EXPLICIT MODEL PREDICTIVE CONTROL OF FAST DYNAMIC SYSTEM

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Abstract

Explicit Model Predictive Control approach provides offline computation of the optimization law by Multi Parametric Quadratic Programming. The solution is Piece wise affine in nature. It is explicit representation of the system states and control inputs. Such law then can be solved using binary search tree and can be evaluated for fast dynamic systems. Implementing such controllers can be done on microcontroller or ASIC/FPGA. DC Motor Speed Control - one of the benchmark systems is discussed here in this context. Its PWA law obtained, simulation of closed loop e-MPC is presented and its implementation approach using MPT toolbox and other such toolboxes is shown in brief.

Index Terms: *Model Predictive Control, explicit, Piece-wise Affine, and Multi Parametric Toolbox*

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1. INTRODUCTION

Model Predictive Control is Optimal Control Strategy based on the dynamic system model. Since its advent it has become popular in industries for constrained control of the systems. Anti wind-up using PID can at the most provide constraints on the control input. Its advantage over conventional PID control schemes is its constraints handling capacity for control inputs as well as system states.

In Model Predictive Control at each sampling time, starting at the current state, an open-loop optimal control problem is solved over a finite horizon. At the next time step, the computation is repeated starting from the new state and over a shifted horizon, leading to a moving horizon policy. The solution relies on a linear dynamic model, respects all input and output constraints, and optimizes a quadratic performance index. Thus, as much as a quadratic performance index together with various constraints can be used to express true performance objectives, the performance of MPC is excellent.

The big drawback of the MPC is the relatively difficult on-line computational effort, which limits its applicability to relatively slow and/or small problems.

Chemical Processes and other processes which have longer time constants in the system and can be considered as slow dynamic systems are thus implemented using Model Predictive Control. Consider the discrete-time linear time invariant system given by

$$x(t+1) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

While fulfilling the constraints,

$$y_{min} \le y(t) \le y_{max} \;,\; u_{min} \le u(t) \le u_{max} \tag{3}$$

For such system optimization problem is,

$$U \triangleq \min_{\{u_t,\ldots\ldots\ldots u_{t+N_{u-1}}\}} \Big\{ J(U, x(t)) = \; x'{t+Ny|t}Px{t+Ny|t}+ \; k{=}0Ny{-}1[x'{t+k|t}Qx{t+k|t}+u'{t+k}Ru{t+k}] \tag{4}$$

Subject to,

$$y_{min} \le y_{t+k|t} \le y_{max} \;, k = 1, \ldots Nc\big(Nc \le N_y - 1\big)$$

$$u_{min} \le u_{t+k} \le u_{max} \;, k = 0,1, \ldots Nc,$$

$$x_{t|t} = x(t),$$

$$x_{t+k+1|t} = Ax_{t+k|t} + Bu_{t+k} \;, k \ge 0,$$

$$y_{t+k|t} = Cx_{t+k|t} \;, k \ge 0,$$

$$u_{t+k} = Kx_{t+k|t} \;, N_u \le k \le N_y$$

Where, Ny, Nu and Nc are Output, Input and Constrain Horizons respectively and K is some feedback gain. [1]

2. EXPLICIT MPC [1]

The idea of explicit MPC is to solve the optimization problem (4) off-line for all (*t*) within a given set *X*.

Placing $x_{t+k/t} = A^k x(t) + \sum_{j=0}^{k-1} A^j B u_{t+k-1-j}$
in (4) it becomes

$$V\big(x(t)\big) = \frac{1}{2}x'(t)Yx(t) + \min_U \frac{1}{2}U'HU + x'(t)FU \tag{5}$$

