of this work was to contribute to the identification of mechanical properties of some materials by the vibratory analysis. We have modeled by the finite elements method through the virtual works theorem, a relation which permits to extract the Bernouilli model in order to realize our measurements. We have verified some hypothesis like which states that the Young's modulus must always stays between the Voigt and Reuss boundaries. From our numerically results, we can conclude that, the use of both analytical grandeurs melt with the self throb can lead us to evaluation of elastic equivalents coefficients of some materials with weak error.

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