APPLICATION OF STOCHASTIC MODELING IN GEOMAGNETISM

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Abstract

The objective of the present study is the identification of the characteristics of monthly Sq variations for geomagnetic components D, H and Z for a time series at four Indian geomagnetic observatories, namely, Alibag (ALB), Hyderabad (HYD), Pondicherry (PON) and Visakhapatnam (VIZ). To study the behavior of Sq monthly variation, a two state Markov chain model is employed. With the help of Markov transition probabilities, one can guess the behavior of a time series data.

Index Terms: *Geomagnetic field – Geomagnetic Sq variation– Stochastic process – Markov chain model – Bernoulli trials*

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1. INTRODUCTION

Significant contribution to research in geomagnetism started from India as back as in 19th century with the pioneering work of Brown and Chambers and Moos. The geographical location of India plays a pivotal role with the latitudinal coverage existing from equator to the focus of the low latitude Sq current system.

Variations in the natural magnetic field are measured at the Earth's surface and elsewhere in the Earth's magnetosphere (for example, at the geostationary orbit). These are field changes with periodicities from about 0.3 second to hundreds of years. (These boundaries are set to distinguish geomagnetic variations from the quasipermanent field and higher - frequency waves). Many of these observed variations from-very short periods (seconds, minutes, hours) to daily, seasonal, semiannual, solar-cycle (11-years), and secular (60–80 years) periods - arise from sources that either are external to the Earth (but superposed upon the larger, mainly dipolar field) or internal to the Earth (the magnetic-dipole and higher - harmonic trends and variations on the scales of hundreds and even thousands of years). The daily and seasonal motions of the atmosphere at ionospheric altitudes cause field variations that are smooth in form and relatively predictable, given the time and location of the observation. During occasions of high solar–terrestrial disturbance activity that give rise to aurorae (northern and southern lights) at high latitudes, very large geomagnetic variations occur that even mask the quiet daily changes. These geomagnetic variations are so spectacular in size and global extent that they have been named geomagnetic storms and sub storms, with the latter generally limited to the Polar Regions.

Solar-terrestrial-physics associated studies were mainly utilizing long series of geomagnetic field observations at the Indian Observatories and also worldwide network of geomagnetic data. The Geomagnetic Observatory data were also used for studies on Interplanetary Magnetic Field (IMF) associations, Solar flare effects etc. The continuously recorded data from the Institute gives an opportunity to decipher the long-term secular changes as well as the daily variations of the magnetic components that is basically the reflection of the ionospheric and magnetospheric changes occurring over the region. Thus, the variations in the geomagnetic field can be used as a diagnostic tool for understanding the internal structure of the Earth as well as the dynamics of the upper atmosphere and magnetosphere.

A stochastic process is the mathematical abstraction of an empirical process whose development is governed by probabilistic laws. Markov chain is a class of stochastic process which has got certain applications. Feller (1968) applied stochastic processes in several situations. Lawless (1982) established Markov and statistical models for life time applications. Anderson (1976) used Box-Jenkins approach to study stochastic models. Chatfield (1977) developed some stochastic models for forecasting, such as, queuing models, renewal process, etc. Recently researchers developed switching models to analyze the behavior of time series. Milkovitch (1977) compared semi-Markov and Markov models in forecasting. Colin (1968) estimated Markov transition probabilities for certain data.

With the help of Markov transition probabilities, one can guess the behavior of a time series data. Kaplan (1975) has studied the ergodicity of a Markov chain.

2. OBJECTIVE:

The objective of the present study is the identification of the characteristics of monthly Sq variations for geomagnetic components D, H and Z for a time series at four Indian geomagnetic observatories, namely, Alibag (ALB), Hyderabad (HYD), Pondicherry (PON) and Visakhapatnam (VIZ).