$subj.to\ GU \leq W + Ex(t)$
Defining Auxiliary variable,

$$z \triangleq U + H^{-1}F'x(t) \tag{6}$$

Gives optimization problem for variable z

$$V_z(x) = \min_z \frac{1}{2}z'Hz \tag{7}$$

$$subj.to\ Gz \leq W + Sx(t)\ where\ S \triangleq E + GH^{-1}F'$$

And to make the dependence of $u(t)$ on $x(t)$ *explicit*, rather than *implicitly* defined by the optimization procedure that solves problem (4). It turns out that such a dependence is piecewise affine in most of the formulations so that the MPC controller defined by (4) can be represented in a totally equivalent way as

$$u(x) = \begin{cases} F_1x + g_1\ if\ H_1x \leq k_1 \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ F_Mx + g_M\ if\ H_Mx \leq k_M \end{cases} \tag{8}$$

Consequently, on-line computations are reduced to the simple evaluation of (8), which broadens the scope of applicability of MPC to fast-sampling applications.

The explicit MPC solution (8) pre-computed off-line is a lookup table of linear feedback gains. The right gain is selected on-line by finding the region $\{x: H_i\ x \leq ki\}$ of the polyhedral partition where the current state x (t) (or, more generally, the current vector of parameters) lies.

There are various methods adopted to find the polyhedral region like Binary Search Tree Algorithm, Logarithmic Solution and Active Set Partition. [2]

3. E-MPC ON FAST DYNAMICS SYSTEM

Explicit Model Predictive Control to be implemented on Benchmark Control Problem of DC Motor Speed Control.

System Model of a Permanent Magnet DC Motor with Speed Controlled by Armature Voltage is obtained through Mathematical Model using Parameters given by Manufacturer Data Sheet. Here Typical Motor Data of standard Portescap Motor are used to obtain the PWA model of the motor for the specified constraints on Matlab with the help of Multi Parametric Toolbox.

For this system, input is the voltage source (V) applied to the motor's armature, while the output is the rotational speed of the shaft dθ/dt. The physical parameters of the Motor used are:

Rotor Moment of inertia (J)	71.4e-7
kg.m^2Motor viscous friction constant (b)	1.5e-6 N.m.s
Electromotive force constant (Ke)	0.0254 V/rad/sec
Motor torque constant (Kt)	0.0254 Nm/Amp
Electric resistance (R)	0.85 Ohm
Electric inductance (L)	0.1e-3 Henry

The motor torque is proportional to the armature current i by a constant factor Kt as shown in the equation below. This is referred to as an armature-controlled motor.

$$\mathrm{T} = \mathrm{K_t i} \tag{9}$$

The back emf, e, is proportional to the angular velocity of the shaft by a constant factor Ke.

$$\mathrm{e} = \mathrm{K_e}\dot{\theta} \tag{10}$$

In SI units, the motor torque and back emf constants are equal, that is, Kt = Ke; therefore K is used to represent both the motor torque constant and the back emf constant.

$$\mathrm{J}\ddot{\theta} + \mathrm{b}\dot{\theta} = \mathrm{Ki} \tag{11}$$

$$\mathrm{L}\frac{\mathrm{di}}{\mathrm{dt}} + \mathrm{Ri} = \mathrm{V} - \mathrm{K}\dot{\theta} \tag{12}$$

In state-space form, the governing equations above can be expressed by choosing the rotational speed and electric current as the state variables. Again the armature voltage is treated as the input and the rotational speed is chosen as the output.

$$\frac{\mathrm{d}}{\mathrm{dt}}\begin{bmatrix}\dot{\theta}\\ \mathrm{i}\end{bmatrix} = \begin{bmatrix}-\mathrm{b/J} & \mathrm{K/J}\\ -\mathrm{K/L} & -\mathrm{R/L}\end{bmatrix}\begin{bmatrix}\dot{\theta}\\ \mathrm{i}\end{bmatrix} + \begin{bmatrix}0\\ 1/\mathrm{L}\end{bmatrix}\mathrm{V} \tag{13}$$

$$\mathrm{y} = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\dot{\theta}\\ \mathrm{i}\end{bmatrix} \tag{14}$$

Along with the state space model, typical constraints for States Speed and Armature Current and Control Input Armature Voltage are specified.

Speed: °θmin = 0 rad/sec and °θmax = 700 rad/sec
Armature Current: imin =0 and imax =1.0 amp.
Armature Voltage: Vmin =0 Vmax = 18Volts

4. CONTROL LAW COMPUTATION AND SIMULATION:

Given a reasonably accurate discrete time state space system model, sampling time, and constraints on the system Piece wise affine law of the system is computed. Multi Parametric Toolbox (MPT)[2] developed by M.Kvasnika et.al is Matlab Toolbox which provides efficient computational means to obtain feedback controllers for constrained optimal control problems. It's a free toolbox with GNU license. Computation of control law is done with the help of various multi parametric programming solvers, which need to be installed along with this toolbox. By multi-parametric programming, a linear or quadratic optimization problem is solved off-line. The associated solution takes the form of a PWA state feedback law.

To derive the explicit Model Predictive Control law for the DC motor Speed Control, its discrete time State Space Model is loaded on the GUI Interface of the toolbox, or it can be exported from previously saved Matlab file or from workspace. Sampling time for the system is selected as per the open loop response obtained from the DC Motor Speed control. Constraints on control input armature voltage, and states speed and armature current are specified. Toolbox validates the model then other parameters like Finite Horizon solution, Prediction Horizon, Control Objective of Regulation to output Reference are specified. Controller for Fixed Reference value of 400 rad/sec and Current of 0.5 Amp is given. With these data the controller computed.

As shown in Fig.1 the whole state space range of both states divided in partitioned space called regions or more generally are called polytopic partitions. Here for 28 such regions are developed. It is 2-dimension hyper-rectangular plane which contains values of states distributed over 28 regions.

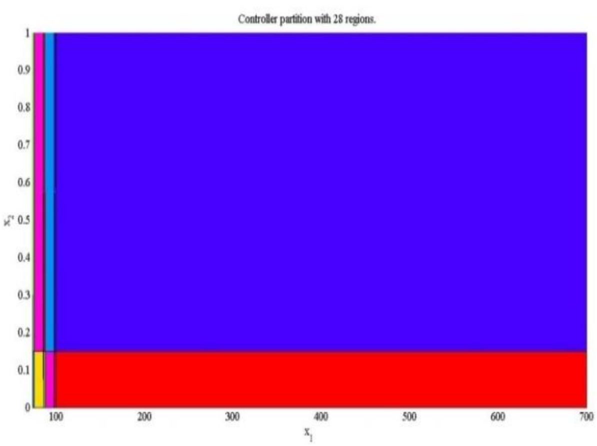

Fig.1: Polytopic Partitions of the system states

Fig.2 shows the control law for DC Motor speed control. Here along with states, control input armature voltage is shown. It is clear from the plot that for increasing speed the armature voltage is increasing.

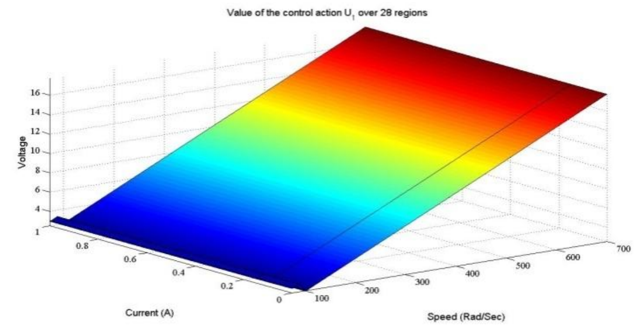

Fig.2: Piecewise Affine Law of System.

For different values of speed and current the region is specific. Once this control law is computed for any system offline, only its region search and output evaluation is required. It thus removes the limitation of conventional model predictive control to compute the control law on line at each sampling instant. Here on the generated law toolbox also specifies whether closed loop system is stable or not.