2.1 DATA USED:

This research work is based on the data of Indian geomagnetic observatories only. Data for monthly variations of the geomagnetic components D, H and Z, from January 1995 to December 1997, for Alibag, Hyderabad, Pondicherry and Visakhapatnam observatories have been obtained from the volumes of Indian Magnetic data.

The monthly Sq variations for geomagnetic components D, H and Z for a time series of 36 months from January 1995 to December 1997 at four Indian geomagnetic observatories, namely, Alibag (ALB), Hyderabad (HYD), Pondicherry (PON) and Visakhapatnam (VIZ) is considered.

3. APPLICATION OF STOCHASTIC MODELING TO GEOMAGNETIC Sq VARIATION:

A TWO STATE MARKOV CHAIN MODEL:

A Markov chain is considered which has two states namely 0 and 1.

3.1 BERNOULLI TRIAL:

A Bernoulli trial is an experiment with only two possible outcomes namely success and failure which are denoted respectively by S and F. Example of such an experiment is testing the quality of a finished product and determining whether it is defective (F) or non – defective (S). One may denote S and F respectively by 1, 0.[5]. The probability distribution of the random variable in this case according to Bernoulli trial is provided below.

3.2 PROBABILITY DISTRIBUTION

Random Variable x	0	1	
Probability p(x)	1-p	p	Total = 1

Bernoulli trial involves two states. One of the states is considered as success and denoted by 1. The other state is considered as failure and denoted by 0.

3.3 DEPENDENT BERNOULLI TRIALS:

In the dependent Bernoulli trials, the probability of success or failure at each trial depends on the outcome of the previous trial.

3.3.1 TRANSITION PROBABILITIES FOR DEPENDENT BERNOULLI TRIALS:

Consider dependent Bernoulli trials. If the nth trial results in failure then the probability of failure at the (n+1)th trial is taken as (1- α) while the probability of success at the (n+1)th trial is assumed to be α. Similarly if the result in nth trial is success, then the probabilities of success and failure at the (n+1)th trial are taken as (1-ß) and ß respectively. i.e., if the system is in state 0 at time n, then the probability of being in state 0 at time (n+1) is taken as (1- α) and the probability of being in state 1 at time (n+1) is taken as α. Similarly if the system is in state 1 at time n, then the probability of being in state 1 at time (n+1) is taken as (1- ß) and the probability of being in state 0 at time (n+1) is taken as ß. These probabilities are called transition probabilities.

They can be written in the form of a matrix as follows.

(n+1)th trial

nth trial state 0 state 1

$$P = \begin{matrix} \text{state 0} \\ \text{state 1} \end{matrix} \begin{pmatrix} (1-\alpha) & \alpha \\ ß & (1-ß) \end{pmatrix} \qquad (1)$$

The matrix provided by equation (1) is called the matrix of transition probabilities. The element in (i,j)th position of the matrix denotes the conditional probability of a transition state j at time (n+1), given that the system was in state i at time n.

n – STEP TRANSITION PROBABILITIES:

It is assumed that the initial probabilities for the system to be in state 0 or 1 are given by the row vector p(0) = (p0(0) , p1(1)). Let the row vector p(n) = (p0(n) , p1(n)) denote the probabilities for the system to be in state 0 or 1 at time n. The latter is referred to as the vector of the nth step transition probabilities.

RECURRENCE RELATION

A relation of the form

f(n+1) = k f(n)

For n= 1, 2, 3……, where k is a constant is called recurrence relation. This relation provides the link between f(n+1) and f(n). Using this relation, one can find the value of 'f' at the stage (n+1) by means of the value of 'f' at the stage 'n'.

Consider the event of the system being in state 0 at time n. This event can occur in two mutually exclusive ways: either state 0 was occupied at time (n-1) and no transition out of state 0 occurred at time n. The probability for this to happen is p(n-1) (1- α); alternatively state 1 was occupied at time (n-1) and a transition from state 1 to state 0 occurred at time n; this has the probability p(n-1) ß. These possibilities can be represented in the form of a recurrence relation as follows:

$$p0(n) = p0(n-1)(1-\alpha) + p1(n-1)\beta$$

$$p1(n) = p0(n-1)\alpha + p1(n-1)(1-\beta) \qquad (2)$$

The equation (2) determines a dynamical system. This system is discrete in nature. The equation (2) can be rewritten in the form of a matrix equation as follows:

$$\begin{pmatrix} p0(n) \\ p1(n) \end{pmatrix} = \begin{pmatrix} 1-\alpha & \beta \\ \alpha & 1-\beta \end{pmatrix} \begin{pmatrix} p0(n-1) \\ p1(n-1) \end{pmatrix} \qquad (3)$$

Equation (3) provides an expression for the matrix P(n) in terms of the matrices P and P(n-1) . By referring to the equations (1) and (3), it is seen that

$$P(n)=PP(n-1) \qquad (4)$$

Equation (4) shows how P(n) can be defined recursively.
On iteration, one gets

$$P(n-1) = PP(n-2)$$

From this, it follows that

$$P(n) = PP(n-2) = P2\,P(n-2)$$

By successive application of the process of iteration, P(n) can be expressed in terms of P(0) as follows.

$$P(n) = P(n)\,P(0) \qquad (5)$$

STATE OCCUPATION PROBABILITIES:

Given the initial probability matrix P(0) and the matrix of transition probability P, one can find the state occupation probabilities at any time n using the equation (5). Denote the (i,j)th element of P(n) by Pij(n) . The following two cases arise.

CASE (i): If the system is initially in state 0, then one has

p(0) = { 1,0}

And , p(n) = { p00(n), p01(n) }

CASE (ii): If the system is initially in state 1, then one has

p(0) = { 0, 1 } and p(n) = { p10(n), p11(n) }

i.e., pij(n) = probability { state j at time n / state i at time 0 }
The quantities pij(n) provide the n – step transition probabilities.

CHARACTERISTIC ROOTS OF THE MATRIX P:

Let I denote the identity matrix of order 2. The characteristic polynomial of the matrix P is the determinant of the matrix P-λI . On simplification, this polynomial is obtained as

$$\lambda 2 + (\alpha + \beta - 2)\lambda + 1 - \alpha - \beta$$

The characteristic equation of the matrix P is

$$\lambda 2 + (\alpha + \beta - 2)\lambda + 1 - \alpha - \beta = 0 \qquad (6)$$

The roots of the equation (6) are called the characteristic roots of the matrix P. The following two cases have to be considered.

CASE (i): $\alpha + \beta = 0$
In this case, the equation (6) reduces to

$\lambda 2 - 2\lambda + 1 = 0$

i.e., $(\lambda - 1)2 = 0$

Thus, the characteristic root in this case is 1, with a multiplicity of two.

CASE (ii): $\alpha + \beta \neq 0$
In this case, the roots of the equation (6) are given by

$$\lambda = \frac{(2-\alpha-\beta) \pm \sqrt{(2-\alpha-\beta)2 - 4(1-\alpha-\beta)}}{2} \qquad (7)$$

The expression within the radical sign reduces to $(\alpha + \beta)2$. With the positive sign in equation (7), one obtains λ = 1. Taking the negative sign in equation (7), λ is obtained as $(1 - \alpha - \beta)$. Thus, the characteristic roots of the matrix P are obtained as

$$\lambda 1 = 1$$
$$\lambda 2 = (1 - \alpha - \beta)$$

Since, $\alpha + \beta \neq 0$, it follows that $\lambda 1 \neq \lambda 2$
Thus in this case there are two distinct characteristic roots of P.