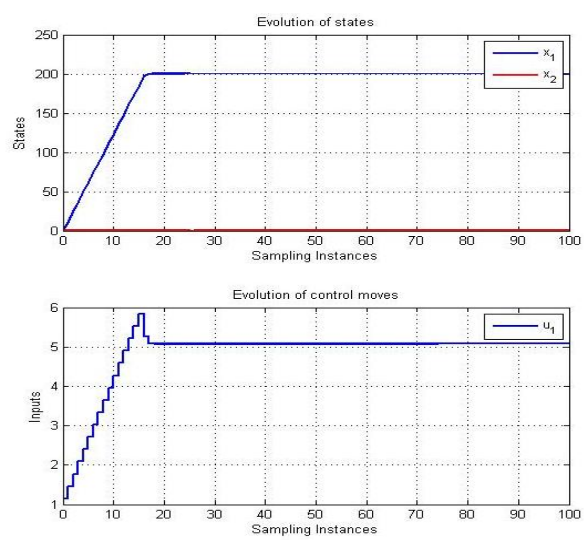

Fig.3: States and Control Input for Speed of 200 rad/sec

It is checked by evaluating it for two different reference points - (1) 200 rad/sec and (2) 400 rad/sec motor speed. The initial condition is assumed [0,0]. The simulation results obtained are shown in the figures 3 and 4 respectively.

As can be seen from the simulation results obtained for different speed values, the control law takes more sampling instance to reach the higher reference point. Also the armature voltage is increased in linear fashion to meet the desired speed.

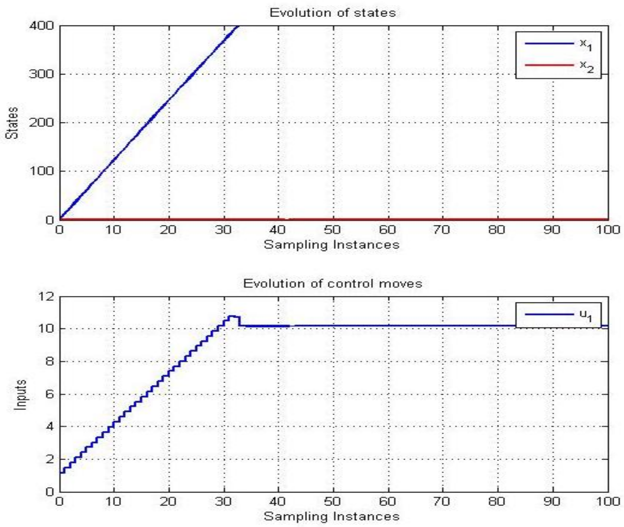

Fig.4: States and Control Input for Speed of 400 rad/sec

5. IMPLEMENTATION OF EXPLICIT MPC:

Hardware implementation of the Explicit Model Predictive Control using Multi Parametric Toolbox can be done by generating its C-Code. The generated C-Code contains the Polytopic partitions. At each sampling instant the states are measured and are compared with the each set of the polytopic partition and the correct region is searched and for that region required control input is given to the plant.

The reduced online computation makes the Explicit Model predictive Control application possible for fast dynamic systems. Survey on Explicit Model Predictive Control and its applications done by Alessandro Alessio and Alberto Bemporad[4] in 2009 shows it can be implemented for the systems with 1-50 ms sampling period systems, 1-2 manipulated inputs.

6. FURTHER SCOPE:

Implementation of the e-MPC on hardwares like controllers can be further extended to the Field Programmable Gate Array (FPGA) Modules and Application Specific Integrated Chip (ASIC) with the help of co-generation tools like MPT toolbox.

Recently a toolbox named Moby-DIC[5] developed by Alberto Oliveri et.al provides such co-generation facility to implement the explicit model predictive control on FPGA and ASIC Hardwares. Thus Explicit Model Predictive control is gradually increasing its applicability in power electronics and other fast dynamic systems.

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