THEOREM (Paria, 1992):

When an 2 X 2 matrix ‘a’, has distinct characteristic roots λ1 and λ2, there exists an invertible 2 X 2 matrix ‘b’ such that

$$a = b \begin{pmatrix} \lambda 1 & 0 \\ 0 & \lambda 2 \end{pmatrix} b\text{-}1 \qquad (8)$$

EXPRESSION FOR P^n:

Consider a matrix P with the property α + ß ≠ 0. By the above theorem, there exists a matrix Q such that

$$P = Q \begin{pmatrix} \lambda 1 & 0 \\ 0 & \lambda 2 \end{pmatrix} Q\text{-}1 \quad (9)$$

One obtains

$$Q = \begin{pmatrix} 1 & \alpha \\ 1 & -\beta \end{pmatrix} \quad (10)$$

Its inverse is obtained as

$$Q\text{-}1 = \frac{1}{\alpha+\beta} \begin{pmatrix} \beta & \alpha \\ 1 & -1 \end{pmatrix} \quad (11)$$

From this, it follows that

$$P = Q \begin{pmatrix} 1 & 0 \\ 0 & (1-\alpha-\beta) \end{pmatrix} Q\text{-}1 \quad (12)$$

The quantities α and β being probabilities satisfy the inequalities

$$0 \le \alpha \le 1 \text{ and } 0 \le \beta \le 1$$

So one obtains

$$0 \le \alpha + \beta \le 2$$

Consequently one has

$$|\lambda 2| = |1 - \alpha - \beta| = |1 - (\alpha + \beta)| < 1$$

Thus if follows that

$$|\lambda 2| < 1 \quad (13)$$

Hence one obtains

$$Pn = \frac{1}{\alpha+\beta} \begin{pmatrix} 1 & \alpha \\ 1 & -\beta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & (1-\alpha-\beta)^n \end{pmatrix} \begin{pmatrix} \beta & \alpha \\ 1 & -1 \end{pmatrix} \quad (14)$$

$$= \frac{1}{\alpha+\beta} \begin{pmatrix} \beta & \alpha \\ \beta & \alpha \end{pmatrix} + \frac{(1-\alpha-\beta)^n}{\alpha+\beta} \begin{pmatrix} \alpha & -\alpha \\ -\beta & \beta \end{pmatrix}$$

(15)

With any initial probability vector P(o) one can use equation (5) and (12) to find P(n). The first term in equation (15) namely,

$$\begin{pmatrix} \frac{\beta}{\alpha+\beta} & \frac{\alpha}{\alpha+\beta} \\ \frac{\beta}{\alpha+\beta} & \frac{\alpha}{\alpha+\beta} \end{pmatrix}$$

As a constant and the second term in equation (15) is

$$\begin{pmatrix} \frac{\alpha}{\alpha+\beta}(1-\alpha-\beta)^n & \frac{-\alpha}{\alpha+\beta}(1-\alpha-\beta)^n \\ \frac{-\beta}{\alpha+\beta}(1-\alpha-\beta)^n & \frac{\beta}{\alpha+\beta}(1-\alpha-\beta)^n \end{pmatrix}$$

Because of the inequality (13), as n → ∝, it is observed that

$$(1-\alpha-\beta)^n \to 0$$

Therefore the second term in equation (15) tends to zero. Consequently, if follows that

$$Pn \longrightarrow \begin{pmatrix} \frac{\beta}{\alpha+\beta} & \frac{\alpha}{\alpha+\beta} \\ \frac{\beta}{\alpha+\beta} & \frac{\alpha}{\alpha+\beta} \end{pmatrix} \quad \text{as } n \to \propto \quad (16)$$

Denote $\frac{\beta}{\alpha+\beta}$ by ∏0 and $\frac{\alpha}{\alpha+\beta}$ by ∏1 . Then it is seen that

$$Pn \xrightarrow[n \to \propto]{} \begin{pmatrix} \prod 0 & \prod 1 \\ \prod 0 & \prod 1 \end{pmatrix} \quad (17)$$

In view of this fact, one has

$$\lim_{n\to\infty} P_n = \begin{pmatrix} \Pi_0 & \Pi_1 \\ \Pi_0 & \Pi_1 \end{pmatrix} P_0 \quad (18)$$

As a consequence of proceeding discussion, one is led to the following result. Let { xt } be a given time series data, following Bernoulli trails.

Let P be the 2 X 2 matrix of transition probability associated with the series { xt }.
Suppose

$$P = \begin{pmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{pmatrix}$$

with $\alpha + \beta \neq 0$ and $|1 - (\alpha + \beta)| < 1$

Then,

$$\lim_{n\to\infty} P_n = \begin{pmatrix} \Pi_0 & \Pi_1 \\ \Pi_0 & \Pi_1 \end{pmatrix} P(0)$$

where,

$$\Pi_0 = \frac{\beta}{\alpha+\beta} \text{ and } \Pi_1 = \frac{\alpha}{\alpha+\beta}$$

MARKOV CHAIN MODEL FOR Sq MONTHLY VARIATION DATA:

To study the behaviour of Sq monthly variation data, a two state Markov chain model is to be employed. It is assumed that the probability of the reading, in a particular day increasing or decreasing from the previous day depends on the condition of previous days. The model has two conditional probabilities as its parameters described below.

α = prob (of being in state 1 today/previous day being in state 0)

β = prob (of being in state 0 today/previous day being in state 1)

No other factor is taken into account to explain the occurrence or non– occurrence of the change in the Sq monthly variation data as described above.[9].

TRENDS IN A TIME SERIES:

There are two types of trends in a time series: positive trend and negative trend. Positive trend means that the time series is increasing, whereas negative trend implies that the time series is decreasing. In a time series the number of periods in which it is increasing or decreasing compared to the previous day is taken into account.

The transition matrix for the occurrence of an increasing or a decreasing trend in a time series data is given by:

$$A = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 1-\alpha & \alpha \\ 1 & \beta & 1-\beta \end{array}$$

The n - step transition probabilities are given by the elements of the matrix An where

$$A^n = \frac{1}{\alpha+\beta}\begin{pmatrix} \beta & \alpha \\ \beta & \alpha \end{pmatrix} + \frac{(1-\alpha-\beta)^n}{\alpha+\beta}\begin{pmatrix} \alpha & -\alpha \\ -\beta & \beta \end{pmatrix}$$

Substitute

$$\Pi_0 = \frac{\beta}{\alpha+\beta} \text{ and } \Pi_1 = \frac{\alpha}{\alpha+\beta}$$

Then one has

$$\lim_{n\to\infty} A^n = \begin{pmatrix} \Pi_0 & \Pi_1 \\ \Pi_0 & \Pi_1 \end{pmatrix}$$

APPLICATION OF STOCHASTIC MODELING TO MONTHLY VARIATIONS OF THE COMPONENTS D, H AND Z FOR THE YEARS 1995, 1996 AND 1997:

The question of transition probalility matrices for the components D, H and Z at the four places ALB, HYD, PON and VIZ is now taken up.

ASSUMPTION IN MODEL BUILDING:

While constructing a model, certain reasonable assumptions have to be made. Some important aspects of the real life situation have to be identified and incorporated in the model. As regards to the data, during certain months, there is neither an increase nor a decrease, compared with the previous month. The number of such months for the 3 components is noted below.

FOR COMPONENT D:

Monthly Variations	ALB	HYD	PON	VIZ
No:of Months	3	1	4	2

FOR COMPONENT H:

Monthly Variations	ALB	HYD	PON	VIZ
No:of Months	3	0	1	2

FOR COMPONENT Z:

Monthly Variations	ALB	HYD	PON	VIZ
No:of Months	0	2	2	4

Since this number is very less for each monthly variation data, it is reasonable to restrict the attention to only those months for which either positive signs or negative signs are noticed. Thus a two state Markov model is assumed for the the monthly Sq variations data.

VERIFICATION OF BERNOULLI TRIALS:

It has been verified that the Markov chains arising from ALB, HYD, PON and VIZ series for the three components D, H and Z follow Bernoulli trials. Because of the fulfillment of this condition, one may go ahead with the construction of the transition probalility matrix for each component in each place. These matrices are determined in the sequel.

CALCULATION OF TRANSITION PROBABILITY MATRIX FOR COMPONENT D :

The number of positive and negative signs are counted for the four places ALB, HYD, PON and VIZ for the component D.
In the case of Alibag(ALB),

	Today t		Total
	-	+	
Previous day t-1	5	7	12
	6	10	16
Total	11	17	28

From this table, the transition probability matrix for ALB series is obtained as

$$P = \begin{pmatrix} 5/12 & 7/12 \\ 6/16 & 10/16 \end{pmatrix}$$

In this case, $\alpha = 7/12$, $ß = 6/16$ and hence $\alpha + ß \neq 0$. So there are two distinct characteristic roots for the matrix P. It is noticed that

$$|\lambda 2| = |1 - (\alpha + \beta)|$$

$$= 0.042 < 1$$

Hence the condition $|\lambda 2| < 1$ is fulfilled.

Preceding the same way, the transition probability matrix for HYD, PON and VIZ series is obtained as follows.

The transition probability matrix for HYD series:

$$P = \begin{pmatrix} 5/14 & 9/14 \\ 10/17 & 7/17 \end{pmatrix}$$

The transition probability matrix for PON series:

$$P = \begin{pmatrix} 4/10 & 6/10 \\ 6/16 & 10/16 \end{pmatrix}$$

The transition probability matrix for VIZ series:

$$P = \begin{pmatrix} 6/15 & 9/15 \\ 8/15 & 7/15 \end{pmatrix}$$

In all the cases, it is observed that $\alpha + ß \neq 0$ and the condition $|\lambda 2| < 1$ is fulfilled.

CALCULATION OF TRANSITION PROBABILITY MATRIX FOR COMPONENT H:

The number of positive and negative signs is counted for the four places ALB, HYD, PON and VIZ for the component H and the results are presented as follows.

The transition probability matrix for ALB series is

$$P = \begin{pmatrix} 8/15 & 7/15 \\ 7/14 & 7/14 \end{pmatrix}$$

The transition probability matrix for HYD series is

$$P = \begin{pmatrix} 8/16 & 8/16 \\ 10/16 & 6/16 \end{pmatrix}$$

The transition probability matrix for PON series is

$$P = \begin{pmatrix} 10/18 & 8/18 \\ 8/14 & 6/14 \end{pmatrix}$$

The transition probability matrix for VIZ series is

$$P = \begin{pmatrix} 9/17 & 8/17 \\ 7/13 & 6/13 \end{pmatrix}$$

In all the cases, it is observed that $\alpha + ß \neq 0$ and the condition $|\lambda 2| < 1$ is fulfilled.

CALCULATION OF TRANSITION PROBABILITY MATRIX FOR COMPONENT Z:

The numbers of positive and negative signs are counted at the four places ALB, HYD, PON and VIZ for the component Z and the results are presented as follows.

The transition probability matrix for ALB series is

$$P = \begin{pmatrix} 10/20 & 10/20 \\ 9/14 & 5/14 \end{pmatrix}$$

The transition probability matrix for HYD series is

$$P = \begin{pmatrix} 5/13 & 8/13 \\ 8/16 & 8/16 \end{pmatrix}$$

The transition probability matrix for PON series is

$$P = \begin{pmatrix} 10/16 & 6/16 \\ 8/14 & 6/14 \end{pmatrix}$$

The transition probability matrix for VIZ series is

$$P = \begin{pmatrix} 6/13 & 7/13 \\ 8/14 & 6/14 \end{pmatrix}$$

In all the cases, it is observed that $\alpha + ß \neq 0$ and the condition $|\lambda 2| < 1$ is fulfilled.

DETERMINATION OF TRENDS:

Let Pij denote the probability of transition from state i to state j after 'n' months. For any positive integer n, the matrix Pn = (Pij)n of the n – step transition probability can be obtained. The limiting matrix Pn as n→∝ indicates that the probability of finding an increasing trend is $\frac{\alpha}{\alpha + ß}$ and of noticing a decreasing trend is $\frac{ß}{\alpha + ß}$. The values of α and ß for each place are calculated for the 3 components D,H and Z. Using them, the probabilities for increasing and decreasing trends are determined. These results are provided in the following table.

LIMITING PROBABILITIES FOR THE 4 PLACES ALB, HYD, PON AND VIZ:

CASE (i): FOR COMPONENT D:TABLE 3:

PLACE	α	ß	PROB. INCRE-ASING TREND	FOR DECRE-ASING TREND
ALB	0.583	0.375	0.6	0.4
HYD	0.643	0.588	0.5	0.5
PON	0.6	0.375	0.6	0.4
VIZ	0.6	0.533	0.5	0.5

CASE **(ii): FOR COMPONENT H:TABLE 4:**

PLACE	α	ß	PROB. INCRE-ASING TREND	FOR DECRE-ASING TREND
ALB	0.467	0.5	0.5	0.5
HYD	0.5	0.625	0.4	0.6
PON	0.444	0.571	0.4	0.6
IZ	0.471	0.538	0.5	0.5

CASE(iii): FOR COMPONENT Z:TABLE 5:

PLACE	α	ß	PROB. INCRE-ASING TREND	FOR DECRE-ASING TREND
ALB	0.5	0.643	0.4	0.6
HYD	0.615	0.5	0.6	0.4
PON	0.375	0.571	0.4	0.6
VIZ	0.538	0.571	0.5	0.5

RESULT AND ANALYSIS

From case (i) it is seen that the monthly variations for component D, at the places HYD and VIZ possess equal chance to have increasing trend or decreasing trend in the short run. In the case of ALB and PON, these probabilities differ. For ALB the difference in probabilities is 0.2, which is the same as the case of PON. From case (ii), it is observed that the monthly variations for component H, at the places ALB and VIZ possess equal chance to have increasing trend or decreasing trend in the short run. In the case of HYD and PON these probabilities differ. The difference in probabilities in the 2 places is 0.2. From case (iii), it is noticed that the monthly variation for component Z at VIZ possess equal chance to have increasing trend or decreasing trend in the short run, whereas in the case of ALB, HYD and PON, the difference in probabilities comes to 0.2.

CONCLUSION

The daily variation in the magnetic field at the Earth's surface during geomagnetic quiet periods (Sq) is known to be associated with the dynamo currents driven by winds and tidal motions in the E-region of the ionosphere known as atmospheric dynamo. Besides the atmospheric dynamo, other sources of electric field and currents at equatorial region contribute to Sq variations on different components of geomagnetic field observed at the ground level. Daily range of the geomagnetic field is an important parameter measuring the magnitude of diurnal variation. Being dependent on the daily maximum and minimum field values, the parameter fluctuates from day to day in accordance with the variability of both these values. A continuous recording of any of the components of the geomagnetic field typically exhibits two types of variations: a smooth, regular variation, known as Sq, the solar quiet day variation and a rapid irregular fluctuation. The Sq variations are the most regular of all the geomagnetic field variations. Here geomagnetic quiet day (Sq) variations have been analyzed through the application of Graph Theoretic Modeling.

The application of Stochastic Modeling for pattern recognition and classification have been used for numerous applications in astronomy, meteorology, cartography, satellite data analysis, artificial intelligence etc., Here it is used to study the identical pattern of geomagnetic variations at Indian observatories. Generally, for a huge volume of data in a complicated analysis this technique yields accurate results. As a result of this study, it is expected that future usage of this technique may be appropriate for exploring some new results in geomagnetism.

It is concluded that there seems to be good ground for expecting a two state Markov chain model to describe the increasing or decreasing trend in the series of monthly Sq variations at the four observatories, in a short run.

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